LEARNING WITH PLEASURE AND SENTIMENTS – TO BE OR NOT TO BE?

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The phrase "to be or not be" in creative education is deeply connected with the main question - what exactly a student is able to understand immediately and especially what he does not. It demands constant verifying of a student's sentiments and feelings. Having such sure notions or at least some commonly acceptable ideas we are to move to structuring of the situation. We must also give some explanation about the way that student should be instructed in order to help manage the situation. And an optimist would also immediately add the condition "sine qua non" – the student ought to learn with the pleasure.

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THE LEARNING WITH PLEASURE AWAKENING SENTIMENTS

And by "pleasure" what the author means in this case is not the typical pleasure, which we try to achieve simply by saying „oh, how brave you are", but more profound pleasure that you, for instance, must unavoidably feel after you have finished building a house (or cracking the problem, that you were not able to yesterday, or a week or even a year before).

That problem - what exactly a student is able to achieve and how to prepare him for that - is the most subtle question, because we may so quickly come to a conclusion that even in early years a person can achieve practically everything and understand even more – when properly instructed.

There is a common view that the student can understand, even invent, everything what might be expressed using very few words, avoiding special terms and tricky ideas. For the tricky ideas the mankind has founded the universities.

First of all we strongly believe that a student – just like any other normal person – especially likes to do what at least for a time appears to him as especially difficult and, most importantly, attractive. Many problems of that kind are the problems dealing with numbers, arrangements of numbers and combinatory.

Instructing a person to start solving something that appears attractive for him is not very difficult. The next step is assisting him – or maintaining the situation. It helps a lot that a normal student is used to operate with numbers, even with these huge ones. Just as many people, it seems, believe even more that, consciously or not, a person is eager to somehow demonstrate for all those concerned, that the difference between him and any adult is considerably smaller than many try to believe or have experienced it to be.

So let us first believe: a normal student is not afraid of any numbers and, first of all, he is very eager to demonstrate that he is not that little. He is also able to proceed constructively; quite often in astonishingly logical way, and instead of making long theoretical reflections he prefers concrete actions.

We would like to discuss with you how to deal with some problems. When speaking about solving them, we strongly believe that we would be able to make some insights about effectively stimulating the challenging and inspiring process of reaching a solution.

The first problem the author will present, could, we believe, be presented in practically any age – after some instruction, of course. We frankly admit that we possess some experience how to instruct and prepare for the process of solution starting from persons, who are 10+ years old.

Besides the original formulation, we will also try to present to the reader that which might appear to him more attractive and then we will report our attempts to reaching the solution for it. We believe that attempts to formulate the problem more attractively so that more people would be interested are good for people of other age groups, besides children.

As the author has seen many times, if something is suitable for a normal child then the same problem is suitable for any normal university student as well.

THE USUAL REPRESENTATION OF PROBLEM

The standard formulation asks us to find the minimal positive integer which is possible to represent in two different ways as a sum of three addends in order that all these six addends mentioned would be all different.

Find means detect, and detect is something what Mr. Sherlock Holmes with Dr. Watson used to do every day (and with great success).

This is not a difficult problem, but nevertheless let us look for some „more attractive representation“, as we just did by adding two popular names that for a normal child even in the United Kingdom may not add anything worth mentioning.

What could be proposed as an attraction to the child as a multitude, which might be split into three parts in two different ways? What a multitude of the kind does the child sees and enjoys every day. We may choose and propose some possibilities of that sort – e.g., it could be the clouds in the sky. Any natural number may be represented as the separate group of clouds. So we may have the whole set of clouds in the sky divided into separate groups of clouds. And then we may ask how to apply that language in order to present another representation of the very same natural integer or the same multitude of clouds. How could we do that? The answer is simple, because the winds are blowing and when wind is blowing the skies are changing their configuration. We might point to your attention the fact that we silently assume that no cloud disappears, but the multitude of skies may still easily change its configuration.

As told before, we deeply believe – because such is experience of our life - that any smart child is able to count prominently and, moreover, is eager not only to add numbers, but also split them. Also, he would be very fond of us if we were able to present it in good style.

So, in order to start properly with finding a solution, we might ask him for an example and the child will immediately respond. The child in any age is strong when answering a concrete question. So, we may ask him: do you know such a number, which possess the miraculous ability and could be split in two different ways into sum of three different addends, so that all six numbers mentioned would be different? The examples are not difficult to find, they are there, they are everywhere to be found, for instance, such is the very regular number 111 and just as that regular, but more usual because of its size, is the number 21:

$$111 = 100 + 10 + 1 = 104 + 4 + 3,$$

$$21 = 1 + 9 + 10 = 2 + 4 + 15.$$

Now it remains only to detect the smallest number among all such “chameleonic“ integers.

Children are natural investigators by their constructive nature and are always curious to detect what integer is he first to lose that property?

The author has gained some experience by teaching that problem, especially at the very end of solving it. Here, the end of solution is rather interesting because at that place some kind of abstract reasoning or something of proof-like thoughts would be highly welcomed. But, according to the author’s experience of the recent year in Grade 3, that abstract end was taken more as the bare statement and as the authority of the teacher. Because the teacher told that the process must naturally take stop at some number and, moreover, the teacher indicated we will find that number when we constructively wouldn’t be able to have that double split into these three addends. Frankly speaking it was the reasoning of the type: it must be because it is. And nothing more. It might be enough in the Grade 3. And in the Grade 4 at the same school on the very same day it was possible to achieve much more: evidently, approximately a good third of pupils present were able to understand even that abstract philosophical part or the proof that 10 is that number. After gathering the concrete experience with numbers the proof that with 10 such double split is impossible followed. The usual words of proof or abstract reasoning were pronounced. These words were pronounced and they were understood by a good third or possibly even a half of audience.

These words were: take six smallest positive integers, these are

$$1, 2, 3, 4, 5, 6.$$

Because their sum

$$1 + 2 + 3 + 4 + 5 + 6 = 21$$

(and this is more than two times bigger than 10) so the number 10 clearly couldn't possess such double-thrice split or representation. It means that the smallest integer with that double split into three addends might be 11. And 11 is indeed such a number because 11 can be written either as

$$2 + 4 + 5 = 11 = 1 + 3 + 7$$

so 11 is indeed the smallest such integer and the rest follows.

After the natural joy of the teacher that his students are so smart, the "repetitio est mater magistrum" followed. Because these pupils of the form 4 also understood that they have achieved something – and then it was high time to repeat that main sentence, which at the same time is the short proof, several times. For the first time it was done immediately after and for the second time - after some 5 minutes. Then again the clouds were mentioned and the wind that scatters clouds remembered.

The second example is about the magic of a suitable instruction. This ought to convince the reader that the subtle advice might indeed appear to be extremely magical.

ONE ROTATION IN THE SAINT-PETERSBURG'S PRORBLEM

which might for the time be connected with our plans to plunge 9 numbers

$$15, 16, 17, 18, 19, 20, 21, 22 \text{ and } 23$$

into 9 entries of 3 x 3 table in such a way that for each pair of numbers, the entries of which happen to share common side, the sum of both integers in any pair is always different. Actually, if we are reading the text of the problem, it is indeed not immediately understandable why this problem might be proposed – and was proposed – in the Saint-Petersburg math Olympiad. And it was not presented in the lowest grade either. What could be reason for that?

Looking back to the proposed problem, we must mention that it is completely clear that this problem is not too difficult. We can simply start with the dull attempts of plunging numbers; after several attempts each child would have an unavoidable feeling that he must proceed applying some system of plunging and by no means letting the things run chaotically.

Let us not be that dull and write down the numbers according to the most common way starting from the smallest one up to the biggest one or

15	16	17

18	19	20
21	22	23

Remembering that we must pair wisely to add all the numbers sharing the common side we start with that adding of neighboring integers getting consequently:

15 + 16, 16 + 17, 15 + 18,

We immediately notice that we are not able to fulfill our requirement and this is already bad because we have just started with calculations and yet have got the number 33 twice already. So the most common manner proved itself to be unsuitable.

Now we would like to ask: is it possible and how should we proceed in order to provide an effective support for our pupil, yet formally doing very little. What kind of impulse might we give? Are there some wonderful means for that?

The effect of achieving much by undertaking practically nothing is quite memorable - the child in each age is very fond of rapid success and, moreover, he never forgets it. Rapid success and how to deal with it, all this might be a really valuable chapter in the theory of effective and especially of joyful teaching.

In our situation all that possible effectiveness is due to the idea of slight rotation of the whole table around its centre. Nothing more is needed at this time. After that very small rotation the idea comes to mind: help the diagonals be at least at bit more like the horizontals. After that the same standard plunging will prove itself to be a solution of the problem. But before let us help the formal diagonals looks like the horizontals and then it's cleared that the best mean for that is the 45º rotation. After the back-rotation we'll have the answer also in the usual form or

15	17	20
16	19	22
18	21	23

Let us note that now indeed all 12 of neighboring pairs prove to provide different sums:

15 + 17 = 32, 17 + 20 = 37, 15 + 16 = 31, 17 + 19 = 36, 20 + 22 = 42, 16 + 19 = 35, 19 + 22 = 41, 16 + 18 = 34, 19 + 21 = 40, 22 + 23 = 45, 18 + 21 = 39, 21 + 23 = 44.

You might sometimes be lucky enough and prove yourself to be able to express infinitely much by saying practically nothing – as just enjoyed in our previous case when advising the rotation. And this was enough for the complete success when dealing with the proposed problem.

LAST CANGAROO PROBLEM IN STUDENT'S SECTION OF LITHUANIAN ORIGIN

According to our opinion, this is the problem that could be shown for any child which can read the text in order to illustrate the truth that the math problem can be ready for reading, yet not necessarily immediately ready for the solving. There are some sorts of problems who are as nice as the fairy tales in the literature. According to the author's opinion, this expresses something that could be formulated using the words, that the mathematical problem might be more that barely mathematical problem. Such a formulation might appear unusual in the math education but, on the other hand, the expressions of that sort and kind are well enough known in art, literature and in other branches of human activities (including sport).

Every cat in Wonderland is either wise or mad. If a wise cat happens to be in one room with 3 mad ones it turns mad. If a cat happens to be in one room with 3 wise ones it is exposed by them as mad. Three cats entered an empty room. Soon after the 4th cat entered, the 1st one went out. After the 5th cat entered, the 2nd one went out, etc. After the 2012th cat entered, it happened for the first time that one of the cats was exposed as mad. Which of the cats could both have been mad after entering the room?

(A) The 1st one and the 2011 one (B) The 2nd one and the 2010th one (C) The 3rd one and the 2009 one (D) The 4th one and the last one (E) The 2nd one and the 2011 one

This is the usual Kangaroo stile for the answers – 5 alternatives are proposed with exactly one of them being the right one.

From the first glance, the proposed task does not appear to be easy; moreover, it introduces some structure in world, likes in fairy tail. That structure is exotic, but the play is arranged following some predictable rules. From that point of view the presentation of the problem that is rather long for the Kangaroo problem is not too long because of the magic of the presentation. As the whole it looks more like fairy tale but not a mathematical problem. The heroes mentioned therein, or wise and mad cats, immediately presuppose tension. The described possibilities of transition and exposition guarantee additional excitement for the fantasy of a child. So, let us all try to touch or otherwise at least mention all remarkable circumstances that could inspire us to recommend these problems for radically young minds. As mentioned before, simply by reading the text of problem we might be predisposed to think that the problem possesses a mystical flavor. At the same time, the action lasts for quite a while and for a remarkably long time one cat enters and another leaves. So, as told, cats are not only filling the scene, but also only the cats are performing on that scene – following given rules. These cats, a lot of them, forms a long line of cats, are acting under strictly fixed rules making that magic situation partly predictable and considerably more attractive at the same time. But we must mention that – just like in fairy tales – the circumstances that allow the hero to win or the situation take the suitable turn must be analyzed rather carefully.

That special turn of the situation in our case the is the moment, when for the first time the child must understand, that he deals with a situation when he sees the three wise cats and the fourth, that is mad, enters.

In many of such cases, when dealing with problems with a bit of magic, attractive and yet not that easy to start solving, we may raise the following question: what is the first remarkable observation that the child could make regarding that given situation? Such first observation very often in child's mind very often appears later to be the main idea of solution or proves itself otherwise to be very important.

That is, the first thought of child dealing with good problems very often is excellent or otherwise worth mentioning.

And, like in every fairy tale, the child is eager, sometimes even unconsciously, hoping for the happy end, which in our case also includes the essential understanding of the whole situation. Usually from that moment the desired solution follows. Sometimes this is where we, who believe that us and the child as well, are able to understand the whole process of solving so clearly, that we start to think that after suitable instruction this problem could be so good for many or even for all who might feel interested or otherwise involved.

So what could be the first remarkable observation of the child in our interesting concrete solvers dance of understanding and explaining of the problem? What could be the first words that the child might find worth pronouncing after some consideration? Sometimes the first words are words that describe or investigate some of „peculiar cases".

What case might be the worst in our situation? First, the one who eliminates the situation to which the problem is devoted for.

Such situation might of course be the situation „with three consecutive mad cats happening somewhere in that long long line of 2012 cats". The situation with 3 "consecutive" mad cats would completely and definitely change and influence the situation. These 3 mad "consecutive" cats would make all other entering cats to be all-mad even if before entering they weren't. We believe that the clear understanding of that might be the first possible serious thought of the child and at the same time the first remarkable stone upon which the solution will be based. The situation with three consecutive mad cats is clearly that "drastic" case after which as end as predicted is the problem is already impossible. We dare to imagine, suppose and believe that the clear notion of that situation is not too complicated for any sensible child of every possible age.

So let’s assume that the child clearly understands that three consecutive mad cats somewhere in that long line of cats eliminates the outcome as described in the problem and this is what the child must always take into account.

The child is also expected to be aware of the following circumstance, which is similar to every good fairy tale, and that is that the most serious things are often revealed in the last few sentences. In our case, the most optimistic reality occurs, when a cat is for the first time exposed as mad only after the 2012th cat enters. It means that it never happens before. What are the circumstances that allow the delay for so long?

Returning to the initial situation - the child sooner or later makes a decision to examine the situation with three first cats more carefully (immediately before the fourth one appears). Three (or even four) cats won’t be too much for any child – if they all are not from the very same flat.

So, let the child examine the situation with the first 3 cats. If all of them would be mad, oh, this would bring the bright child to already regarded bad case. That is quite clear because it’s had been already regarded.

We will repeat it for the sake of completion. So if the 4th cat is mad, then everything remains as bad as it is. And if the 4th cat isn’t mad when entering, it would very quickly be. So, it is of no importance which cat is leaving. It quite obvious, that the 1st cat, that is leaving, is mad. And so the situation with three mad cats, if it once occurred once, remains for all times. So, it is clear that all 3 of consecutive cats cannot be mad.

Now, the child could regard the “opposite situation”. What if all three consecutive initial cats were all wise ones? If all of them were wise ones, then the fourth cat, when entering or before entering, couldn’t in any way be mad, because otherwise, immediately after entering, he would be exposed as mad. But we are informed in the text of problem that this happened much later. So according to the fibula of the whole story if three initial cats are all wise and if in no case madness was exposed until 2012th cat entered, then this means that „intermediate“ entering cats were all wise ones. And we understand now that 2012th entering cat is supposed to be mad – then everything would happen as described.

And what then? The situation is indeed possible but none of proposed 5 alternatives – none of them, we patiently repeat – suites the situation. But this is not necessarily required, because in the text of problem we read: „which of cats could both have been mad after entering the room?“ And “could” doesn’t mean “must be”, does it?

It allows us (and the child, only sometimes much more quickly) to drop off both extremely situations of the type: all three initial cat “mad” or all of them “wise”. In the second case the situation is possible but none of mentioned 5 choices as answers suits there.

Now we are expected to regard the „intermediate situation" when not all 3 initial cats are either all wise or, to the contrary, all mad. That means that among these 3 first cats there is at least one mad and at least one wise cat. We must admit that after the 4th cat entering (and the 1st leaving) the situation couldn't be allowed to go to extremes. These cases are already investigated.

The 4th appearing cat must create the situation "2 versus 2" situation because it is still too soon for the "3 versus 1" situation. It means that earlier, before the 4th cat entered, the situation was precisely "2 versus 1". Now it just remains to be mentioned that all situations "2 versus 2" are contained in the following line -

MMWW, MWMW, MWWM, WMMW, WMWM, WWMM.

Now we must realise that the 5th in that line must be of the same sort as that 1st one, otherwise the "3 versus 1" situation would appear and for that it's still too soon. Similarly, the 6th must be of the same sort as that 2nd one and so on until the entering of that fatal 2012th cat happens. Entering, the 2012th cat must change the situation to one, which has never happened before, or for the first time "3 wise cats versus 1 mad one". It means that the 2012th entering cat must be wise with previous cats modulo 4, that is these 2008th, 2004th,…, 8th and 4th being, to the contrary, all mad ones, and with 3 previous cats before all of them, creating the situation "2 wise cats against 1 mad". Such are exactly the situations MWW, WMW and WWM. Needless to say that only that initial "WMW case" suits as exactly one of these proposed answers, namely, that which name is B.

The problem itself is created by a Lithuanian problem composer dr. Aivaras Novikas, once an IMO bronze and silver medalist.

In the Kangaroo competition in Lithuania, that problem in 2011 was proposed to 4708 competitors. The data of the answer distribution is given below:

No answer	Answer A	Answer B	Answer C	Answer D	Answer E
1964	496	377(right)	521	958	392
41.72%	10.54%	8.01%	11.07%	20.34%	8.32%

As mentioned before, the right answer was that least popular one.

The author has some experience in explaining the idea of how to manage the solution of a problem without saying anything, only making two "moves".

That experiment was repeated several times with different audiences, starting from students of Grade 3 and 4 and finishing with students of Vilnius University. The problem itself was reported to the author by the famous Latvian professor, problem composer and educator Agnis Andžans.

Firstly, the lecturer asks how to distribute 12 identical coins between two purses putting in one purse two times as much coin as in the other. The clear answer – 4 and 8 – in all audiences was given immediately, without the slightest delay.

And now the lecturer kindly asks to change slightly the numbers and take 10 instead of 12 and achieving exactly the same – that in one purse there would be two times more coins than in the other. It needs to be added that to divide coins into parts is not permitted.

It is good to let the audience feel that the situation demands something that might be called legal, yet not straightforward action. It is very good when the lecturer is able to behave in such a way that the audience doesn't lose interest as well as hope to achieve the solution. Then the lecturer is able to announce that he will prove that the audience is genial or at least so infinitely smart that he now will make some two "moves" and some people will immediately, unavoidably present the answer.

And so the lecturer makes the two "moves" – with the first movement imitating some rather small circle and with another – another circle, at that time possibly large one.

Immediately, some listeners tell you what you are expected to do: One purse with 5 coins must go in another purse with the remaining 5 coins. Nothing more is needed and everything is as clear as day.

The author spent many years teaching how to solve simple and yet not that simple problems, has written several books in three languages trying to report some of his impressions. Some of these books are mentioned in the list of references.

References

Kangaroo problems for the Year 2012

Mazanik S.A. (together with Barabanov E.A., Bernik V. I., Voronovich I. I., Kaskevich V. I.), *Problems of the Minsk city math Olympiad for younger grades*, Minsk, "Konkurs" association, 2005;

Problems of Saint-Petersburg Mathematical Olympiad 2010, Editors: Berlov S.L., Ivanov S.V., Khrabrov A.I., Kokhas' S.P., Petrov F.V., Smirnov A.V., SPb, Nevski Dialekt Ed., 159 p., ISBN 978-5-7940-0147-1;

Kašuba R., *Once upon a time I saw a puzzle,* Part I, Riga, University of Latvia, 2008, 59 p., ISBN 978-9984-45-045-2;

Kašuba R., *Once upon a time I saw a puzzle,* Part II, Riga, University of Latvia, 2008, 67 p., ISBN 978-9984-18-102-8;

Kašuba R., *Once upon a time I saw a puzzle,* Part III, Riga, University of Latvia, 2009, 88 p., ISBN 978-9984-45-13;

Kašuba R., *Kak reshat' zadachu, kogda nie znajesh kak* (*How to solve the problem when you do not know how*), Moscow, 2012, Prosveshchenije Publishing House, p. 174, ISBN 978-5-09-023583-9.