ABOUT MATHEMATICAL MODELING IN THE REALITY OF THE CYBERNETIC WORLD

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In this article, we explore the relation between mathematical modeling (MM) and the cybernetic world, understood as any environment produced with digital technologies. We seek to provide evidence that the MM practiced in the reality of the cybernetic world has singular characteristics that interfere in the process of constructing models. Toward this end, we discuss reality from a theoretical perspective, in order to understand the reality of the cybernetic world. This understanding is related to data collected while working with university-level mathematics students as they develop electronic games using the Scratch programming environment. We conclude that the reality of the cybernetic world is one dimension of reality with specific characteristics related to space, time, and plasticity, which opens up new possibilities for using MM in mathematics education.

INTRODUCTION

Research and teaching practices that link mathematical modeling (MM) and digital technologies, pointing to a synergy between the two, are increasingly common (Campbell 2010; Siller & Greefrath 2010; Greefrath 2011; Borba & Soares 2012). According to Kaiser (2011) and Dalla Vecchia & Maltempi (2012), despite the existence of different conceptions of MM, the issue of reality is always present, leading us to assume that there is an intrinsic relation between MM and reality. The nature of reality is a delicate question, however, which is further complicated in the context of digital technologies, generating descriptors such as: reality of the cybernetic world, augmented reality, hyper-reality, virtual reality, mundane reality, physical reality, etc. Thus, we question whether the reality to which MM refers encompasses the reality of the cybernetic world, and seek to identify the specific characteristics of MM in the context of this reality.

Seeking answers to these questions, we present a theoretical discussion regarding reality followed by results of a study conducted with university-level mathematics students, aiming to understand mathematical modeling as it is practiced in the cybernetic world.

REALITY AND REALITY IN THE CYBERNETIC WORLD

In this article, we assume the perspective of reality proposed by Bicudo (1999, p. 31, our translation), understood as:

> a dynamic, temporal, historical whole, perceived in the man-world encounter, not separate from the perceiver, who speaks of it and interprets it, building a web of meanings in the intersubjectivity upon sharing experiences and communicating interpretations.

Thus, reality can be understood as lived reality that occurs in specific times and spaces, constituted in the natural field where all thoughts, actions, and perceptions of the different subjects who live in it originate. We ask, however: Can the cybernetic reality be understood as a dimension included in this reality?

Research by Bicudo & Rosa (2010) suggests that it can. However, considering the cybernetic world as reality requires conceiving of it from a perspective that differs from that of modern science, which speaks of physical, objective reality by referring to the place and time where measurable entities, which can be physically manipulated, are located or placed. As Bicudo and Rosa point out, from this perspective, the cybernetic world cannot be considered real, given that the "whereabouts" of this world presents itself in a different manner. In philosophy, however, we find that the real, and consequently, reality, can have distinct ways of showing themselves, involving, among other aspects, the actual and the virtual (Granger 1994).

The actual can be considered that situation or entity which shows itself to the observer who contemplates the here and now, i.e. that which appears in mundane reality. Actualization is the process by which something moves from a situation of potentiality to the actual state, in which potentiality signifies "characteristic of what is potent, of that which has the force to be, which carries within itself the potential to become" (Bicudo & Rosa 2010, p. 24, our translation). According to these authors, the opposite of actual is the non-actual, i.e. that which was not actualized and is in a state of potentiality, including the virtual, which "refers to the way, in general, that it could become actualized through actions that are together with the available materials and the techniques, in particular applications, explanations of the empirical, etc." (p.27, our translation). In other words, the essence of the virtual lies within itself, and is not necessarily linked to the actual, although it may contain situations that may become actualized. For this actualization to occur, the act and the material must occur at the same time, rendering it dependent on them, in such a way that the nature of the material conditions the manner in which the actualization takes place. In addition, according to Granger (1994), the reference is responsible for the relation between the actual fact and the virtual fact, and in the reality of the cybernetic world, this reference is based on the scientific and technological device being used, which supports the actualizations that occur in it (Bicudo & Rosa 2010).

Thus, the virtual is real but not actual, because it has not yet happened through acts and the material at hand. According to Granger (1994), the sciences (mathematics in particular) do not deal directly with facts presented to the observer in mundane reality, i.e., actual facts; rather, they deal with virtual facts. In this way, we could say that mathematics is virtual, but has the potential to branch out into actual facts. Physics and chemistry, for example, avail themselves of mathematics, but they use it as a tool in their theoretical constructions, which are not exact like mathematical theory, since no matter how theoretical they are, they operate with mathematical apparatuses (mathematical knowledge, formulas, ways of operating, etc.) and the empirical dimension in which our acts take place. According to Bicudo & Rosa (2010), computer science also operates with mathematics (which, as we said, is virtual) and, unlike physics and chemistry, deals with another dimension of available "material and form", namely: communication, information, and the creation of programs within a landscape pre-established by mathematical knowledge. And upon actualizing the virtual (the potentiality present in the mathematical apparatus) through form and the available material and effectuated acts (decisions), reality is being constituted which, therefore, also encompasses that which is addressed in the cybernetic world.

Thus, actualizations in the sphere of the cybernetic world emerge from the relation of the human being with the scientific apparatus, through commands, languages, and actions that occur in the encounter between human and computer as well as in the communication between humans and others subjects that is made possible by the system of technological reference. In this broad dimension of reality, which opens up to the experience of the subject, space and time are of a different nature, being more branched out and fluid, such that actions may not actualize in the same manner as they do in everyday lived reality without digital technologies. Thus, we see the reality of the cybernetic world as assuming a plasticity that enables the construction of environments in which the physical relations established in the mundane can either be experienced or totally ignored.

Understanding the cybernetic world as reality, as a modality of lived reality, implies the emergence of a set of possibilities that can be investigated, in the philosophical as well as educational fields. Based on the above, it is our understanding that the essential aspects of the reality of the cybernetic world are associated with singular time, space, and plasticity, constituting a basis for investigation of the mathematical modeling process when the situations involved address this broad dimension of reality. Thus, we assume MM to be a dynamic and pedagogical process of building models that refer to and aim to address problems of any dimension encompassed in reality.

MATHEMATICAL MODELING IN THE CONSTRUCTION OF ELECTRONIC GAMES

The cybernetic world is a singular space where the actions taken may not be actualized in the same manner as they are in everyday lived reality without digital technologies. Aiming to investigate MM in the reality of the cybernetic world guided by qualitative research methodology, we ministered a course entitled "Construction of Electronic Games". We believe that the construction of electronic games paves the way for the locus of the happenings that are related to the situations of the game to be the reality of the cybernetic world. This enables investigation of MM in which the "where" of the happenings is a space that differs from the classical physical notion, thus making possible advancements in understanding of the nature of MM practiced in the reality of this world.

Eight university-level mathematics students participated in the course, which took place in eight 4-hour periods from May to July, 2009. The set of information composed of their words and gestures and their interactions with each other and with the software and other media constituted a fundamental part of the data analyzed. Data was collected using cameras and mainly by means of the Camtasia software, which makes it possible to capture simultaneously the image of the computer screen and images and audio of the students as they interact with them.

The main software program used was Scratch, a free software, developed at the Massachusetts Institute of Technology (MIT), which was conceived based on many ideas from Logo. It is a visual programming language that allows users to construct, interactively, their own stories, animations, games, simulators, songs, and art. The commands are composed of blocks that are dragged to a specific area and connected, creating a program that can be executed. Some excerpts from a program made with Scratch are presented in Figure 1.

Within the perspective of MM adopted, we consider a program, or part of a program, made with Scratch to be a model, as we assume that it is the result of an association of situations from reality (actual facts) with concepts related to mathematics (virtual facts), by means of a reference. In this case, Scratch assumes the role of reference which apprehends the situation being investigated, enabling the manipulation of concepts and symbols according to logical formal rules (propositional calculus). The set of commands that can be actualized in the cybernetic world is limited, but they can be interwoven to configure more complex commands. This reference makes it possible to work with a set of pieces of knowledge associated with science and that branch out and become interwoven as users/authors interact with the situations they wish to represent using the Scratch language.

RESULTS AND ANALYSIS

One pair of students decided to create a game in which a car, controlled by the player, would navigate around obstacles that appeared on a road. At first the students created a model (part of a program) responsible for the movement of the car that allowed its vertical movement over the entire window (area used to execute the program). However, the car´s movement extrapolated the region occupied by the road in the image, giving the impression that the object was floating. Laura, one of the students, then posed the question: *And how do we get it to stop floating?*

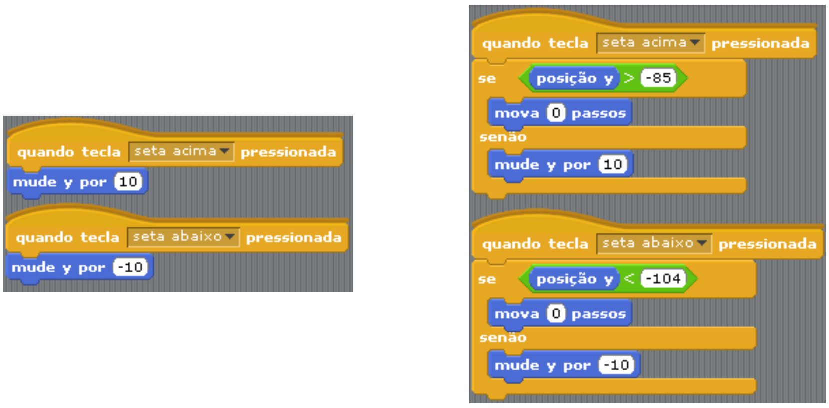
The effort to re-create a movement similar to an automobile in mundane reality was what guided the consequent steps to solve this problem, as can be observed in Laura´s description of her objective: *I want to... like, if it reaches this height here, this (x,y), it stops. It moves zero steps. Otherwise it will go there, up above. I don´t want it to go beyond this height; in this case, I can´t go above or below the road. I* *want it to go only on the road.* Thus, it can be observed that the student´s reference is clearly a fact that actualizes in the mundane world, which is the movement of the car in this dimension of reality.

On the other hand, the thing that generated all this questioning was not a specific situation that occurred with an automobile and experienced by the student in mundane reality, but rather actions that involved the manipulation of Scratch while creating a game, and which were actualized on the computer screen. The situation, in this case, only revealed itself to be problematic to the student within the reality of the cybernetic world, since it allowed the structure that was built to become actualized in a different way compared to everyday experience. Thus, there was a sort of tension created between what was imagined by the pair of students and what was actualized in cyberspace, creating a problematic field that led to various actions by the students.

With this, one can see how the cybernetic world opens the way for “realities that are possible, projected, and invented” (Bicudo & Rosa 2010, p. 20), and that this creative process can configure itself in the tension generated between aspects that refer to situations inspired by mundane reality and the potentiality provided by the plasticity of the cybernetic world which, through actualization, makes it possible for occurrences experienced in this space to extrapolate the physical situations of the everyday.

The model for the movement of a car, created by this pair of students using Scratch, ceases to be something imagined or thought and becomes a fact that is subject to manipulations of the language itself. This manipulation of the language involves a web of concepts, notably supported by mathematics, which allows the situation being investigated/modeled, when apprehended by language, to be altered or modified, enabling actualizations in the cybernetic world that are distinct from those that occur in mundane reality (a floating car, for example).

To achieve their objective, the pair of students made changes in the model associated with the movement of the car throughout the construction process. In the first model, presented in Figure 1a, there are in fact no limits to the movement of the car. In Figure 1b, however, the model includes conditioners that allow movement only within stipulated limits. Thus, although the player decides how to move the object, all the actions are previously established by the model. Unlike mundane reality, the fact actualized by the model in the reality of the cybernetic world is completely determined by the model itself, which is also characteristic of this dimension of reality in the face of the model constructed.



(a) **(b)**

Figure 1: Initial model (a) and final model (b) made using Scratch

In this sense, we understand Bicudo & Rosa (2010, p. 28) when they state that "[…] reality in cyberspace is virtual because it is based on the sciences, notably mathematics", i.e. it is a dimension of reality, but it lends itself to virtual adjectification because it is totally supported by the scientific apparatus. This common base enables a complete determination of the operations, which are actualized in the reality of the cybernetic world.

According to Bicudo & Rosa (2010), this determination is compensated by the multiplicity of ways in which these actualizations can take place. In the specific case of MM, which involves a process of construction, this multiplicity does not reveal itself only in the possibilities that the (ready-made) model offers, but also in the very construction of the model itself, allowing a multiplicity of possible paths which influence the final structure, depending on the way they are actualized in the reality of the cybernetic world, and the objective of those involved. This aspect can be observed at two different moments during the interaction between the Laura and the researcher:

Laura: And how do we get it to stop floating?

Researcher: You need to put some conditioners, right? It´s going to stay stopped there, right? [Referring to the horizontal movement].

Laura: It stops.

Researcher: Does it stop or... does it "move" zero steps?

Researcher: Or... stop moving. So you can put, change X to Y, you can put an "if". An "if-else"...

Laura: Put an "if-else", then, in the middle of this here [of the commands]?

Researcher: Right. "if-else" or "if", I´m not sure. You would have to test it there to see what works.

The researcher´s last suggestion indicates that one of the ways to find a solution is to test the different possible paths, indicating that the idea of multiplicity is present not only in the actualization of the game by the player, but also in the process of constructing the game itself. The following excerpt indicates a new multiplicity of paths to be followed:

Laura:	When the *y* mouse is equal to, in this case, to this point *y* here [pointing again with the mouse to the position of the car and observing that it is located at y = -65]
Assistant:	Equal or less. I´m not exactly sure what the objective is there.
Laura:	OK, so I´m going to experiment.

In this excerpt, the research assistant expresses doubt regarding the path to be taken, indicating two paths ("equal or less"). These distinct possibilities are taken up by Laura, and her subsequent actions show that the use of the word "experiment" was associated with the use of all three symbols that the command could assume, thus presenting a multiplicity of paths.

In this way, we affirm that the MM activity presented involved a problem that resulted from the tension created between what was imagined by the students and its actualization in the cybernetic world, which revealed aspects that differ from the actualization of an automobile in mundane reality. This difference, in the analysis presented, occurred by means of the apprehension of the phenomenon by the reference, which is given by the Scratch language. Because it has the same structure as the cybernetic world, the constructions of the model and of space itself are interwoven, showing that, in the reality of the cybernetic world, the model (which is the apprehension of the phenomenon by the reference) completely determines the possibilities of the player´s action. In addition, the follow-up actions related to the problem of the car were associated with the change and adaption of the model to the situation desired by Laura, which consisted of keeping it within the limits of the road. This process involved experimentation with various different commands. When the car failed to behave as desired, changes were made. These changes in the model continued until the model was adapted to the desired situation.

This is consonant with some views of MM that see the effort to adapt the model to the situation as an important stage in the modeling process, as in research carried out by Bassanezi (2004), Biembengut & Hein (2007), Borromeu Ferri & Blum (2010), and Kaiser, Schwarz & Tiedemann (2010). According to these authors, the adaptation of the model, or of the solution the model provides for the situation to which it refers, is denoted by validation, and can be consolidated through comparison of the data obtained using the model to the empirical data. If the comparison is unsatisfactory, the model is re-formulated to find a better solution.

When another pair of students in the course was analyzed, however, we observed that they did not proceed to reorganize their model when faced with a situation that failed to behave as they wanted. Rather, they sought to re-organize the entire visual environment in which the model was referenced and would become actualized.

These students decided to create a game in which the objective was to navigate on a map of a fictitious city by changing two variables, denoted “meters” and “degrees”. The map was initially created based on an image the students obtained from the Internet (Figure 2).

Figure 2: Map in the Scratch programming environment

When the MM addresses a problematic situation from the cybernetic world, it can be influenced by mundane reality, as discussed above. Nevertheless, it is still possible to see aspects related specifically to the context of the cybernetic world. The experience of this second pair of students is an example of this, in which the “where” characteristic of reality in cyberspace shows itself decisively in the way they develop their work, broadening the mundane possibilities. It is precisely based on this potentiality of creation and re-creation of spaces that we believe the MM process can manifest itself differently compared to cases where the reference derives from the everyday, since it is supported by a reality that can be constructed, imagined, thought and re-thought, showing an environment that is plastic and transforms itself.

Some of the problems faced by this pair of students involved the relation between the scenario (map) and the object that moved in it. The object was supposed to return to the stipulated starting point upon touching the edge of the streets, but this was not happening. The following excerpt shows the researcher attempting to help the students and illustrates one of the singular aspects of the cybernetic world, plasticity:

Researcher: So I think the best thing would be to put a stronger border here. [Referring to the edge of the streets in the image represented on the map]

Fernanda: Do it with Paint, maybe. [Referring to the software Paint, which can be used to edit images. This idea was later abandoned in favor of using the software Excel].

The researcher shows that one possible solution to the problem presented by the model created by the students would not involve changing it, but rather changing the background scenario that limited the action of the object. The researcher´s suggestion reveals a plastic view of the cybernetic world that was immediately corroborated by the student. This agreement led the students to change the scenario. Initially, they aimed to totally deconstruct the model, i.e., to build a completely new one. However, the use of the Excel software allowed them to re-build it based on the original map. It was at this moment that the student, Fernanda, again showed that she considered the space where the model was being constructed to be malleable, revealing an aspect that is very distant from a reference to the physical, mundane reality which is already actualized. This aspect can be observed mainly when the students say: *We can make it thin, like this, and afterwards we can augment it, OK?*

Our interpretation is that there was a concern regarding the borders and the thickness of the streets, perhaps influenced by the researcher when he suggested making the border “stronger”. However, when the student says, “We can make it thin, like this”, we believe the student was, at first, attributing importance to the outline of the streets in the construction of the map; but the rest of her comment, “and afterwards we can augment it”, revealed that the malleability of the construction of the environment was already incorporated. In other words, the student was not initially concerned about all the details that constituted the map and could influence the commands used, since the environment constructed could be modified to fit the desired format.

This observation strengthens the idea of the plasticity of the reality of the cybernetic world and reveals a singular aspect when we perceive the problematic situations of this construction from the perspective of mathematical modeling. In fact, when the reference is mundane reality, the model is normally “refuted”; however, in the reality of the cybernetic world, it is possible to refute the “reference”, and maintain the main idea. With this, one observes the existence of a potential space where it is possible to consider naturally situations in which a model was invented by the students. The plasticity of the cybernetic world, allows, at least potentially, the creation of a space in which imaginative aspects can be actualized, thus representing a differential for the practice of mathematical modeling itself.

NOTES

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