MATHEMATICAL CREATIVE SOLUTION PROCESSES OF CHILDREN WITH DIFFERENT ATTACHMENT PATTERNS

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In research about mathematical creativity seldom the early childhood is taken into count. The paper investigates the mathematically creative solutions of Kindergartners (in Germany children between 3-6 years attend Kindergarten) who have social/emotional difficulties. Starting point is the longitudinal study MaKreKi (mathematical creativity of children) in which theories of mathematics education and psychoanalysis are amalgamated for the investigation of mathematical creativity. In this paper two episodes of two children between 4 and 5 years old are presented, while they cope with a mathematical task. It focuses on the mathematical creative ideas emerging in the interaction between the involved children and the accompanying person and how the children deal with the mathematical instructions.

Keywords: mathematical creativity, attachment theory, early childhood mathematics

INTRODUCTION

Attempts to define mathematical creativity seem to lead to more than 100 contemporary definitions of creativity (Mann 2006), but mathematical creativity in early years is rarely examined. Thus the central research question is, how does mathematical creativity express itself at the age of preschool and how is it observable?

Following Urban (2003) a theory of creativity has to consider the "4P-E Structure" of creative thinking and acting, which embodies the interactive structure of the factors: problem, person, process, product and environment. The existence, the range and recognisability of possible problems to be solved creatively are determined by meta- environmental factors like evolutionary and social-historical developments, macro-environment like economic, material, cultural and political conditions and micro-environmental factors such as socioeconomic conditions of the family (Urban 2003). From a socio-constructivist point of view, the individual ability of mathematical creativity develops in the course of many interactions with other members of the culture. The paper focuses on the mathematical process while children are working on mathematical tasks in mathematical situations of play and exploration. Therefore it highlights on the negotiations of meanings (Brandt & Krummheuer 2001) between the children and the accompanying person during the interactive process while coping with mathematical task and in a specific cultural system. Beside this situational, micro-sociological access the paper also refers to psychoanalytically- based attachment theory, in which a function of the culture is understood as an aspect of the relationship between mother and child. Bowlby's

theory of attachment suggests that children come into world biologically pre-programmed to form attachments with others, because this helps them to survive (1969). The neonate develops special relationships with her parents. In the first years of life the child develops an 'inner working model' through child-parents-interactions (Bowlby 1969). This 'inner working model' contains the early individual bonding experiences as well as the expectations, which a child has towards human relationships, derived from these experiences. They conduce to interpret the behavior of the caregiver and to predict his or her behavior in certain situations. So the attachment between mother and child has a great impact on the social- emotional and cognitive development of the child. After the first year of life this 'inner working model' becomes more and more stable.

Children are confronted with mathematical tasks and contents from different domains of mathematics as they appear in their everyday life. In the MaKreKi project these contents are presented in form of mathematical situations of play and exploration (Vogel 2012) for the children regarding their assumed mathematical competencies. Within in these situations a competent adult guides the children. He has some knowledge about the mathematical contents, which are intended and a minimal set of instructions like questions or allegations. The competent adult also has some hints of possible reactions and expressions of the children, so that he is somehow prepared for possible mathematical tasks and solutions emerging in the context of the mathematical situation of paly and exploration. All these information for the accompanying person are documented in 'didactical design patterns' (ibd.). On the situational level the presentation of these mathematical situations of play and exploration initiate processes of negotiations of meanings, which necessarily do not be in in accord with the described mathematical domain nor the activities that are expected in the design pattern. In the MaKreKi project the children often shows unusual associations and solutions for mathematical tasks, which can be also seen as some legitimated ways and perspectives in the given mathematical context.

Following Urbans idea of the "4P-E Structure" this article examines macro-environmental factors like cultural conditions e.g. the intended mathematical domains/contents and the expected mathematical tasks and solutions presented in the mathematical situations of play and exploration as well as the interactive negotiations of these themes (micro-social factors). Beside this the paper also involves psychodynamic aspects of early childhood development (micro-environmental factors). In the research design of the MaKreKi project children are dealing with mathematical problems and tasks guided by a competent adult. This adult can be seen as a representative for the parents, because it might be reasonable to assume that children shows similar behaviour in this situation like they would with their parents, because of their stable 'inner working model'.

THEORETICAL PERSPECTIVES

The following section introduces theoretical perspectives of mathematical creativity in early childhood and offers psychoanalytical considerations about attachment theory and the connection to creativity.

Mathematical Creativity

Mathematicians and researchers in mathematics education as well as psychologists have examined mathematical creativity under their various scientific viewpoints (Hadarmard 1954, Sriraman 2004). A clarification of concepts of creativity is difficult and additionally complicated by its relationship to the concepts of intelligence, giftedness and problem solving.

With respect to the relative lack of current research the following analysis deal with the following four aspects of mathematical creativity (Sriraman 2004):

• *Choice*: Poincaré (1948) described as a fundamental aspect of mathematical creativity the ability to choose from the huge number of possible combinations of mathematical propositions a minimal collection that leads to the proof. With regard to the age group of interest under this *choice* aspect of mathematical creativity the production of (unusual) relations between mathematical examination and experiences and the playful contact with mathematical methods is understood.

• *Non-algorithmic decision-making*: According to Ervynck (1991), mathematical creativity articulates itself not when routine and/or standard procedures are applied but when a unique and new way of solving a problem emerges. With regard to the age group one is able to shift the accentuation and speak of the "divergence from the canonical" (Bruner 1990, p.19).

• *Adaptiveness*: Sternberg & Lubart (2000) characterize creativity, as the ability to present an unexpected and original result that is also adaptive.

• *De-emphasizing detail*s: In his study, in which he investigates the ideas and thoughts to mathematical creativity from famous mathematicians, Liljedahl (2008) has discovered that detail does not play any role during the incubation phase of creativity. Many of the participants mentioned how difficult it is to learn mathematics by attending to the details, and how much easier it is if the details are de-emphasized.

Attachment theory

Attachment theory originates from Bowlby (1969) and postulates the central role of attachment behaviour for individual development. Bowlby perceives the attachment system as the central source of motivation. In his approach the antagonism between attachment and exploration has a highly relevant explanatory power. Both systems cannot be simultaneously activated. If a child feels secure, it can activate his exploration system and explore his surroundings. If it perceives a danger, the

attachment system is activated. The child interrupts its exploratory behaviour and seeks safety by its parent. Four attachment pattern are described (Ainsworth et al. 1978): *Insecure-avoidant*: The 'insecure-avoidant' child (A) experiences that its mother feels best when it shows no intense affects itself and behaves towards her in a controlled, distanced manner with a minimum of affect. *Secure*: The securely attached child (B) has, thanks to its sensitive mother, a chance to build up a secure relationship to her in which the whole spectrum of human feelings in the sense of communication with another, that can be perceived, experienced and expressed. *Insecure-ambivalent*: The ambivalently attached child (C) has spent its first year with a mother, who sometimes reacts appropriately, and is at other times rejecting and overprotective, i.e. on the whole, inconsistent and for this reason she reacts in a way that is unpredictable for the child. *Insecure-disorganized*: The disorganized/ disoriented attached child (D) could not build up a stable inner working model, as its mother (or father) suffered under the consequences of an acute trauma (for example, the dramatic loss of an important person). They were psychically so absorbed by this loss that they could hardly take up a coherent relationship with their infant.

Relating this approach to the topic of mathematical creativity of young children, the results of empirical attachment research point to the fact that the shaping of domain-specific (mathematical) creativity can not only be localized in the potentially stimulating mathematical contents in the child's milieu but also in the type of attachment of the child to its parents. Grossmann describes the link between the attachement pattern of the child and the 'successful cooperation' (in german: gelingende Gemeinsamkeit) in a child-parent play situation more detailed (Grossmann 1984). The 'successful cooperation' of this play situations correlates with the delicacy feeling of the mother and a more delicacy feeling leads very often to a secure attachment pattern of the child (ibd.). Mothers of secure attached children seem to be more reserved, gentle and they show more efforts in handing over the lead to their children in play situations instead of mothers of insecure attached children. Mothers of insecure attached children are often strict and controlling and they have more instructional ratio in play situations than mothers of secure attached children (ibd.). Significant differences between children with a secure attachment pattern and children with an insecure attachment pattern in play situations are also described in the study of Grossmann. Secure attached children are more often initiators of the common play and they seem to be rather extroverted instead of insecure attached children, who wait for instructions.

METHODOLOGY

Regarding the theoretical considerations and the attempt to identify mathematically creative moments in mathematical interactions of preschool children, in the following there is conducted an analysis of interaction, which based on interactional theory of learning (Brandt & Krummheuer 2001). It focuses on the reconstruction of meaning and the structure of interactions. Therefore it is proper to describe and

analyze topics with regards to contents and the negotiation of meaning in the course of interactional processes. The negotiation of meaning takes place in interactions between the involved people. These processes will be analyzed by an ethno methodological based analysis, in which is stated that the partners co-constitute the rationality of their action in the interaction in an everyday situation, while the partners try constantly to indicate the rationality of their actions and to produce a relevant consensus together. This is necessary for the origin of own conviction as well as for the production of conviction with the other participating persons. This aspect of interaction is described with the term 'accounting practice' (Lehmann 1988, p. 169). To analyze these 'accounting practice' of children in mathematical situations, the reconstruction and analysis of argumentation of Toulmin (1969) have proved to be successful. Four central categories of an argumentation are "data", "conclusion", "warrant" and "backing". The general idea of an argumentation consists of tracing the statement to be proven back to undoubted statements (data).

FIRST INSIGHTS

The following section presents extracts of interpretations of two children Nina and René of the MaKreKi project while they are participating in the mathematical situation of play and exploration called 'Ladybug'. Both children are examined paired with one of her/his closer friends and at least one adult person, who acts as nursery teacher.

The 'Ladybug-situation'

In this situation the children can differentiate between similar objects, which differ according to their size and color. The objects are pictures of ladybugs, which differ in size (small and large), in color (red, green, yellow), and in the spots on ladybugs in three ways (shape, amount of spots, size). The design pattern suggests the following mathematical activities to the children through material and designated instructions and impulses: Counting and determination of quantity; Arrangement and comparing of sets e.g. in respect of the number of elements on the back of the ladybugs; Identifying mathematical structures. The 'Ladybug-situation' consists of two parts. In the first part the children are dealing with little ladybug cards. Typical instructions of the accompanying person are: 'Look what I brought.' ‚Put together all ladybugs which belong together'; ‚Can you find further groups or families of ladybugs?'; ‚Why do these ladybugs belong together?' In the second part the children are dealing with big ladybug-cards, which have small and large spots on their backs. Usually the accompanying person offers a triplet of big ladybug-cards and asks: Which one does not belong?

Case study one: René's solution process in the 'Ladybug-situation'

René is a four years and 9 months old boy who lives with his parents and his older sister in a small city. His father works fulltime in a computer firm and his mother remains at home. He shows insecure-avoidant attachment pattern (A). The

attachment pattern is measured by the MCAST (Green 2000).

Beside René there are two persons involved: Lisa, a four years old girl from René's preschool and a member of our research team, who conducted the conversation with the two children. The following interpreted episode refers to the end phase of a collective processing of the task. René, Lisa and the member of the research team invented a familial system of description: The small ladybugs represent kid-bugs and the big one mom-bugs, dad-bugs, or parents-bugs. All little ladybugs lie around the carpet.

The guiding adult has put a triplet of 3 red ladybug cards in the centre of the carpet:

Figure 1: Triplet of ladybug-cards

René has mentioned that the ladybug with 19 little triangles and Marie has mentioned that the bug with seven big triangles does not belong to the group:

René comes up with the solution that *both* bugs with many and small triangles do not belong. His justification has two aspects:

- Comparing the figures of the small and the big ladybug-cards, he concludes, that the bugs of the small cards should also only possess small figures on their tops.
- The two cards with the many and small triangles cannot exist in the system of the cards at all.

If one understands the figures of the ladybugs to be people's hands, René's argument is that parents do not have hands of the size of kids, this is impossible. They cannot be parents and children "at the same time", as he says. With respect to the three aspects of mathematical creativity mentioned one can conclude: René's solution is based on a surprising choice of a familial system of description for the comparison of the ladybugs. Hereby he does create a somehow non canonical combination of size and family-members. Furthermore on the level of speech, he expresses this unusual choice by a linguistic adaption of the size of ladybugs by using a familial metaphor. He says that the big ladybugs would be "already big". The wording of "big" can appear in the size-system of description and in a familial system of description. The guiding adult seems to have difficulty in comprehending René's approach. Possibly she shares Marie's solution. So she asks René for an explanation two times ('why' and 'what do you mean by saying this'). With respect to the interactional setting it is René, who takes the part as the competent partner and explains his position to his counterpart. In this situation he presents a very deep argumentation as the toulmin scheme shown:

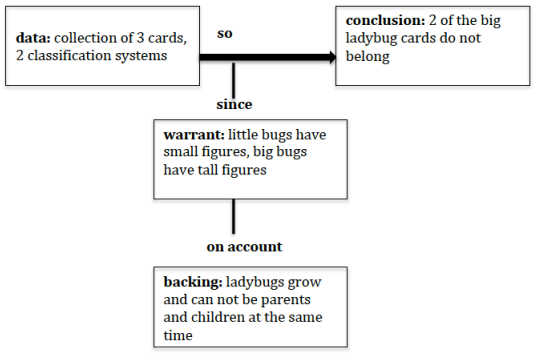

Figure 2: Toulmin scheme of René's argumentation

In René's argumentation one can see that he connects the first part (finding family members, making groups of little ladybugs because of their relationships regarding their spots (amount, shape) or their colors) with the second part (separate big ladybugs, which do not belong together) of the mathematical situation. So he transforms two operations into one under disregarding the detail that only one ladybug does not belong to the triplet. In the end the guiding adult forces an agreement and asks if it is all right for René to take Marie's solution.

Case study two: Nina's solution process in the 'Ladybug-situation'

Nina is a five years and five month old girl who lives with her mother in a German major city. Her parents are divorced. Nina shows secure attachment pattern (B).

Beside Nina there are two other persons involved: Samira a five years old girl from Nina's preschool and an accompanying person from the research project. At first the children and the adult person have dealt with the little ladybugs. They have discovered various families of ladybugs where the color and the number of spots determine to which family a ladybug posse. At the end of this phase Nina mentioned that all ladybugs of the same family are grown in the same stomach. After that the little ladybug cards are moved to the edge of the table. The presented scene begins with the second part of the ladybug situation: The guiding adult has put a triplet of big yellow ladybug cards on the table and asked: Which one does not belong?

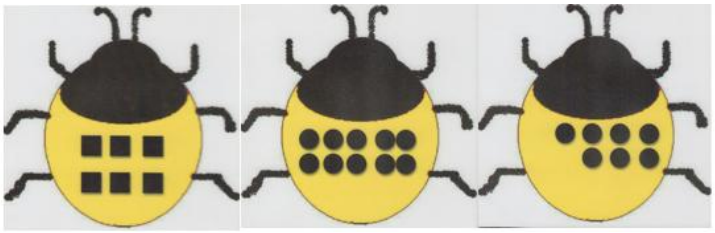

Figure 3: Triplet of ladybug-cards

Nina comes up with the solution that two big ladybug cards are wrong and only one ladybug card is right. The right one has ten circles on his back, which correspond to the number of little yellow ladybugs. She creates a non-canonical solution. Her surprising choice includes aspects of relationships similar to a mathematical function: The number of elements on the back of the ladybug and the quantity of

babies, which appertain to the bug. Each spot represent one of his babies. She extends her functional relationship by determining the color as a feature of the functional relationship between the ladybugs: The big yellow bug can only have little yellow ladybug kids. By reconstructing Nina's argumentation it is obvious that she connects the first part of the mathematical situation (finding families/groups of ladybugs) with the second part (which one does not belong?):

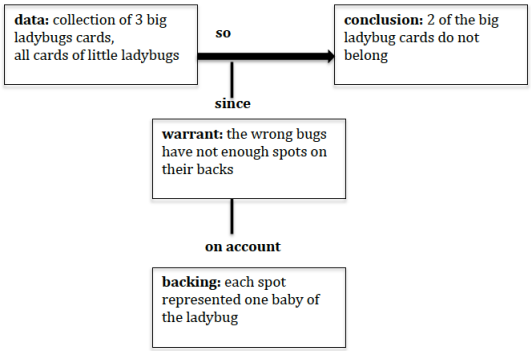

Figure 4: Toulmin scheme of Nina's argumentation

Nina disregards the detail, that only one ladybug does not belong to the triplet. On the level of speech Nina is able to formulate her non-canonical solution with appropriate expressions, so the identification of the little yellow ladybugs as babies of the big yellow ladybug can be seen as a linguistic achievement to describe the functional relationship between the little and the big ladybugs in an adaptive way. Samira shares Nina's solution, but the guiding adult seems to be surprised. Similar to René, Nina has to give an explanation of her one card solution, what is expected by the guiding adult.

Summary and Prospect

Two cases of the MaKreKi study were discussed and it has been showed that the approaches mentioned in section 2 are useful to describe creative mathematical processes of young children at kindergarten age. The cases illustrate that children who can be seen as mathematical creative are able to change the perspective on a mathematical task, although a clear instruction from the guiding adult focuses another perspective. Nina as well as René offered an one card solution instead of the expected two card solution, because of the connection they have discovered between the two parts of the ladybug situation. Comparing René's and Nina's solution with solutions of other children in the erStMaL project (erStMaL project examines the mathematical development in early childhood), who attended also in the ladybug situation, exhibits their mathematical creative potential. Other children do not discover a kind of connection between the two parts and offer only the expected one card solution. Instructions may have a strong impact on children's interpretation of mathematical tasks and so only mathematical creative children are able and have the confidence to see more possibilities and perspectives than the canonical solution, which is forced by the comments of the guiding adult. Both children are initiators

(Grossmann 1984) of their non-canonical solutions. In case of Nina as a child with a secure attachment pattern this observation is in accord with Grossmann's results. Following Grossmann children with an insecure-avoidant attachment pattern like René are often less autonomous in play situation. In the mathematical situations of play and exploration this is not the case for René. He often is the initiator, too. So the behaviour of children in play situations may be linked to the context of the situation as well as to their attachment pattern. René as a mathematical creative child is able to take the part of the competent partner because of his mathematical creativity and his great interest in mathematical situations. Therefore further research is necessary, in case of René it would be interesting to analyse his behaviour in play situations without a mathematical context to see if he reacts similar autonomous like in the ladybug situation or more reserved like other children with an insecure avoidant attachment pattern.

From the adult's perspectives, which are in line with the didactical design pattern of the ladybug situation, there were canonical solutions by comparing the colors of the ladybugs, the shapes on their backs or their number of spots on their backs. In this perspective it is easy to determine two ladybugs which belong together because of their equal features, which the third ladybug do not posses. From the situational perspective the final definition of the problem situation is a matter of the negotiation of meaning in the concrete situation of interaction. In both scenes the children provide a non-canonical solution by finding equivalence classes between the big and the small ladybugs. The two examples show that some kind of instructions especially instructions which expected arguments and reasons supports mathematical creative potential of young children. Regarding the construction of children's mathematical thinking it is important to understand and honour also their non-canonical solutions, which might be the first steps of the development of mathematical creativity in early childhood. Additional analysis of young children's non-canonical solutions can help to describe, understand and identify the mathematical potential of young children. Therefore a conceptual framework has to develop which examines the cultural and the situational impact as well as the influence of the attachment pattern on the development of mathematical creativity in early childhood and connects with creative mathematical abilities of young children.

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