WRITER IDENTITY AS AN ANALYTICAL TOOL TO EXPLORE STUDENTS' MATHEMATICAL WRITING[1]

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Learning to communicate in, with and about mathematics is a key part of learning mathematics (Niss & Højgaard, 2011). Therefore, understanding how students' mathematical writing is shaped is important to mathematics education research. In this paper the notion of 'writer identity' (Ivanič, 1998; Burgess & Ivanič, 2010) is introduced and operationalised in relation to students' mathematical writing and through a case study it is illustrated how this analytic tool can facilitate valuable insights when exploring students' mathematical writing.

INTRODUCTION

In recent years the notion of *identity* has received increased attention in the field of mathematics education research (for an overview see e.g. Steentoft & Valero, 2009). Despite of this Steentoft and Valero concludes that *"... identity is still only emerging in mathematics education research and is far from explored in full."* (2009: 76). As argued by Sfard and Prusak (2005) a key challenge in this endeavour is to provide operational definitions of the notion of identity itself.

How writers construct their identities in texts has been the focal point of several studies in the field of writing research. In fact in his overview of writing research Hyland (2009) notes this as one of the key issues to be addressed in writing research and point to the works of Ivanič (1998) as a major contribution in this context.

In this paper I will argue that Ivanič's notion of *writer identity* (Ivanič 1998; Burgess and Ivanič 2010) can in fact provide a useful way of operationalizing the notion of identity in relation to students' mathematical writing. In order to do so, I will initially introduce the notion of writer identity as outlined by Ivanič and elaborate on how this can be operationalised. Following this I will demonstrate the usefulness of these analytical tools by applying them to two different mathematical texts written by the same student. The center of attention will be how, and why, the student changes his way of representing himself as a writer of mathematical writing.

To sum up, the two research questions leading this paper will be: (1) *how can the notion of identity be operationalised in relation to students' mathematical writing, and (2) what kinds of insights can an identity perspective on students' mathematical writing provide?*

THEORETICAL FRAMEWORK – THE NOTION OF WRITER IDENTITY

Ivanič distinguishes between four interrelated aspects or meanings of *writer identity* and characterizes these as '*ways of thinking of a person's identity in the act of writing*' (Ivanič, 1998: 23-30). As such the notion of writer identity is a *social* notion, which concerns the many ways in which writers position themselves through their use of semiotic resources.

Discoursal self. This aspect of writer identity is concerned with the impression that writers convey of themselves in a particular text. Writing mathematical texts is not just about getting the calculations right. A written text, mathematical or not, also leaves the reader with an impression of who the author is or perhaps wants to be. The discoursal self is constructed by the text characteristics of the particular text, but is closely related to the values, beliefs and power relations embedded in the discourses[2] that are present of the social context of the writers.

Authorial self or *self as author* as Ivanič prefers to call it, is the way writers appear as authors in particular texts. This involves in which ways, and to what extent, writers attribute the choice of content and form of the texts to themselves or to other authorities. Some writers of mathematics present the content of their writing as platonic truths while other writers present it as being the work of their own. As both the discoursal self and the authorial self are identities that are inscribed in particular texts they can both be analyzed using text analysis.

Autobiographical self. The third aspect focuses on the personal stories the writers brings with them to the act of writing. This involves norms, values and beliefs related to the writing of mathematical texts and as such this aspect will be shaped by the writers' previous encounters with writing events that involved mathematics. Obviously this aspect of writer identity cannot be disclosed through text analysis alone, but can instead be further explored through interviews with the students.

Possibilities of selfhood. In any social context *possibilities of selfhood*, or *subject positions,* will be available to the writers in the act of writing. Students' mathematical writing in most cases takes place in the institutional context of school and is as such embedded in the various possible discourses of school mathematics. These discourses in turn shape the possibilities of self-hood available to the writers. This aspect of writer identity can be explored through classroom observations, studies of institutional texts such as curricula and by interviewing both students and teachers.

METHODOLOGY

Underlying the notion of writer identity is an understanding of writing as a *social act of meaning*. This fundamental assumption has methodological consequences both on the level of research design and for the use of analytical tools. In both cases the goal is to minimize the gap between text and social context by using context sensitive approaches when exploring students' mathematical writing. On the level of research design Lillis (2008) argues that this can be done by adopting ethnography as a methodology. The two texts analyzed below are drawn from a one year long ethnographic longitudinal study of students' mathematical writing conducted along the lines suggested by Lillis.

Analytical tools for text analysis

Understanding students' mathematical writing as a social act of meaning has in this case led to a text analysis based on systemic functional linguistics (e.g. Halliday, 1978). A significant contribution as to how such analytic tools can be applied to mathematical texts is developed by Burton and Morgan (2000), Morgan (1998; 2006) and O'Halloran (2005). As noted by Burton and Morgan (2000: 430)

> Although sometimes seen to be peripheral to the main mathematical content natural language serves in the construction of the identities of the author and reader and of the epistemological and ontological assumptions underlying the writing.

Therefore, I will in this case restrict the focus of the text analysis to the *mathematical language* of students' texts thus leaving out the *visual mediators* such as tables, diagrams, algebraic notation etc. (Sfard, 2008) from the analysis.

Textual characteristics of the discoursal self

Characterizing writers' discoursal selves involves analyzing how the writers represent, or portrays, themselves in their mathematical texts. According to Hyland:

> An example is the extent to which a writer takes on the practices of the community he or she is writing for, adopting its conventions to claim membership. (Hyland, 2009; 73)

An integral part of this would be to explore in what ways and to what extent writers make *use of mathematical vocabulary and conventional forms of mathematical language* (Morgan, 1998; 97), when we analyze how students' try to claim membership to some sort of mathematical community in their texts. Another important aspect of how writers construct their discoursal self concerns their use of *personal pronouns* Using the personal pronoun *I* is unusual in most academic mathematical texts (Burton and Morgan, 2000) but can indeed be meaningful in school settings where students' often have to demonstrate their mathematical competencies to a teacher (see also Rowland, 1999). Moreover, writers can direct the attention to themselves in their mathematical texts is by *referring to their own actions and cognitive processes*. As with the use of the personal pronoun *I* this text feature can, I suggest, in a school setting be understood as students' way to accommodate to a demand of *showing their own understanding* of the mathematical issues or problems elaborated in the texts.

Textual characteristics of the authorial self

One way that writers can convey their authority is by using words or phrases that signal different kinds of *ownership*. As noted by Burton and Morgan (2000) this can be studied in mathematical texts by attending to the use of *modality*, which includes the use of *adverbs* (e.g. *almost, always, certainly, clearly, easily, nearly, potentially, possible*), *adjectives* (e.g. '*the derivative is easily calculated in this case.*') or *modal auxiliary adverbs* (e.g. *must, can, may, could*). Another, perhaps more subtle, way of expressing authority is by making it clear what is *not* in the particular text. Purposefully *omitting* more or less obvious parts of calculations or symbolic manipulations, and perhaps even underlining that this is the case, is a well known way of claiming authority in the field af mathematics. Yet another way of signaling authority in a text can be to explicitly express different kind of *choices* that has been made by the author. This could, for example, include choices regarding the mathematical content of the text. As the authorial self is closely connected to the writers' willingness to get behind claims or arguments that are put forward in the text, again, like in the case of the discoursal self, studying the writers' use of *personal pronouns* can provide valuable insights (Hyland, 2009).

THE CASE OF CHRISTOPHER

Christopher is a student at a Higher Technical School (htx), which is one of the four upper secondary school programs in Denmark (grade 11-13). A review of the mathematical texts Christopher has produced as a student at htx shows that he has gradually found a way to construct his identity as a writer of mathematics that is quite *stable* with regard to how he represents himself in his texts. The extract of text shown below in figure 1 is chosen because it illustrates how he typically does this.[3]

The different assignments that Christopher works with during his time as a htx-student can roughly be divided into three categories: (1) *Home assignments* that typically consisted of a collection of separated word problems, (2) *Project assignments* that usually were constructed around one guiding problem, and (3) *Presentation assignments* that were tasks where the primary goal was to present some sort of mathematical topic or idea. The two text extracts below is taken from Christopher's responses to a home assignment (figure 1) and a presentation assignment (figure 2) respectively. In both cases the text was produced at the end of the school year and had to be handed in to the teacher who evaluated it afterwards

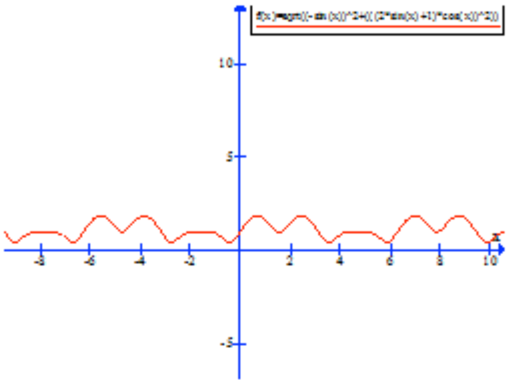
In several interviews I talked to Christopher about his perception of the different kinds of assignments in order to get some insight in to his *autobiographical self*. Home assignments are for Christopher associated with *training* and *standardization* and he describes these as *controlled* by the teacher. Regarding project assignments the converse seems to be the case. Christopher characterizes these with expressions like *independence* and *a possibility of putting the mathematics into a larger perspective*. In both cases the relations between type of assignment and the associated discourse seems locked for Christopher. This might be an important part of the explanation as to why he had continuously constructed similar types of discoursal and authorial selves in his mathematical texts in the last year of htx.

The home assignment. The home assignment consisted of a set of word problems or tasks, which had been posed at a written national examination in mathematics some years before. In the extract we see Christopher's answer to one of the tasks. The purpose was to determine the maximum speed of a moving particle whose motion is described by a vector-valued function. When we enter Christopher's text he has just found the derivate of the vector-valued function – the velocity vector. Notice how Christopher is clearly present as an acting agent in his own text.

1 Now where I have to find the maximum speed, I should start by calculating the length of my
2 velocity vector, because the speed is defined like this.

3 $|v(t)| = \sqrt{x'(t)^2 + y'(t)^2} \Leftrightarrow |v(t)| = \sqrt{(-\sin(t))^2 + ((2\cdot\sin(t)+1)\cdot\cos(t))^2}$

4 By inserting it to Graph[1], I can get an overview of the various possible vertices.



5

6 This doesn't look all wrong, because I can imagine that the two big 'sharp' turns that appears in the
7 first graph is what gives the two turns on this graph. The spaces between the two major vertices
8 correspond to the major loop before the turn's returns on the original graph. By taking the
9 derivative of the speed function I can find out where the vertices can be found.

10 $|v'(t)| = \sqrt{(-\sin(t))^2 + ((2\cdot\sin(t)+1)\cdot\cos(t))^2} = \dfrac{2\cdot\cos(t)\cdot((2\cdot\sin(t)+1)\cdot(\cos(t))^2 - 2\cdot(\sin(t))^2\cdot(\sin(t)+1))}{\sqrt{(4\cdot(\sin(t))^2 + 4\cdot\sin(t)+1)\cdot(\cos(t))^2 + (\sin(t))^2}}$

As this should be taking the derivative equal to zero, then the numerator has to be 0 before the whole fraction can and therefore I can just solve t in the numerator.

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The equation I solve is this:

14 $2\cdot\cos(t)\cdot((2\cdot\sin(t)+1)\cdot(\cos(t))^2 - 2\cdot(\sin(t))^2\cdot(\sin(t)+1)) = 0$

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I get 5 solutions, which indicates for which value of t there is a turning point:

16 $t = 0{,}695 \quad t = \frac{\pi}{2} \quad t = 2{,}44 \quad t = \frac{3\pi}{2} \quad t = 5.88$

-- By evaluating my graph I found out that t=0,695 and t=2,44 both were vertices. Those values have the same kind of vertices, and therefore, I can just insert one of the values in my speed function:

19 $|v_{max}(t)| = \sqrt{(-\sin(0.695))^2 + ((2\cdot\sin(0{,}695)+1)\cdot\cos(0{,}695))^2} = \underline{\underline{1{,}865}}$

Figure 1: Extract from Christopher's written answer to a home assignment

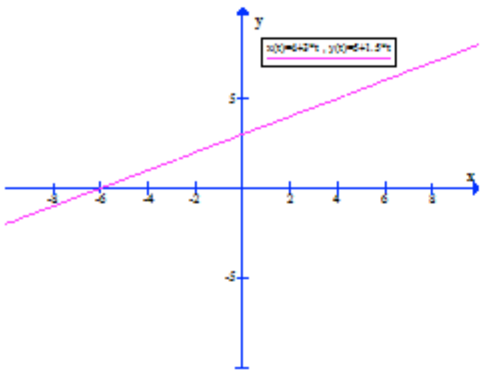
The presentation assignment. In the presentation assignment each of the students was assigned a specific mathematical topic, and their job was to present it in the best way possible to the other students. Christopher was assigned the topic *vector-valued functions*, and the text shown in figure 2 is taken from a paragraph where he writes about *the straight line.* As we enter Christopher's text he has just sketched a graph of *f(x)=½x+3* in a coordinate system.

As an example, we look at the line drawn earlier on. The line is passing through two points A=(1;3,5) and B=(4;5). These points are used to describe a direction vector:

$$\vec{r} = \vec{O}B - \vec{O}A \Leftrightarrow \vec{r} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3{,}5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1{,}5 \end{pmatrix}$$

Using the point B as P_0 we can write the vector function:

$$\vec{O}P(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 1{,}5 \end{pmatrix} \cdot t = \underline{\underline{\begin{pmatrix} 4+3t \\ 5+1{,}5t \end{pmatrix}}}$$



If we want our parametric equation for the line to look like the original function for a straight line y=ax+b, then we have to eliminate the parameter t.

Here, the prior example is used as a basis:

$$f(t) = \begin{pmatrix} 4+3t \\ 5+1{,}5t \end{pmatrix}$$

To eliminate t, we isolate t in either the x-part or the y-part:

$$x = 4 + 3t \Leftrightarrow t = \frac{x-4}{3}$$

This is then inserted to the y-part:

$$y = 5 + 1{,}5t \Leftrightarrow y = 5 + 1{,}5 \cdot \left(\frac{x-4}{3}\right) = \underline{\underline{0{,}5x + 3}}$$

Figure 2: Extract from Christopher's written answer to a presentation assignment

Construction of identities in the two texts

Due to limited space, the analysis will focus only on the most significant *differences* between Christopher's two texts. These are displayed in table 1. Numbers in brackets refer to the line numbers in figure 1 and 2 respectively.

Textual characteristics	Home assignment text (Figure 1)	Presentation assignment text (Figure 2)
Use of personal pronouns	Example – use of 'I' and 'my': '*The equation I solve is this:*' (13) '*… my velocity vector…*'(1-2)	Example – use of 'we' '*As an example, we look at the line drawn earlier on.*' (1)
Use of conventional forms of mathematical language	Example - *colloquial* language: '*As this should be taking the derivative equal to zero, then the numerator has to be 0 before the whole fraction can and therefore I can just solve t in the numerator.*' (11-12)	Example - *textbook* language '*If we want our parametric equation for the line to look like the original function for a straight line y=ax+b, then we have to eliminate the parameter t*' (7-8)
Reference to own actions and cognitive processes	Example – actions '*By evaluating my graph I found out that t=0,695 and t=2,44 both where vertices*' (17) Example – cognitive processes '*This doesn't look all wrong, because I can imagine that…*' (6)	No explicit reference to own actions and/or cognitive processes
Explicit signals of authority	Example - *omitting*: '*… therefore I can just solve t in the numerator.*' (12)	No direct use of words and phrases that signals authority

Table 1: Summary of significant differences between the two texts

So what kind of identities does Christopher seem to construct in his two texts? In the first text the discoursal and authorial selves are characterized by an explicit presence of Christopher appearing as an active and visible agent in his own text. It is Christopher himself as a person who appears as the source of the mathematical actions performed and the new knowledge presented. Christopher seems to construct an identity as *a writer who wants to make clear to his reader: (1)how he himself as a person are able to solve the task he is given, and (2) what he thinks while doing this.* It is a very different discoursal and authorial self we encounter in Christopher's second text. His own clear presence as a person in the text is gone, and instead an identity as a *neutral mathematical communicator* using the voice of a mathematical textbook is constructed.

One of the most remarkable differences in the way Christopher constructs his identity in the two texts is the alternating use of personal pronouns. In an interview at the end of the school year I asked Christopher to comment on this difference between the two texts.

1	Christopher:	I believe it is when you are thinking that you are writing some mathematics to somebody else who understands mathematics, then you are kind of saying 'Now we do this' or … but sometimes I also say 'I' and I cannot really explain why. But in this case, I think I can …
2	Interviewer:	With these notes [*the presentation assignment text*[4]]?
3	Christopher:	Yes, because now it's not for one person, that I have to imagine that this is for, but generally for somebody who doesn't understand it, and then it might sound a bit stupid to say: 'I'm doing this' because it's not me, who does it. It's … generally you can do it like this. That's also why I wrote 'a vector-valued function is described' instead of 'I describe a vector-valued function'. It is, I guess, not me who has found it or … and sometimes I also just think that 'we' is just … It's just a better way of writing it than using 'I'. I think it sounds a little … In the home assignments … there I know that it's me who makes it, I mean calculates this stuff. So there I think it fits better than as for example in this paper, even though it is me who has done it. But that's not what's important for the purpose of this text.

In the interview Christopher gives three different reasons for his replacement of the personal and for him habitual, *I* with the more neutral *we*.

Community with the reader - The use of *we* indicates that both writer and reader are part of the same community, and Christopher does in fact himself point to the fact that an equal relation with the reader ('*... somebody else who understands mathematics…*') give rise to the use of *we* in his texts. At the same time he is aware that in other texts, which are only read by his teacher, he most often uses *I* and not *we* in his wording. This seems to leave him doubtful ('*...I cannot really explain why.*').

The source of knowledge – The use of *we* instead of *I* contributes to a linguistic move of responsibility away from the writer as a person, and this seems crucial to Christopher in this case. He recognizes, of course, that he is the author of both texts, but underline that in the case of the text on vector-valued functions this is not essential in relation to the purpose of the text ('*...it is me who has done it. But that's not what's important for the purpose of this text*').

Sounding in the right way – The third reason is affective in nature. Christopher states that '*It's just a better way of writing it than using 'I'.*' and continues '*I think it sounds a little ...*' referring to the use of *I*. What is meant by '*just a better way'* is of course not transparent, but it seems plausible that Christopher in this case, consciously or unconsciously, identifies with some sort of mathematical community, where the use of *we* would be regarded as more appropriate than the use of *I*.

CONCLUSIONS

In this paper one way of operationalizing the notion of identity in relation to students' mathematical writing has been presented. When producing mathematical texts students need to be able to decode and navigate in the purposes and possibilities of selfhood associated with writing in the mathematical classroom, and to construct identities that are perceived as *appropriate* in the given context. As pointed out by Morgan (2006: 239):

> While establishing appropriate identities is of importance to participants in any situation, it is of critical importance to students at all levels whose oral and written productions are to be assessed.

Different types of writing assignments offer different possibilities of selfhood to the students in the mathematical classroom and Christopher in the presented case reacts by constructing different kinds of identities in his two texts. For him *purpose* of the writing assignment and *audience* of the text seems to have a specific influence on the way he construct his identity as a mathematical writer.

In this way the case illustrates how varying the writing assignments of the mathematical classroom hold a potentially strong possibility of forming students' ability to develop their style of writing in mathematics. The analysis show that an identity view on students' mathematical writing can provide valuable insights about, what students want to achieve with their writing, how they understand the role and function of writing in mathematics education, and how this understanding can shape the way they express themselves in their mathematical texts. Such insights can be an important step towards providing better opportunities for students to develop linguistic and communicative competences in mathematics.

NOTES

1. The study presented in this paper is part of the research project *Writing to learn, learning to write – Literacy and disciplinarity in Danish upper secondary education* supported by the Danish Agency for Science, Technology, and Innovation. For additional information please go to www.sdu.dk/fos.

2. Throughout the paper I use the notion of discourse as Gee (1996: 131) defines *Discourse* (with a capital *D*).

3. In both text extracts the original layout has been changed slightly. Christopher inserts a line break when he uses the sign for material equivalence $\Leftrightarrow$. For the sake of saving space I have removed these line breaks simply letting the calculations continue horizontally. I do not analyze the layout of Christopher's texts in this paper.

4. My insertion in square brackets.

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