DESIGN AND VALIDATION OF A TOOL FOR THE ANALYSIS OF WHOLE GROUP DISCUSSIONS IN THE MATHEMATICS CLASSROOM

Laura Morera, Núria Planas & Josep M. Fortuny

Universitat Autònoma de Barcelona, Spain

We present an analytical tool to characterise whole group discussions in mathematics classrooms. Following an exploratory study with 8th grade students, our interest lies in identifying and characterising interactional episodes and certain actions that support them. It has been confirmed (Morera & Fortuny, 2012) that the tool helps to detect what we call Mathematical Learning Opportunities –MLO. It shows not only what participants are doing in the course of their interaction, but also which the potential learning contents are. From a methodological point of view, the tool appears to operate in different didactical settings. We provide an example of its use in a teaching and learning setting of mathematical work mediated by a DGS instrument.

INTRODUCTION

Learning can be viewed as one of the ultimate reasons behind the actions of the students and the teacher in the mathematics classroom. In the context of whole group discussion, it is important to identify the various actions that have a role in framing the students' learning. Morera and Fortuny (2012) have argued that the identification of these actions and its development in whole group may serve to reach evidence of learning. While existing research has placed much emphasis on the presentation of practices that teachers can learn in order to improve the didactical effects of classroom conversations (Stein & Smith, 2011), developing systematic research on the potential of whole group discussions is a relatively new endeavour. Further investigation is needed to carry out on the nature of such discussions from an analytical perspective.

The goal of our research is to create and apply an analytical tool to identify and characterise: a) episodes of whole group discussion involving mathematical work, b) actions by the participants in these episodes, and c) opportunities in terms of potential contributions to the students' mathematical learning. This triple examination of episodes, actions and opportunities leads to a more global characterisation of whole group discussions in the mathematics classroom, under the assumption that whole group promotes episodes of mathematical learning (Krummheuer, 2011). We are aware that the underlying notion of Mathematical Learning Opportunities –MLO– is itself critical in that one can imagine almost any situation to be a potential scenario for learning. What is scientifically useful for us is to view the sets of learning opportunities as an initial step toward the search for evidences of effective mathematical learning on the part of the students. This report provides an overview of the process that has been followed to create the tool, including examples with classroom data, and a summary of the current refinement being developed around methodological considerations.

Context of the research

For the research, ten class sessions of an experienced grade 8 teacher were chosen. A sequence of five inquiry problems was designed to study isometries in a collaborative way using DGS. After the students had solved a problem in pairs, the teacher led a 50-minute whole group discussion. DGS was involved in the students working in pairs and the whole group interaction. During the whole group discussions, the teacher and the students were observed and video-taped with three video cameras and several additional voice recorders to capture all actions in detail. The attention was focused on the interactions between the participants and the software as well as on all different series of significant actions produced by both the teacher and the students.

After data collection, qualitative data analysis software was used to organise and codify the whole group videos. The three authors worked together on carrying out the codification. The codification of videos was primarily expected to provide a general picture of the interaction. Since a very early stage of the research, the purpose was to create a well-established analytical tool to better understand how interactions between actions, which are the main components of an episode, support MLO.

EPISODES, ACTIONS, AND MLOs

A theoretical approach has been developed to help determine the relationships between the *actions* of an *episode* that support *mathematical learning opportunities*. In this section we explore the nature of these key constructs, and see them as highly interrelated. On the one hand, when a MLO occurs in an episode, it is appropriate to ask what actions are behind the creation of such opportunity. On the other hand, when certain actions take place in an episode, it is also appropriate to look for the possibility of emerging MLOs. In the interactional and critical approaches to mathematical learning (see, for instance, Planas & Civil, 2010), one of the main focuses is to understand how sequences of actions are to be enacted in ways that promote a sort of correspondence between actions and learning. Although each action is singular in that it is mainly enacted by one participant, its implications do not refer to concrete individuals but rather involve the different participants having a role in the discussion.

Identifying and characterising episodes

It is part of the tradition of interactional theories to take episodes as organised units of data. When deciding what to take into account in the identification of classroom episodes, the organisation of units of data was framed by criteria of topic cohesion. In the context of a whole group discussion, we focus on those episodes most likely to influence the students' learning by promoting qualitative shifts in the mathematical thinking over the course of collective participation around a concrete curricular topic.

The qualitative shifts were considered in the sense developed by Saxe and colleagues (2009) when talking about “the travel of ideas” in the classroom.

To characterise the selected episodes of the whole group discussion orchestrated by the teacher, we take the instrumental and the discursive dimensions. Concerning the instrumental dimension, we draw on the types of instrumental orchestration by Drijvers and colleagues (2010): *Technical-demo*, which refers to the demonstration of artefact techniques by the teacher; *Explain-the-screen*, which refers to whole-class explanation by the teacher, guided by what happens on the computer screen; *Link-screen-board*, where the teacher emphasises the relationship between what happens in the technological environment and how this is represented in conventional mathematics of paper, book and blackboard; *Discuss the-screen*, which involves a whole-class discussion about what happens on the computer screen; *Spot-and-show*, where student reasoning is brought to the fore through the identification of interesting student work during preparation of the lesson, and its deliberate use in a classroom discussion, and *Sherpa-at-work*, where a so-called Sherpa-student uses the technology to present their work, or to carry out actions requested by the teacher. We adapt this typology to broadly classify the nature of the episodes according to the didactical performance of a discussion in which the artefact is not necessarily given by a DGS instrument. Our types are the following: *Explaining the artefact*, *Explaining through the artefact*, *Linking artefacts*, *Discussing the artefact*, *Discovering through the artefact*, and *Experiencing the instrument*. The first three types are dominated by the teacher’s actions, while the last three types are dominated by the students.

Concerning the discursive dimension in the characterisation of episodes, we draw on types that have been grounded throughout the evolution of the process of our research (Morera, Fortuny, & Planas, 2012). After having examined all the mathematical concepts and procedures in all the selected episodes, an effort was made to search for common patterns that might help to understand a generic development of the episodes and the shared particularities. Despite any episode can be distinguished from other episodes of the same lesson, the search for discursive commonalities among them has allowed determining a linear sequence of types for general orientation: *Situation of the problem*, *Presentation of one solution*, *Examination of resolution strategies*, *Examination of particular and/or extreme cases*, *Consideration of cases*, *Examination of different solutions*, *Connections*, and *Generalization*. This linear pattern refers to commonalities arisen from episodes with a specific kind of mathematical problems that foster both particularization and generalization. A different kind of problems would have probably led to a different discursive characterisation. Nevertheless, the sequence above is expected to represent whole group discussion around a set of mathematical tasks that goes beyond the concrete geometrical problems on isometries in our study. As it will be shown later in this report, the characterisation of an episode by means of a series of discursive and instrumental types gives a complex picture of the interaction among participants, along with the contents and the direction of the interaction.

Identifying and characterising actions from the episodes

Episodes have been taken to be instrumental and discursive units of data. But if we go deeper inside their structure, we need to add a complementary characterisation based on the actions that constitute the episodes. In summary, we have first developed a major characterisation of episodes by means of instrumental and discursive types, and then have looked at them in terms of the actions that frame the two types.

To search for significant actions that include the participants' interventions and the instrumented acts related to the use of an artefact, we draw on the mediational approach to the teaching and learning of mathematics (Mariotti, 2012). The emphasis on the relationships between humans and artefacts was initially fostered by our use of DGS environments, but has now turned to be generalised to any context of teaching and learning. We agree that knowledge is socially constructed by subjects involving the media because the participants collaborate to re-organise thinking with a different role than that assumed by written or oral language. The relation between significant actions in an episode may provide opportunities to greatly enhance students' learning (Hershkowitz & Schwarz, 1999). Thus, we classify the significant actions within episodes like: Students' actions considered as *Thinking-Math Interventions*; Teacher's actions, considered as *Didactical Interventions*; and *Instrumented Actions* performed by the participants and centred on their use of artefacts. While the Thinking-Math and the Didactical Interventions are rather located in the discursive dimension, the Instrumented Actions are better thought of as related to the instrumental dimension. The relationship between the instrumental types (from the major characterisation of episodes) and the Instrumented Actions is complex and needs to be explored through the representation of the analytical tool. Similarly, the relationship between the discursive types (from the major characterisation of episodes) and the Thinking-Math and Didactical Interventions are to be illustrated through the analytical tool.

Identifying and characterising MLOs

The identification and characterisation of episodes and actions is followed by the identification and characterisation of learning opportunities (see Yackel & Cobb, 1991, for a classical conceptualisation of learning opportunities as directly resulting from the interactions). We assume that the teaching and learning of mathematics take place in settings in which an important variation in the amount of MLOs exists. Moreover, we assume that such variation has an influence on the achievement of individual students depending on the quantity of MLOs in the specific settings of mathematical practice, together with the social conditions that qualitatively mediate such quantity. By either promoting or reducing the amount of MLOs, a change is made in the students' actual learning experiences. This is part of our argument for organising the search for mathematical learning in relation to the search for learning opportunities.

To make the notion of MLO operational, we take it on the form of opportunities for participation in a classroom discourse in which certain actions are oriented toward the discussion of specific *Mathematical Contents*, *Thinking Strategies*, and/or *Self-Regulating Activities*. Actions by individuals are interpreted as potential contributors to the mathematical learning in groups. Through the three types of actions around 'contents', 'strategies' and 'activities', students develop their knowledge in interaction with other participants and build concrete relationships that help the classroom discussion to focus on the mathematics. Consequently, we see learning as a qualitative change in the contents, strategies and activities developed by a student (i.e. a learner) to become or keep being a participant of a community that has its own institutionalized repertoire, like it is the case with the mathematics classroom.

We structure the analysis of opportunities by establishing these three types that, in turn, may respectively favour conceptual (e.g., the notion of homothecy), procedural (e.g., the practice of conjecturing) and regulative (e.g., the norm of justifying) mathematical learning. The MLOs of an episode are also more broadly characterised through the types of actions that frame the opportunities. This complementary characterisation is helpful in that it relates, for instance, the Thinking Strategy that is to be learnt with the students' and the teacher's actions that are enhancing the consideration of that particular strategy. Furthermore, two MLOs that are equally characterised as Mathematical Content and have the same conceptual content of reference can be distinguished by means of the complementary characterisation in terms of the actions in the episode that frame each MLO. The understanding of a MLO depends on both the contents for potential learning and the actions that contribute to making these contents emerge. On the other hand, the relationships between the MLOs and the actions in an episode point to the impact of the whole group dynamics on the students' learning.

THE RESULTING ANALYTICAL TOOL

The resulting analytical tool that has been created organises the characterisation of the whole group discussion throughout the characterisation of episodes, actions and MLOs. A complex representation of the whole group interaction is developed to inform about the richness of the discussion in terms of the amount of diverse MLOs. In this section, we explain the tool and then exemplify its effectiveness.

The representation of the episodes of a whole group discussion

For a concrete whole group discussion and as it has been argued earlier, the nature of the episodes is characterised through the instrumental and the discursive dimensions. Figure 1 illustrates the structure of the whole group discussion exemplified in this report. We claim that all the episodes identified in a whole group discussion can be represented in a two-dimensional matrix that suggests a coordinate system. Each episode is located in the system with two coordinates that determine its position, but the position is not uniquely determined as more than one episode may own the same two coordinates. The use of this representation allows whole group discussion to be interpreted in terms of a sequence of episodes with changing particularities concerning the use of artefacts and the interaction with the mathematical task.

Once the episodes have been defined and the whole group discussion has been structured as a sequence of characterised episodes, an in-depth study of the actions involved in each episode is required.

Whole-group discussion (Problem 3)	Situation of the problem	Presentation of one solution	Examination of resolution strategies	Examination and/or extreme cases	Consideration of cases	Examination of different solutions	Connections	Generalization	***(Discursive types)***
Explaining the artifact									
Explaining through the artifact							**(e_{10})**		
Linking artifacts		**(e_3)**							
Discussing the artifact			**(e_5)**					**(e_9)**	
Discovering through the artifact					**(e_7)**				
Experiencing the instrument	**(e_1)**	**(e_2), (e_4)**		**(e_6)**		**(e_8)**			
(Instrumental types)									

Figure 1: Representation of the episodes (e_i) involved in a whole group discussion

(Subscript *i* helps to follow the episodes chronologically)

The representation of the actions within each episode

The actions involved in the episodes are defined only by one element. As presented in the theoretical framework, the nature of this element differs depending on the agent (i.e. students' actions considered as Thinking-Math Interventions –TMI; teacher's actions considered as Didactical Interventions –DI; and Instrumented Actions –IA– performed by the participants and centred on the uses of particular artefacts). It is not intended to place each action under one of the groups, TMI, DI or IA. Nevertheless, our analysis has led to only a few non-classified actions (i.e. the actions that we have not been able to include in any of the three groups above). When an action has been characterised by considering the agent, it is codified to describe its nature in more detail. Apart from the chronological succession of actions, the relationships between them also have to be taken into account. Thus, we add oriented segments to connect the actions that are influenced by others. Finally, a structure is created to summarise all this information: a) the nature of all actions, b) the participants who are performing each action, c) the time sequence when the actions occur, and d) the oriented segments that relate the different actions (Figure 2).

We exemplify the use of the tool in the characterisation of the actions of one of the episodes in Figure 1. The episode comes from the 50-minute whole group discussion around the third problem of the sequence. The problem asks to find the centre of a rotation mapping two line segments given in the plane. We present the analysis of the first episode identified in the discussion. The episode presented (e_1) is characterised as “Experiencing the instrument”. Two students are using the DGS technology to present their work and to carry out actions requested by the teacher. In the transcription, we can observe the participants involved and the different nature of their actions. Each action is linked to one of the three types presented above.

Student 1: We decided to do a perpendicular bisector between a point and its homologous. Then we tried this intersection and we realised that it coincided and that it was the right rotation centre.

TMI: To explain a procedure to reach a solution

Student 2: [While Student 1 explains their solution, she makes a DGS construction]

IA: To complete an explanation with visual construction

Teacher: Now that you know that this point is right, could you argue why?

DI: To ask for a mathematical argument

Student 2: If two points, when we rotate a piece, coincide, it means that they are at the same distance from that point. So if they are at the same distance, for example, the perpendicular bisector is the locus point of all equidistant points between the two original ones.

TMI: To elaborate on a deductive justification

Teacher: The locus.

DI: To reformulate technical vocabulary

Student 2: Ah, yes! The locus.

TMI: To correct technical vocabulary

Student 1: [While Student 2 expresses the justification, he uses visual DGS figures that are on the screen for his explanation]

IA: To draw on visual DGS figures

After having identified the types of actions involved in the episode, the situation is represented in a visual diagram that incorporates preliminary oriented segments (Figure 2). It is particularly important to have these segments triangulated.

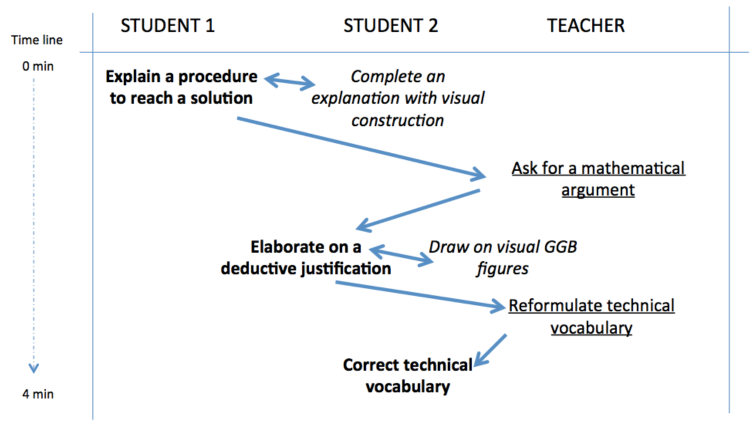

Figure 2: Visual representation of episode 1 (e_1)

From a descriptive point of view, we can observe that the participants involved in this four-minute episode are two students and the teacher. If we focus on the nature of the actions, we observe that the students' Thinking-Math Interventions (bold) are central. We also observe that the Didactical Interventions (cursive) by the teacher are equally important. In his first intervention, he asks for an argument and in the second one, he makes a vocabulary correction of the expression "locus". The students' use of the software is crucial too. In the transcription, we see that they first use the software to make a construction and then, point to the screen and show the completed construction, which are considered Instrumented Actions (underline).

The representation of the MLOs within each episode

The final use of the analytical tool is to identify rich situations that can influence the students' learning process in a middle-term perspective. In the selected episode, three situations that can enhance the students' mathematical learning are interpreted in terms of MLOs. The first two situations are derived from connections between TMI and DI, and the third one is derived from connections between TMI and IA. Following the characterisation of MLOs in three possible types, for this episode we find a dominance of Self-Regulating Activities. The contents of the interaction suggest the type of learning involved in the regulation of the task performance. The three MLOs below are not based on the mathematically correct performance of explanations, but rather on the production of consciousness around the importance of certain practices.

MLO_1 –Importance of dual explanations with communicative and technological skills

The fact that the students use software every time they want to explain or show something to the class, as occurs twice in this episode, is a significant relationship between the Thinking-Maths Intervention and the Instrumented Actions around the uses of DGS. The students complement their explanations by drawing on visual DGS figures: Student 1 explains the procedure to reach the solution and Student 2 develops a deductive justification. We consider that this situation gives importance to the dual explanation through a combination of communicative and technological skills. In further research, we will have to assess the possibility of being influenced by this episode when a student later uses the DGS to complement a written or oral solution.

MLO_2 –Importance of reasoning and learning the specific justification

The fact that the students are asked for a mathematical argument after the presentation of incomplete solutions is illustrated in this episode by Student 1, who points out the importance of arguing the solutions for any mathematical problem. Moreover, in this situation, the correct justification is given by Student 2. This action also enhances the potential for learning because it provides specific knowledge (the correct argumentation) that may influence the understanding by other participants that have listened to the contribution made public by Student 2.

MLO_3 –Importance of correct use of the mathematical language

The fact that the teacher corrects the mathematical expression “locus point” after its inappropriate use by Student 2 during his argumentation creates a rich situation in which the importance of using correct vocabulary is modelled. Student 2 reacts to by correcting his expression and showing a more accurate use of the mathematical language. Thus the positive influence of the intervention is evident. Although the teacher merely says the concept without giving any further explanation and we cannot guarantee conceptual learning of “locus point” because the student may be just repeating a word, the acquisition of technical vocabulary is at play. Anyhow, it might happen that what we have considered as a potentially rich situation led to non-learning responses, and this might happen for all the MLOs that have been identified.

We have identified three potentially rich situations that can be seen as MLOs. It might have been advantageous to describe more precisely only one MLO, but we have preferred to illustrate the plurality of MLOs that may be linked to a single episode. We have shown that analysing the episodes with our tool provides an overall view of how the actions of an episode interact. This overview facilitates the detection of potential situations emerging as a convergence of different agents involved in the episode. On the other hand, the identification of learning would have required to look for a sequence of episodes as evidence of reaching learning responses throughout the experience of a sequence of MLOs. However, it is not the aim of this report to trace the sequential development of the students’ mathematical learning during class discussion.

DISCUSSION

For the investigation of whole group interaction in the mathematics classroom, accurate and systematic analytical methods are needed. In this report we have presented a newly-developed analytical tool to facilitate this investigation. We have summarised the design and validation of the tool, whose main objective is to identify rich situations that may enhance the students’ mathematical learning during whole group discussions. Its effectiveness has been illustrated by applying it to a transcript of a whole group discussion. The tool analyses discussions from multiple perspectives. On the one hand, the framework of the instrumental orchestration by Drijvers and colleagues (2010) provides a clear episode-structure of the discussions. On the other, the findings about the actions involved in the episodes support the humans-with-media theoretical approach that takes the subject and the tool involved in a mathematical activity into account (Mariotti, 2012). These results are consistent with Hershkowitz and Schwarz’s (1999) findings that students’ progress is caused by verbal interactions, but also by artefact manipulations and communicative nonverbal actions.

We are concerned with the problem of applying the tool to any setting of teaching and learning mathematics. The applicability of the tool can be expected to be improved by, for instance, reconstructing some of the types for the instrumental and the discursive dimensions from a more global perspective. Nevertheless, it is not easy to broaden the definitions of these dimensions to include situations in which the artefacts are less visible, or the mathematical tasks are less argumentative. Therefore, the analytical tool still has limitations that constrain its generalisation. The tool has emerged from the analysis of certain whole group discussions with particular teacher, students and problems. Despite these limitations and others that may appear, we conjecture that the tool could also be applicable, after minor adaptation, outside the scope of this study with a design experiment involving different contents and technical environments. It would be worthwhile to investigate the effectiveness of the tool in various contexts such as discussions involving different teachers and students, other kinds of problems and artefacts. Further research will help to refine the tool and examine its potential and reliability to explore whole group discussions in the mathematics classroom.

Notes

Projects EDU2011-23240, EDU2009-07113 and EDU2012-31464, and Grant FPI BES-2009-022687, Spanish Ministry of Economy and Competitivity.

References

Drijvers, P., Doorman, M., Boon, P., Reed, H., and Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, *75*, 213-234.

Hershkowitz, R., and Schwarz, B. (1999). The emergent perspective in rich learning environments: Some roles of tools and activities in the construction of sociomathematical norms. *Educational Studies in Mathematics*, *39*, 149-166.

Krummheuer, G. (2011). Representation of the notion "learning-as-participation" in everyday situations of mathematics classes. *ZDM –The International Journal on Mathematics Education, 43*(1), 81-90.

Mariotti, M. A. (2012). ICT as opportunities for teaching-learning in a mathematics classroom: The semiotic potential of artefacts. In T.-Y. Tso (Ed.), *Proceedings of the 36th International Conference of the Psychology of Mathematics Education Group* (vol. 1, pp. 25-42). Taipei: PME.

Morera, L., and Fortuny, J. M. (2012). An analytical tool for the characterisation of whole-group discussions involving dynamic geometry software. In T.-Y. Tso (Ed.), *Proceedings of the 36th International Conference of the Psychology of Mathematics Education Group* (vol. 3, pp. 233-240). Taipei: PME.

Morera, L., Fortuny, J. M., and Planas, N. (2012). Momentos clave en el aprendizaje de isometrías en un entorno de clase colaborativo y tecnológico. *Enseñanza de las Ciencias, 30*(1), 143-154.

Planas, N., and Civil, M. (2010). Discourse processes in critical mathematics education. In H. Alrø, O. Ravn & P. Valero (Eds.), *Critical Mathematics Education: Past, present and future* (pp. 145-160). Rotterdam: Sense Publishers

Saxe, G., Gearhart, M., Shaughnessy, M., Earnest, D., Cremer, S., Sitabkhan, Y., Platas, L., and Young, A. (2009). A methodological framework and empirical techniques for studying the travel of ideas in classroom communities. In B. Schwarz, T. Dreyfus, & R. Hershkowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 203-222). New York: Routledge.

Stein, M. K., and Smith, M. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Yackel, E., Cobb, P., and Wood, T. (1991) Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education, 22*(5), 390-408.