THE USE OF THEORY IN TEACHERS' MODELLING PROJECTS – EXPERIENCES FROM AN IN-SERVICE COURSE

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We discuss how to support teachers' use of theory on modelling and on learning mathematical concepts in an in-service course on project work and modelling for upper secondary teachers. The course is centred on the teachers' experiments with modelling projects in their own classes. The paper is a case study on how to facilitate the interplay between theory and development of teaching practice. Our analyses focus on modelling as a means for learning mathematics, and on theories on the learning of mathematical concepts. The theories have potentials for improving the teachers' practice, but they need concretisation and re-contextualisation in the teachers' projects in order to be helpful for the teachers.

INTERPLAY BETWEEN THEORY AND PRACTICE IN MODELLING

The research question is: *How to integrate theories on modelling and on the learning of mathematics in an in-service course on modelling for teachers at upper secondary level so as to be helpful for the teachers' development of their own practice?*

In general, theories in the area of teaching and learning mathematical modelling have developed – primarily during the last three decades – in quite close connection with the inclusion of modelling in secondary mathematics curricula, and through interplay with the development of practices of teaching in this area. As a common foundation, these theories rest upon the definitions of key concepts related to modelling such as the notion of a mathematical model, the cyclic modelling process and modelling competency. The theories include justifications of mathematical modelling as an important element of mathematics curricula in general education and suggestions for design of teaching in different contexts. As a tendency in more recent research, specific learning potentials and difficulties related to modelling and specific ways of enhancing the students' learning of mathematical concepts through modelling have being researched (Zbiek and Conner, 2006), (Blomhøj and Kjeldsen 2006, 2010b). This development opens for a closer interplay with theories on mathematical learning in general. However, also in this area as in mathematics education research in general, there is a scarcity of studies investigating concretely how theories can be of use for teachers in developing their practice.

In general, research in mathematics education tends to focus more on the development of theories than on how these theories can really be put into play in the process of improving teaching and learning. Hereby, the process of doing research is separated from the process of applying theory in developing practices of mathematics teaching. It is a challenge to find ways to bring theory into play in the practice of teaching. Action research seems to be a fruitful approach to teachers' professional development in that respect (Krainer, 1999). Boaler (2008, p 103) found that the teachers' involvement in experimenting with their own teaching seems to be a very important factor in successful use of theory in practice. The development of our in-service course and our related research can be seen as a form of developmental research practice with exactly this aim (Blomhøj and Kjeldsen, 2006).

THE STRUCTURE, ORGANISATION AND PURPOSE OF THE COURSE

Before presenting and discussing the use of theory in the in-service course, we provide the reader with information on the course structure and organisation. The course is a 7.5 ECTS course (1/8 of a full year of study) designed and taught by the authors in collaboration. Typically, the participants in the course are 12-20 mathematics teachers with a variation of teaching experiences. The teachers normally have only a small reduction of their teaching load, while taking the course. The course begins with a seminar over three full days. Here the participants are introduced to mathematical modelling and problem oriented project work, as well as to theories on the learning of mathematic as presented in the next section.

During the seminar, the teachers work in groups of two or three developing a modelling project for their own classes covering approximately 10 normal lessons and substituting two written homework assignments for the students (5 hours of homework).

In the first phase of the development of the projects, the groups are asked to emphasize the following four issues: (1) Intentions for their own development. What is it in particular that they want to try out and experiment with as a teacher in their project organised modelling course? (2) What are the main intentions for the students' learning in the course? (3) How to set the scene for the students' problem oriented project work? (4) How to evaluate the students' learning through observations and/or product evaluations? The groups are encouraged to limit and focus their answers to these questions as much as possible. The groups' first proposals are presented and discussed in the first seminar, and then further developed afterwards. Two weeks after the first seminar, the preliminary descriptions and student materials for the modelling projects are distributed to all participants. Within a period of two months the groups finish the detailed planning and each teacher try out his or her experimental course in at least one class. In some cases, it is possible for two teachers working at the same school to observe parts of the course in the colleague's class. This has proven to be supportive for the teachers' reflections and for their further cooperation with improvements of the projects. We therefore encourage participation in pairs of teachers from the same school. A 1-day seminar is held after the period with experimental teaching, with the aim of supporting the teachers in reporting their projects and their related reflections in a form that could be helpful for colleagues wanting to do similar modelling projects. A first version of the reports is distributed to all participants a month later. These are presented and discussed extensively at the final 2-days seminar after which the groups receive written feedback including suggestions for improvements before publication on the web, if the teachers so want. This organisation allows detailed discussion of the teachers' projects and on their relation to the theories introduced at the course.

THE USE OF THEORIES IN THE COURSE

At the first seminar, the teachers are introduced to theories in three different domains: (1) Theory on problem oriented project work (Blomhøj and Kjeldsen, 2010a). Here the emphasis is placed on the importance of formulating a problem, which can guide the students' modelling process, and enable them to take control of the process supported by milestones and supervision. The way in which the scene is set for the project in the class, and the explicit demands for the students' project reports should be the main didactical instruments for steering the students' project work. (2) Theory on mathematical modelling, e.g. justifications for modelling, modelling competency and the modelling cycle (Niss et al., 2007), (Blomhøj and Kjeldsen, 2010a). Here the emphasis is on illustrating and discussing the modelling process, reflections in relation to concrete examples, and on the teachers' ideas for modelling problems for their projects. (3) Theory on the learning of mathematical concepts. This element relates to modelling as a vehicle for supporting students' learning of mathematics (Blomhøj and Kjeldsen, 2010b). Here we present and discuss theoretical ideas related to: The important role of representations for the learning of mathematical concepts (Steinbring, 1987); the process-object duality in concept formation (Sfard, 1991); concept images (Vinner and Dreyfus, 1989); and the RME-idea concerning the change of role of a model from "a model of some particular object/situation" to "a model for the learning of a mathematical concept" described by Gravemeijer (1994). In parallel with diSessa and Cobb (2004), we argue that such theories have a lot to offer for improving teaching experiments.

However, the theories need to be concretised and re-contextualised in relation to the teachers' particular modelling projects, in order to be helpful for the teachers. That is in fact the main reason for the design of our in-service course. In the course, the participating teachers are developing their own modelling projects, and through our interplay with the teachers in this process, it is possible to bring the theories into play. Here we have found it helpful to develop, until now, three different forms of intermediate representations for presenting and discussing with the teachers, the potentials of the various theoretical ideas for the development of their practice of teaching modelling. These are: (1) Detailed and concrete descriptions of the modelling process behind the models in the teachers' projects. Such unfolded modelling processes enable the teachers to see and discuss the potentials for supporting the development of the students' modelling competency in the planned activity and to help students in case of difficulties, without spoiling the essential modelling challenge; (2) Schemes spanning all the different representations of particular mathematical concepts and their interpretations in a given modelling context. Such schemes can help capture the potentials for supporting the students' mathematical learning and they can be a tool for analysing the students' modelling activities in relation to their mathematical learning potential. Figure 2 represent such a scheme for the project presented in the next section; (3) Construction of anticipated dialogues between the teacher and a group of students facing some particular modelling challenges or learning difficulties. Discussing such dialogues can help teachers prepare for supporting the students' learning in modelling activities without taking over the students' tasks. In the present paper we focus on the use of (2) as a tool for spanning the mathematical learning potentials in a modelling activity and as a tool for analysing the students' activities.

Overall, the methodological approach in our research and development of our course is similar to that of critical mathematics education (Skovsmose & Borba, 2004). Here three types of situations (teaching practices) are in play: CS: The current situation, AS: The arranged situation, and IS: The imagined situation. Three different processes combine the three situations: (1) Experimenting; (2) Analysing; and (3) Pedagogical imaging, see figure 1.

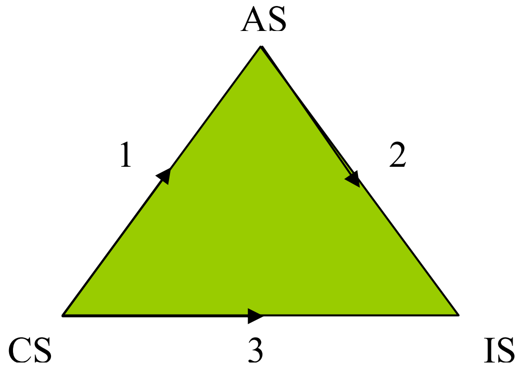


Figure 1: The methodological triangle in critical mathematics education.

In our course, we use theory on mathematical modelling and on the learning of mathematics to establish a shared idea about an imagined situation concerning a concrete modelling project suggested by the teachers (process 3). We help the teachers to use elements of theory as a basis for designing their projects (the arranged situation) (process 1) and for describing their aims for developing their practice of teaching in relation to the imagined situation (process 3). At the final seminar for each project, we analyse the relation between the arranged situation and the teachers ideas about the imagined situation (process 2), which they – to some extent – have described in their reports. Hereby we develop new ideas together about the imagined situation and about how to develop the projects (the arranged situation) for a new run. In relation to these processes, we focus on how to develop our interaction with the teachers in order to support better the teachers' reflections on their projects.

A MODELLING PROJECT ON THE DECAY OF ALCOHOL AND THC

This problem oriented project work in mathematical modelling of the decay of alcohol and THC (tetrahydrocannabinol, the active drug in hash) was developed by four teachers from three different high schools as part of their participation in our in-service course. These teachers took the course as part of their re-education as mathematics teachers coming from other professions such as engineering. However, they already had their own classes and some limited experiences with teaching mathematics in non-permanent positions. The project was implemented in three first year classes (age 15-16 years) in ordinary high schools in Denmark.

Before deciding for this theme for their project, the teachers discussed the ethical aspects of the theme. Most Danish young people age 15-18 drink alcohol at parties, and some drink too much! In the current situation hash is illegal to sale and possess but legal to smoke. In average, one or two students in a class of 30 students can be expected to have had some experiences with hash. The teachers found the theme relevant and motivating for the students. They sought it could make the students reflect critically on their own alcohol habits and possibly prevent them from experimenting with hash. Moreover, they found the theme mathematical relevant since the decay of alcohol and THC is respectively essentially linear and exponential.

The teachers had the following learning goals for the students in the project work. They wanted to use the modelling project to:

1. provide the students with a positive experience on using their mathematical skills to answer interesting and relevant questions from their life world
2. support the students' conception of modelling and applications of mathematics
3. teach students to have a critical outlook on mathematical models
4. support the students' learning of linear and exponential functions
5. develop the students' understanding of the parameters in the two models
6. train the students to communicate mathematics
7. support the students' IT competences

These learning intentions were clearly inspired by the theories introduced at the in-service course, and they fall into three groups: (I) aspects of developing the students' modelling competency (1-3); II) aspects of developing the students' concept images and their mathematical understanding of linear and exponential functions (4-5); (III) aspects of developing the students' IT and communication skills (6-7).

The students worked in groups of three to four and to begin with they were given a set of four exercises and the following task – in our translation:

> "Write an article for students of your own age about the decay of alcohol and THC in the human body. In the article you should also explain the mathematics you have used to complete the exercises. Your answers to the exercises and your graphs should be integrated into your article."

In the first two exercises, the students were presented with a set of realistic, but not authentic data showing the decay with time of some amount of alcohol or THC respectively in a person. The students were asked to draw graphs hereof using T-Inspire or Excel, to describe the graphs in their own words, to determine the time for the amount to decrease to the half two times consecutively – an hereby experience a fundamental mathematical difference between the two cases, to determine the mathematical expression for the functions represented in the graphs, and finally to interpret, in their own words, the significance of the parameters of the functions in the two contexts. The students were encouraged to find data on the web for the decay of alcohol and THC in the human body to compare with their own models developed from the given data. In exercise three, the students were given data for the amount of alcohol in some popular drinks, and asked to model the decay of the amount of alcohol they had consumed at their last party. Finally, they were asked to compare the decay of hash and alcohol.

In the following we will focus the discussion on the second group of learning intentions. We analyse the students' reports and the teachers' related reflections with respect to these learning intentions. Moreover, in the next section, we will illustrate how the theories can help span the potentials of the project for supporting the students' learning of linear and exponential function. Our analyses rely on discussions that took place at the in-service course, the teachers' reports, their design of the project, the tasks given to the students and the articles written by six of the student groups, two groups from each of the three classes. We round off by discussing the relation between the potentials of the projects and what was fulfilled in the project - according to the teachers' and our analysis.

Regarding the first three learning goals, the teachers' evaluations show that aspects of the students' modelling competency were invoked in all three classes. In the articles written by the students, they interpret their linear and exponential graphs for the decay of alcohol and THC, respectively with respect to the modelling context. They translate back and forth between the graphical representation and what it tells us about the amount of alcohol and THC in the body as times go by. Several groups of students reflected upon the validity of the models in the sense that they questioned whether the parameters in a model that was based on data for one particular person are valid for another person. The web reveals that the rate for the decay of alcohol is mainly due to the liver and is only slightly depending on the body mass. A constant rate of 8 g/hour is a good estimate for a normal grown up person. For THC one finds a first order decomposition with a half-life period close to three days.

One group of students extended the model so as to capture a situation where there was an initial amount of alcohol in the body followed by further consumption of alcohol during a couple of hours. The spreadsheet representation of the model made it easy for the students to implement this idea as an instant addition of alcohol at a particular time represented in a cell in the spreadsheet. Such an extension of the model makes it possible to ask new questions. For example: How to plan the alcohol consumption in order to have a good party, and not to have too much to drink?

The students came to reflect upon the differences in the way in which the substances decrease in the two situations through the questions about how many hours it takes for the amount of alcohol and THC, respectively to decrease to half of the initial amount. They used the model with their estimation of the amount of alcohol they had at their last party. Many of the students were surprised to realize how many hours it actually takes according to the model before the alcohol in their body has vanished.

With respect to supporting the students' learning of linear and exponential functions and the significance of the parameters it is unclear whether the students realized the fundamental properties of the two types of functions. As one teacher wrote:

> "The idea was that the students should realize that the half time was a constant in the exponential case and not in the linear case. Many students didn't realize that because they used the graphs [to determine when half of the amount had decayed and when half of the half of the amount had decayed] and they reached two different approximations [for the exponential function]."

The students were able to reach an expression for the linear decrease of alcohol and the exponential decay of THC. They were also aware that the parameters in the linear function measure the initial amount of alcohol and the amount of alcohol that disappears per hour – except that in some of the articles, the students were not accurate in distinguishing between the amount of alcohol and the concentration of alcohol in the body. However, as one of the teachers wrote:

> "Surprisingly many of the students had problems de-mathematizing the parameters a '$y=a\cdot x+b$' and a ' $y=b\cdot e^{ax}$ ', and interpreting the significance of these parameters for the decay of alcohol and THC, respectively. The main problem was the understanding of a indicating the [absolute] amount of decrease of alcohol per hour and a being a number determining the relative decrease of THC per hour as e^{a}. …So next time I will use different names."

The article, the students were asked to write, trained the students' competency in communicating in and with mathematics. In the articles, the students sometimes reasoned with mathematics and other times within the modelling context, e.g.:

> "In all these calculations we have seen that hash is in the body for a longer time than alcohol, and therefore it can damage one's ability to learn if one smokes hash regularly."

The students performed the calculations on the models constructed with the given sets of data, and for these it took much longer for the body to get rid of the THC than the alcohol. However, the model cannot say anything about learning disabilities!

The teachers' intentions for the students' learning were more or less fulfilled. The students used mathematics to model situations from their life world. Nearly all students were able to reflect upon and criticize the models to some extent. The modelling of alcohol and THC created teaching situations in which the teachers became aware that the students had bigger problems than anticipated with understanding and interpreting the parameters in the linear function and the exponential function both mathematical and in the modelling contexts. The modelling project remedied these problems to some extent.

THE USE OF THEORY IN DESIGNING AND ANALYSING THE PROJECT

In our in-service course, we presented, as we said before, to the teachers the model for concept formation in mathematics developed by Anna Sfard (1991) and the basic idea that the access to a mathematical concept goes through the meaning of its representations and their relations (Vinner and Dreyfus 1989), (Steinbring, 1987).

We find that these elements of theory fit very well together and that they can be combined to form a tool for capturing the potentials for learning mathematics through modelling activities. In the schemes in figure 2, we illustrate this idea in relation to the concept of a linear function and the different forms of representations that can come into play in the modelling of the decay of alcohol. In each cell in the scheme, there is a representation of the concrete model of the alcohol decay and of the general linear model in the same form of representation. In each form of representation both the process and the object aspect of the concept can be represented. Moreover, all these representations can be interpreted and be given meaning in the modelling context. The mathematical properties can be pinpointed in the representations and related to the properties of the general linear model or to the general exponential model. Hereby, the modelling context can mediate meaning even to the general mathematical model and to the mathematical concepts involved. We believe that the transition from a process to an object understanding of a mathematical concept can be supported in all forms of representations present in the scheme. However, this needs to be further researched. The important thing in this context is that the scheme can help span the learning potentials in modelling situations with regard to a particular concept, and hereby be of help for the teachers both in designing projects and in the dialogical interaction with the students.

In the design of the project on alcohol and THC, the teachers were introduced to the learning potentials spanned in the scheme at the first seminar in the in-service course. However, the way the scene was set for the students' modelling work in the project did not systematically challenge the students' to work with the different representations. The algebraic and graphic representations are dominant in the students' articles. Moreover, in these two forms of representations, the process aspects are not visible in the students' work.

In the final seminar of the in-service course, the teachers reflected themselves about the unfulfilled learning potentials in the project. Suggestions for how to guide the students to work with the different representations in the scheme the next time were discussed. One way to do this could be to ask the students to explain the model in all five representations from both the process and the object perspective in the articles. Also, the learning potentials in comparing the schemes for the linear function and alcohol model with that of the exponential function and the THC model need to be addressed explicitly in the requirements for the students' articles.

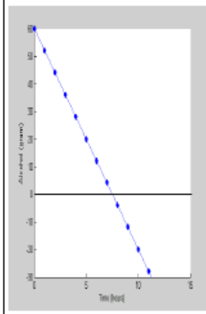
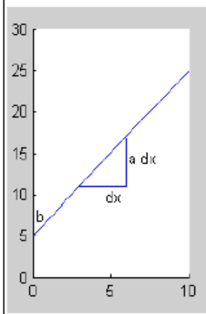
	Natural language	Numerical	Algebraic	Algorithmic (Excel)	Graphic
Process	8 gram of alcohol is removed by the liver per hour. 12 gram is added per drink (beer)	x 0 1 2 y 60 52 44 -8 -8	$f(x+1)=f(x)-8$ $f(0)=60$	B2=-8 A5=0 A6=A5+1…. B5=60 B6=B5+$B$2 ….	
	x times the slope plus the constant yields y. One extra unit of x means a change of the slope in y	x 0 1 2 y b $a+b$ $2a+b$ $+a$ $+a$	$f(x+\Delta x)=$ $f(x)+a\Delta x$; $f(0)=b$	Can be generalised by change of parameter and initial value	
Object	After five drinks and x hours: $y=60-8x$ gram of alcohol is left in the body	x 0 1 2 3 y 60 52 44 36	$y=-8x+60$	B2=-8; B3=60; A5=0 A6=A5+1…. B5=$B$2*A5+$B$3 B6=$B$2*A6+$B$3 …….	
	A linear combination with constant sum.	A tabel of (x,y) with $y=ax+b$	$f(x)=ax+b$	The algorithm is general due to parameters.	

Figure 2. Shows process and object aspects of representations of the alcohol model.

CLOSING REMARKS ON THE THEORY-PRACTICE RELATION

To support the students' learning of mathematics is an important objective for including mathematical modelling in secondary mathematics. However, in order to meet this objective we need to develop the practice of teaching modelling based on theories on the modelling and on the learning of mathematics. This can be done through theory based experimental work in the context of in-service courses or developmental projects. To facilitate the teachers' use of theory in such contexts it is important that we try to develop intermediate representations and modes of collaborations, which can mediate powerful theoretical ideas to teachers, so as to be helpful in their development of their own teaching practice. We think that the representation scheme presented in this paper (see figure 2) is an example hereof.

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