STUDENTS' PICTURE OF AND COMPARATIVE ATTITUDE TOWARDS MATHEMATICS IN DIFFERENT SETTINGS OF FOSTERING

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The fostering of promising students in mathematics can be done in different ways. By the results of a survey in different settings we try to give advices for a suitable way of selecting and fostering potentially mathematically gifted students. Therefore we look at the students' picture of and comparative attitude towards mathematics.

Keywords: mathematical giftedness, fostering, selection, beliefs, attitudes.

INTRODUCTION

There are different ways of selecting promising students for reasons of fostering (in mathematics) consequently leading to different settings:

One way is to offer additional courses or materials for volunteers, which means that the *interest* of the students is the main motivational force. The general importance of this aspect was already identified by Kruteskii (1976), for example, who lists a "general synthetic component", a "mathematical cast of mind", i.e. the necessity of an aptitude and interest for mathematics in order to be successful within this subject. He states "It is expressed in a selectively positive attitude toward mathematics, the presence of deep and valid interests in the appropriate area, a striving and a need to study it, and an ardent enthusiasm for it" (Kruteskii, 1976, p. 345). The mathematician Kurt Devlin appears to have the same opinion when he declares that "whatever it is that causes the interest, it is that interest in mathematics that constitutes the main difference between those who can do mathematics and those who claim to find it impossible" (Devlin, 2000, p. 275).

Another way is to rely on teachers' choices, so it is not only the students' interest driving them to the courses but also a *qualitative* external selection process. Amongst others, this was already pointed out in Linke & Steinhöfel (1986, 1987), for example.

And a third way would be a *quantitative* procedure by testing the students (see Nolte, 2012, Kontoyianni et al., 2011, Hagborg & Wachman, 1992, Bittker, 1991, Löser, 1985, amongst many others) or simply choosing those with the best marks in mathematics (see, to some degree, BSHA in the next section, for example).

In practice, selection processes often are designed as multilevel mixtures of these three ways (see Wagner & Zimmermann, 1986, for example). From a psychological perspective the sketched ways of selecting represent different aspects of the causality between a giftedness potential on the one side and performance or assessment,

respectively, on the other side.We already discussed these aspects in Brandl (2011) and Brandl & Barthel (2012) and gave some first results concerning theoretical insights and corresponding practical consequences there.

In this paper we want to look at different settings of fostering again in order to compare the students' picture of and their comparative attitude towards mathematics. This was already done for just one setting in Brandl & Barthel (2012) where we concluded that there are strong hints that mathematical gifted students should be fostered separately from their (old) classmates. The reason was/is just their absolutely different attitude towards mathematics and therefore the absolutely different learning atmosphere they deserve or create, respectively, for and/or by doing mathematics.

THE DIFFERENT SETTINGS

We did our empirical survey in three different settings. The first one is a German boarding school for high attaining students, where successful candidates are chosen by very high assessments, representing the third way sketched in the introduction ("*quantitative* selection"); a mixture of mostly the first ("*interest*") and a little of the second ("*qualitative* selection") way are the fostering courses at our university. As a contrast to these two settings we added a "normal" class from a regular higher secondary school.

Boarding school for high attaining students (BSHA)

In order to be selected for this German boarding school for high attaining students, applicants have to fulfill several requirements: in the main subjects – German, Mathematics, a foreign language and natural sciences – students need to achieve at least the mark "good"; an average mark "good" in the last two school reports; a general IQ score of approximately 130 points in the intelligence structure test I-S-T 2000 R (Liepmann et al., 2010) and a successful participation in a two-day assessment center concerning social skills. The sample group consists of 8 classes with 113 students from 11^{th} and 12^{th} grade (which represent almost all students within these grades). Their age ranges from 14 to 18 with an average age of 16; the number of boys and girls is balanced. On a scale from 0 (fail) to 15 (excellent) the median of their marks in the subject mathematics in the last school report is 12.

Fostering courses at university (FCU)

The offer of special fostering courses in mathematics at schools or at universities is quite common and has a long tradition. However, there are different kinds of selection processes for these courses, as for example the teacher's choice, special tests for mathematical giftedness, just students' interest or a combination of those, see Nolte, M. (2012) amongst many others, for example. At the University of Passau these kinds of courses have been offered for several years. The participants are split into at least two groups consisting of 13-to-14-year-olds and 15-to-16-year-olds.

There are no tests for their IQ or mathematical giftedness; mostly the students are attending the courses just because of their own interest in the subject. Unfortunately, the numbers of participants (and so the samples) are rather small: 9 "younger" and 6 "older" ones. Typically, the participants of the "higher" course are former participants of the "lower" course.

Regular higher secondary school (RHSS)

In order to compare the results from the data collected at the BSHA and the FCU with a "normal" school, the identical questionnaires were given[1] to 23 students of a class in a 11th grade at a regular higher secondary school ("Gymnasium"). The average age is the same as at the BSHA and the sample consists of 9 female and 14 male participants. Of course, their average mark in mathematics is much lower than that of the sample at the BSHA.

METHODOLOGICAL REMARK

It has to be pointed out that the number of participating students in the survey is different for the three settings BSHA, FCU and RHSS. So, the conclusions drawn from the evaluation of the three groups cannot be interpreted in a general way based on representative quantitative empirical data. However, as *all* students from 11th and 12th grade of BSHA and *all* students from the courses of FCU were chosen, the results may still be representative for the different settings in a *qualitative* way; and as *all* students from one whole class from the 11th grade at RHSS were chosen, the RHSS sample can serve as a representative *qualitative* counterpart. The more or less qualitative results of this survey can so be seen as a starting point or a pre-study, respectively, for a deeper and representative quantitative research.

PICTURE OF MATHEMATICS

The questionnaire given to the students was identical[2] for all three settings. Amongst others it covers questions concerning the notion of mathematics; answers could be given on a bipolar 5-point Likert-type scale. The categories on the horizontal axis in the following figures are:

1: Calculations, rote learning, algorithms

2: Problem solving, proofs, riddles

3: Creative process of building a theory

4: Tinkering[3] with beautiful things

5: Competence to get a safe and well-paid job

6: Modelling, language of nature

7: Symbols, numbers, patterns, formulas, abstract structures

8: Texts, theorems, laws

9: Something for freaks

Boarding school for high attaining students (BSHA)

For the BSHA we got the following profile:

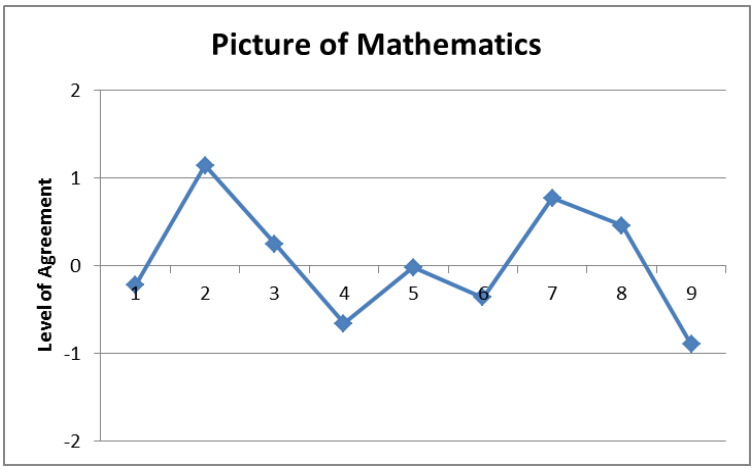

Figure 1: Picture of Mathematics (of 11th and 12th grade altogether) at the BSHA.

The diagram shows an ambivalent picture of mathematics: though the students of the BSHA do not believe that mathematics is something for freaks (9) but is rather problem solving and proofs (2), patterns (7) and theorems (8) instead of rote learning (1), however, they rather don't see it as a tinkering around with beautiful things (4) or the language of nature (6). This might be caused by the fact that the lessons at the BSHA are bound to the general curriculum necessary for the central-posed A-level exams.

Fostering courses at university (FCU)

Participants in the FCU answered like this:

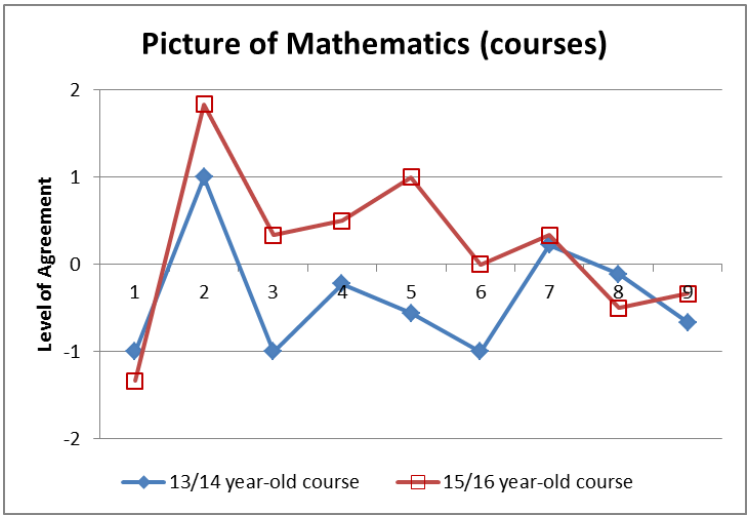

Figure 2: Picture of Mathematics of the 13-to-14-year-old and 15-to-16-year-old ones at the FCU.

Whereas the curve for the younger ones is quite similar (despite being a little bit more negative) to the one at the BSHA, the older ones (fitting better to the group at the BSHA and the RHSS) show a generally more positive picture except for the numbers 1 (rote learning) and 8 (theorems). The main difference to the BSHA diagram can be seen in the far more positive values concerning problem solving (2) and tinkering around with beautiful things (4). This surely reflects the freely chosen mathematical contents of the lessons and the way of dealing with mathematics within these courses.

Regular higher secondary school (RHSS)

The 11th grade at the RHSS delivered the following picture:

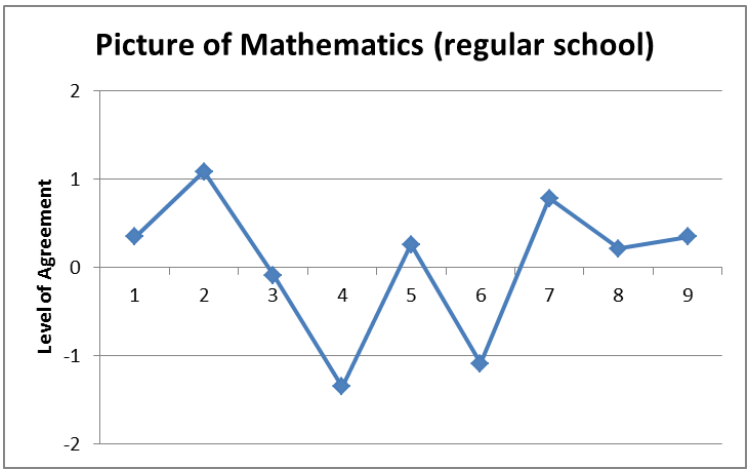

Figure 3: Picture of Mathematics of an 11th grade at the RHSS[4].

First, the line in figure 3 is completely different to the line of the older ones in the FCU, illustrating the diverging look at mathematics of interested students and those who are not. Second, although being (surprisingly) similar in most points to the diagram in figure 1 for the BSHA (items 2 to 8), there is a tendency to see mathematics as some kind of rote learning process (1) and something for freaks (9).

COMPARATIVE ATTITUDE TOWARDS MATHEMATICS

The data for the different attitudes towards mathematics came from answers on a 7-point Likert-scale. The students had to rate their own attitude towards mathematics as well as estimate the one of their classmates at the current school (or the university courses, respectively), their classmates at their – for BSHA: old – school (dispensed for RHSS) and society as a whole. The categories on the horizontal axis in the following figures are:

1: ugly, terrible	------	aesthetical, beautiful
2: hard	------	easy
3: boring	------	interesting, exciting
4: incomprehensible	------	logical

Boarding school for high attaining students (BSHA)

The results are shown in figure 4:

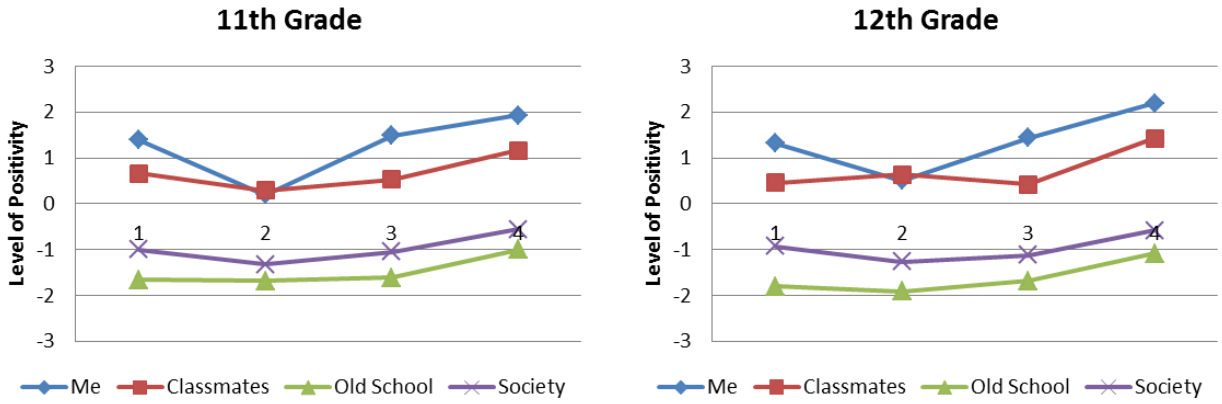

Figure 4: Attitude towards Mathematics, separately for grades 11 and 12, at the BSHA[5].

All students rated themselves more positively than the rest (classmates, old school, society). This might suggest a sort of "I'm sure that the others don't find math as interesting and nice as me"-thinking. Furthermore, there is a remarkable gap between the estimated attitudes of the current classmates compared to the old ones. Of course, all members of the sample were selected because of their very good marks in mathematics. Nevertheless, the difference of the two lines shown in figure 4 indicates that they must have felt like strangers in their old classes. Additionally, the attitude towards mathematics in society is always rated quite low, but higher than that in their old class (see also Brandl & Barthel, 2012).

Fostering courses at university (FCU)

The attitudes of participants of the FCU are shown below:

Figure 5: Attitude towards Mathematics, separately for the 13-to-14-year-old and 15-to-16-year-old ones, at the FCA.

First of all, just like for the BSHA students, the students themselves and their course-mates show a positive attitude towards mathematics in contrast to the estimated

negative one of their "normal" classmates and society. The gap between the current course-mates and the "normal" classmates is quite huge compared to the analogical gap (between "classmates" and "old school") in figure 4. Furthermore, the fact that society's attitude is rated more positively than that one of their "normal" classmates shows up again.

Regular higher secondary school (RHSS)

In contrast, at the RHSS the following rating was given:

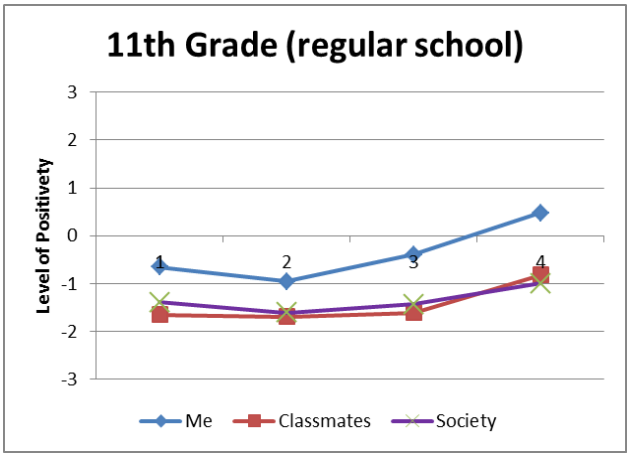

Figure 6: Attitude towards Mathematics at the RHSS[6].

Obviously there is only one slightly positive rating for the individual logical sight ("Me", 4). All the rest is in average negative, which is a big difference to BSHA and FCU. Of course, this results from the fact that now not only the highest attaining or interested ones are asked. Surprisingly, the levels for the estimated attitudes of classmates and society are at almost the same height as it is for BSHA and FCU. So this seems to be very stable. The same holds for the still existing gap between "Me" and all the others, but here the classmates are the current ones. However, the curves for classmates and society hardly differ – a fact that is not the same when asking the exceeding ones (BSHA & FCU).

HYPOTHESES / QUALITATIVE CONCLUSIONS

From the observations in the diagrams from figures 1 to 6 we draw the following qualitative conclusions which may serve as hypotheses for a larger and deeper quantitative study:

- The picture of mathematics of somebody who is interested in mathematics differs essentially from that one averaged over an "ordinary" class; it seems[7] also to be more positive than that one averaged over a "high-attaining" class.
- In general, the beauty of mathematics and the possibility of thinking of mathematics as just a tinkering around are only stated by students who were "chosen" for special courses because of their interest in mathematics. So, on the one hand, this stays in line with the findings in Kruteskii (1976) and

Brandl & Barthel (2012) that the strong motivational force *interest* and the ability for *aesthetical sensation with mathematics* are correlated in some way. On the other hand, it seems as if an institution depending on an official curriculum (instead of a free choice of contents) is not able to establish any kind of playful aesthetical enjoyment by doing mathematics in class for a (often large) group of highly different interested and motivated students.

- There is a gap between the (more positive) attitude towards mathematics of *students themselves* and the estimated attitudes of their *current classmates* at BSHA and RHSS. We interpret this as a sort of "I'm sure that the others don't find math as interesting and nice as me"-thinking in all math classes. This psychological fact may be used by the teacher, as the awareness of the students' individual understanding of their potential in and their attitude towards mathematics can help to establish a more comfortable atmosphere in order to foster the students according to their individual needs.
- The levels for the estimated attitudes of their old (BSHA) / "normal" (FCU) / current (RHSS) classmates and society in all three settings always are at almost the same (negative) height; so this seems to be quite stable. This could reflect the "negative image" of mathematics in society which perhaps is projected onto or adapted by the classmates. Furthermore, this may be a signal that the "mathematical climate" in ordinary math classes at regular (secondary) schools may be not supportive for establishing successful learning processes in mathematics. This is underlined by the fact that in the case of BSHA and FCU the estimated attitude of the old / "normal" classmates is rated even lower than that one of society.
- In the context of actual fostering by separation of the particular students there is also a significant gap between the (more positive) attitude towards mathematics of *students themselves* and the estimated attitudes of their *old* (BSHA) and "*normal*" (FCU) classmates, respectively. It is biggest when looking at the interested ones (FCU).

  Probably the environment of an ordinary math class (like at RHSS) is not suitable and supportive for promoting mathematics as something beautiful, challenging and joy-bringing. Students from BSHA / FCU may have felt / feel some kind of alienated in their old / "normal" classes. Based on these preliminary results, separating them from their old / "normal" classmates seems to be a promising way to give them an appropriate surrounding for performing and learning mathematics.

  However, in individual problem-centered interviews with all the (eight) mathematics teachers at BSHA almost all teachers confessed that the main problem when it comes to the students' performance (especially in mathematics) seems not to be their more or less existing giftedness potential,

but the psychological hindrance of a *narcissistic wound* / *shock* that comes from being confronted with just best-of-students in class and the eventual loss of this status for oneself. So, apart from aspects more or less related to (mathematical) beliefs and attitudes discussed in the previous paragraphs, pedagogical and psychological issues strongly connected with performance and assessment (And not the giftedness potential in the first place!) have to be considered seriously and may even be seen as decisive if a separation/selection is considered.

Hence, additional non-performance orientated courses (as realized in FCU, for example) seem to be suitable settings.

We hope that these preliminary and qualitative results can serve as guiding hypotheses in order to be confirmed in a deeper quantitative research.

NOTES

1. This survey was done in the context of the exam thesis of Christian Barthel, which was supervised by the author of this paper. So the results described within this paper concerning the RHSS are based to some part on Barthel, C. (2011).

2. The questions related to the estimation of the comparative attitudes of the students themselves in relation to other groups were modified slightly to fit the context of BSHA, FCU and RHSS (example: 'classmates' for BSHA was changed to 'course-mates' for FCU).

3. This is motivated from the description/definition of mathematical giftedness in Ruelle (2007).

4. Figure 3 is related to figure 20 in Barthel (2011, p. 53).

5. Figure 4 is partly taken from figure 2 in Brandl & Barthel (2012) and appeared first in Brandl (2011c).

6. Figure 6 is based on figure 26 in Barthel (2011, p. 64).

7. Probably the sample size of FCU is too small to come to a stronger conclusion.

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