PRINCIPLES OF TASKS' CONSTRUCTION REGARDING MENTAL MODELS OF STATISTICAL SITUATIONS

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In this paper we focus on the construction of tasks which is a specific topic within our on-going research into the development of a diagnostic tool for analysing young students' mental models when they act within simple statistical situations. For this, we discuss tasks from a theoretical perspective concerning statistical content, statistical thinking processes and – as the main aspect in this paper – the representation of tasks. We deduce principles of the tasks' development partly from statistics education research literature and partly from our previous research.

INTRODUCTION

About 60 years ago, Piaget and Inhelder (1975; original 1951) published their seminal work referring to children's development of thinking with probabilities that has a strong impact on stochastics school curriculum until now (Jones & Thornton, 2005). The set of tasks which Piaget and Inhelder had used aimed to analyse the thinking of children of different ages, who were not schooled in probability. Taking into account the diversity of children's responses referring to different probabilistic problem situations Piaget and Inhelder (1975) developed their stage model of children's probabilistic thinking development. From the perspective of a diagnostic teaching the tasks which Piaget and Inhelder gave to the children could be treat as diagnostic tool to analyse the children's initial position of learning probability (Hußmann et al., 2007).

After Piaget and Inhelder subsequent research gave a lot of empirical evidence of which instructional conditions the children's probabilistic thinking could be developed (Jones & Thornton, 2005). For example, Fischbein and his colleagues investigated instructional conditions that facilitate children to come from primary intuitions concerning probabilistic situations to secondary intuitions (e.g. Fischbein, 1975). Nowadays, there exists a huge amount of research that discusses contrarily the Piagetean levels of the student's development of probabilistic thinking with probabilities and, further, investigates instructional settings in which students are able to foster their probabilistic thinking (Jones & Thornton, 2005).

By contrast with probability working on data has a comparatively short tradition in school curricula of most countries (cp. Shaughnessy, 2007; Batanero, Burrill, & Reading, 2011). Nevertheless, by now there also exists a huge amount of research in terms of intervention studies that give evidence of how students learn doing statistics adequately and, further, of how students' competencies concerning different statistical contents are developed. However, we still know little about the reasoning of students without systematic schooling in statistical situations focusing on data (Mokros & Russel, 1995). With regard to this, we start a research aiming to analyse the decision making of young students (without schooling) in realistic statistical situations (Eichler & Vogel, 2012). However, as a consequence to the mentioned research gap, in the educational literature only very few tasks can be found that facilitate to analyse statistics unschooled students' thinking or decision making concerning statistical situations. For this reason, we started to develop tasks representing statistical situations to provoke non-schooled young students' thinking about and decision making in statistical situations. After we had used these tasks in pilot studies we changed the tasks, used the changed tasks in further pilot studies again and so on. In this way our task development is taking place until now.

In this mostly developmental paper we focus in retrospection on both several principles of tasks' development that can be deduced from theoretical constructs of psychology, and some empirical results of our previous studies. Discussing this development, we firstly analyse one well known and well elaborated task of Bakker & Gravemejer (2004) that is used in an intervention study. Afterwards we change this task step for step according to theoretical and empirical based prerequisites of a diagnostic task aiming to investigate non-schooled children's thinking in statistical situations.

FIRST STEP OF THE DEVELOPMENT OF A DIAGNOSTIC TASK CONSIDERING CONTENT AND THINKING PROCESSES

Using existing tasks as starting point for the development

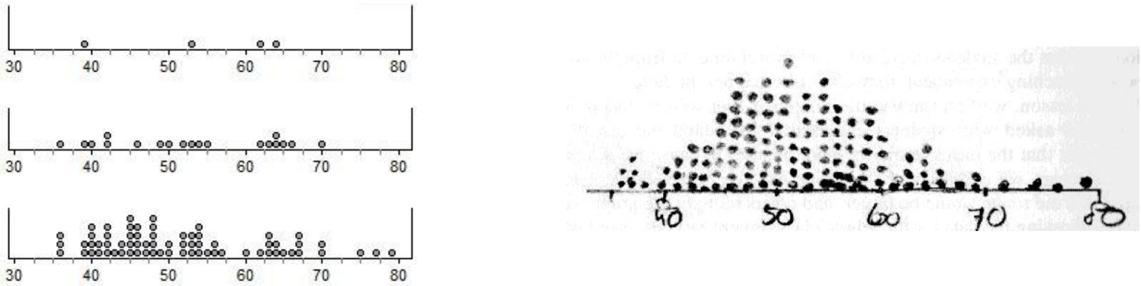


Figure 1: Collections of students' weight and students' prediction of a larger sample

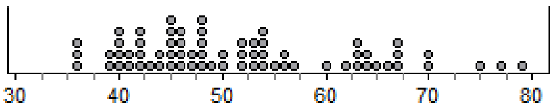
We begin with a task from Bakker and Gravemeijer (2004, p. 158) used in a design research approach aiming to develop students' informal reasoning about distributions. The specific task aims to provoke students' thinking about variation, sampling and representativeness:

> In a certain hot air balloon basket, eight adults are allowed [in addition to the driver]. Assume you are going to take a ride with a group of seventh-graders. How many seventh-graders could safely go into that balloon basket if you only consider weight?

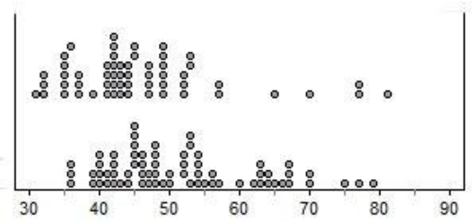
Firstly, the students tried to estimate the average of the weight of both students and adults. Afterwards they collect data to students' weight in their class as basis to make a prediction for a larger sample. It seems obvious that non-schooled children would not be able to think about centre and variation in this task in a way using statistical concepts like range, average, median, and so on since they do not have systematic knowledge of these concepts. However they might be able to think about variation and centre in an informal way. Potentially these children might also be able to think about the sample in an intuitive way that a bigger sample would give more certainty than a smaller sample. However, we restrict our considerations to an informal way of thinking about centre and variation representing two main aspects of statistical thinking (Wild & Pfannkuch, 1999).

Provoking students' thinking

Given that non-schooled children are able to read the iconic representation of the data, they could be potentially able to describe the distribution in an informal way. However, research in stochastics education give a lot of evidence that a task involving a decision making in terms of both a comparison and a prediction would particularly provoke children's comments about a situation (e.g. Yost et al., 1962; Fischbein, 1975). Thus, a task involving decision making in terms of making a prediction – Bakker and Gravemeijer (2004) included a prediction in their task – could foster the students to thinks about a statistical situation (see figure 2, left side). As well, it is possible to enhance the given task leading to decision making process in terms of a comparison.



Another seventh-grader of your school will be weighted. Estimate his weight. Give a rationale for your estimation.



There are the sample of your school and a sample from another school. Which students are more heavy-weight? Give a rationale for your answer.

Figure 2: Variation of the task including decision making

Tasks in this way could serve as a diagnostic task to provoke students' informal thinking about centre and variation. However these tasks presuppose the students' ability to read the iconic representation of the data, i.e. the transnumeration of data into a graph (Wild & Pfannkuch, 1999), which is obviously not common for non-schooled children (e.g. Shaughnessy, 2007). At this stage of task development we expand our theoretical framework with regard to mental model theory. This theory allows for describing the perception and processing of statistical situations that were not represented by elaborated statistical methods like the dot plot. In the next section we outline the very basics of this theory.

THEORETICAL ESSENTIALS OF MENTAL MODEL THEORY

We reduce our theoretical focus on those aspects of mental models, which were fundamental for characterizing our task development.

Johnson-Laird (1983, p. 156) states: “A mental model […] plays a direct representational role since it is analogous to the structure of the corresponding state of affairs in the world – as we perceive or conceive it.” From the perspective of information processing Schnotz and Bannert (1999) describe mental models as being constructed individually according to a task and its requirements within a specific situation. By this, they conclude that a mental model represents *the structure* as well as *the function* of the modelled object in an *analogous relationship*. These are three essential characteristics of mental model theory which were important of being pointed out with regard to the task development:

- *Structure:* An essential process of mentally modelling a situation’s structure is recognising the physical objects of the situation, e.g. a die and its characteristics (or a student and his weight), as well as the relationship of these objects and their characteristics. Given data are also to be seen as being part of a situation’s structure because they represent results of a process having passed.
- *Function:* Concerning the dynamic aspect of mental models, i.e. the function, Seel (2001) suggests that, when coping with demands of a specific situation, an individual constructs a mental model in order to simulate relevant aspects of the situation to be cognitively mastered. Thus, the function of mental models allows for deriving answers via mental simulation of systems by anticipating possible results given for example by throwing dice (or students becoming the weight they have when they were weighed). Mental simulations do not result in quantitatively exact conclusions but in qualitative ideas about the expected outcomes of such simulations (De Kleer & Brown, 1983). These “qualitative simulations” (De Kleer & Brown, 1983, p. 155) require sense making about the system or process that should be simulated, its constituent components and their relationships.
- *Analogous relationship:* This means, a mental model and the corresponding modelled situational object or process coincide structurally at least in some constituting elements. Thus, mental models can principally be inferred from observable information which represents mental modelling of a situation or task, the conditions of a students’ specific situation (experience, pre-knowledge), and students’ outcomes after working with tasks (written responses, videotapes).

These three characteristics become crucial when we analyse the demands on mental modelling of tasks concerning decision making in simple statistical situations, when we construct a hierarchical model of tasks’ complexity and finally, when we construct new tasks on base of this model.

SECOND STEP OF THE DEVELOPMENT OF A DIAGNOSTIC TASK CONSIDERING MENTAL MODEL THEORY

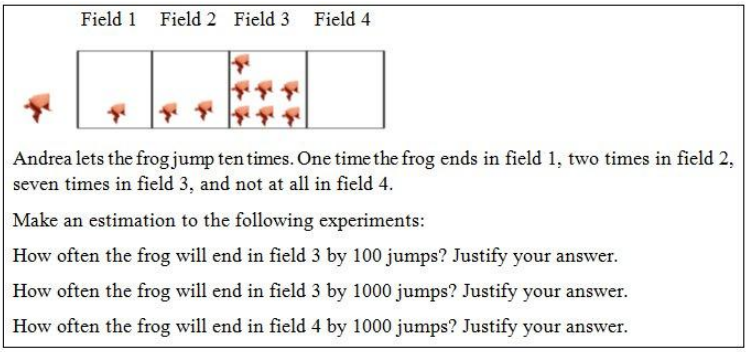
Mental modelling of a situation represented in a task includes perceiving its *structure*, which is a precondition to conduct mental simulation adequately (*function* of the situation). This means that disclosing those elements which impact on a statistics problem situation in respect of its structure and its function would reduce the complexity of a situation represented in a task, especially for young statistics unschooled students. For this reason, the development of tasks has to treat the visibility of a situation's structure as a matter of principle of tasks' representation. The visibility of those elements which constitute the *structure* as well as the *function* of a situation impacts on mental simulation. Such a mental simulation equals the process of data generation when a statistical situation is regarded – it could be seen as a qualitative mental data generation.

Changing the task according to demands of mental modelling

In the task of students' weight the result of data generation (*function* of the situation) is represented by the given data. Thus, the *function* of this situation itself is hidden because having passed as well as the situation's structure including those elements that impacted on the students' weight. Similar tasks, in which both the structure and the function are potentially visible, are the following:

> Two students try to throw coins as close to the wall as possible. Who is the better player? Give a rationale for your answer.
>
> Andrea lets a paper frog jump several times. Estimate, where the next paper frog will end. Give a rationale for your answer.

The two situations include a request for decision making as defined above. Further, the data as result of the situation's function (i.e. process of data generation) could be given in the same way as shown in figure 2. Taken into account that young students were potentially not able to understand the construction of a dot plot, we used a pictogram to represent the data, e.g. the frog jumps (figure 3; Eichler & Vogel, 2011) and changed the situation a little.



Andrea lets the frog jump ten times. One time the frog ends in field 1, two times in field 2, seven times in field 3, and not at all in field 4.

Make an estimation to the following experiments:

How often the frog will end in field 3 by 100 jumps? Justify your answer.

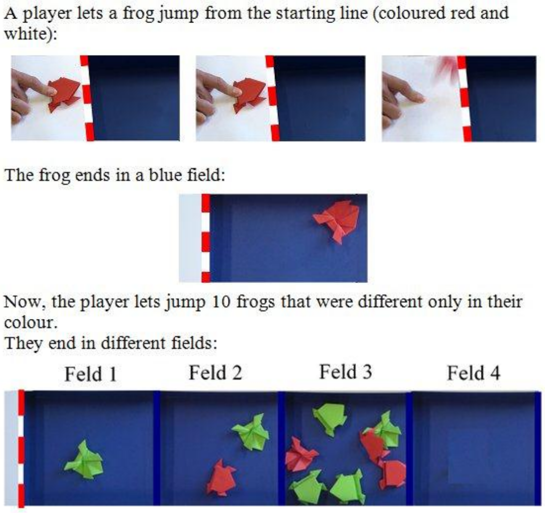
How often the frog will end in field 3 by 1000 jumps? Justify your answer.

How often the frog will end in field 4 by 1000 jumps? Justify your answer.

Figure 3: The frog task (early version)

However, a lot of young students confronted with this task using questionnaires and interview seem to misinterpret the situation. In particular, the equal arrangement of the frogs in the fields 1, 2 or 3 (fig. 3) led some students to the interpretation that those frogs being visible do not represent several jump results in reality. Those students were not able to match the descriptional representation (the text) and the depictional representation (the pictogram) to make inferences from the given information on base of an adequate mental model of the situation (Schnotz & Bannert, 1999).

An auxiliary change of the situation's representation in the task was to use a realistic picture of the situation in contrast to the more abstract representation using a pictogram (Vogel, 2006). Further, according to the findings in research into multiple external representations (Ainsworth, 1999; Mayer, 2001; Seufert, 2003), we combine for each step of the data generation a text with a picture. According to the cognitive load theory (Sweller et al., 1998), the text itself is formulated as simple as possible. Finally, we construct photos as pictures of the situation including all relevant objects representing the structure of the situation (cf. Schnotz & Bannert, 1999). We show in figure 4 only the changed representation of the situation since the task is the same as shown in figure 3.



A player lets a frog jump from the starting line (coloured red and white):

The frog ends in a blue field:

Now, the player lets jump 10 frogs that were different only in their colour.
They end in different fields:

Figure 4: The situation of the frog task (final version)

Findings concerning the changed tasks

Using these special kind of "stochastics problem-storyboards" in an interview design (Eichler & Vogel, 2012) gave some evidence that even non-schooled students (referring to stochastics) were able to understand the situation in the intended way. Thus, after students have worked on the task given in the form described above, they were asked to conduct the experiments with real frogs and fields. In any case, the students were directly able to cope with both the structure and the data generation represented in the task.

Analysing the rationales that students gave in a paper and pencil test (students worked alone on several tasks like the frog task) yield amongst other results (ibd.) that students tend to identify different objects of a situation to be mainly relevant for the data generation. The consequence of this finding was to re-analyse the tasks in terms of characteristics of mental model theory using the terms *data*, *objects* and *mental simulation*. Regarding the frog task we can state with regard:

- to the *structure* of the problem situation that *data* representing the situation are given by the visualised results of 10 different frog jumps as well as the relevant *objects* of the situation. The objects include *human* objects, i.e. the jumper, and

non-human objects, i.e. the frogs, the fields, and the interplay of both sorts of objects;
- to the *function* of the problem situation that the process of *data generation* impacted by the objects and their interplay is visible and a prediction of future results of the data generation necessitates a *mental simulation* of the situation.

The distinction of given information in a task and a possible request to make a prediction to future data by mental simulation yield a model of a hierarchy of tasks' complexity that we could back up in a study based on eight tasks that are based on the same principles of construction as the frog task (Eichler & Vogel, 2012):

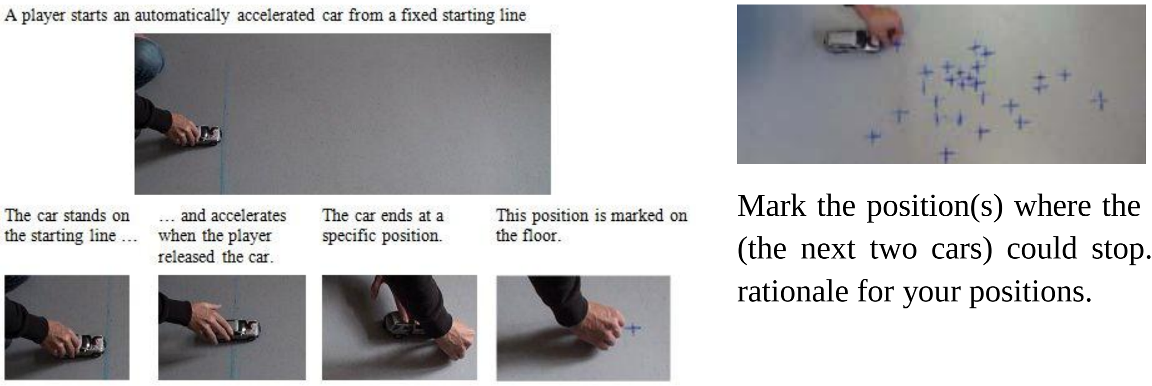
1. Lowest level: data given, human and non-human objects visible, mental simulation not requested (e.g. a task in which the results of coin throws have to be compared without making a prediction to future coin throws, see above);
2. Low level: data given, human and non-human objects visible, mental simulation requested (e.g. the frog task);
3. High level: data given, only non-human objects visible, mental simulation requested (this would be the case in the task concerning the weight of students, if it would be possible to develop a picture of the situation as described above);
4. Highest level: data not given, only non-human objects visible, mental simulation requested (e.g. a die task in which the students have to make a decision between two dice with different shapes to get a specific number).

Although we finished the development of tasks with identifying the levels of tasks' complexity, and used them with a bigger sample of non-schooled students, there were other aspects of the tasks that could be taken into account to develop a valid diagnostic tool for investigating young students' mental modelling of statistical situations.

FURTHER TASK DEVELOPMENTS

Our findings gave evidence that disclosing the impact of a human object on the data generation complicate a task for the students. However, our set of tasks includes one counterexample in which the visibility of a human object hinders the students' understanding of the situation, i.e. the so called car task (figure 5). According to the results of the interview study the car task is the only task with which's situation not all students were able to cope when they were asked to conduct the experiment shown in the task.

Some of the students' responses gave evidence that they make inferences (Schnotz & Bannert, 1999) of both, the formulation "an automatically accelerated car" and the pictures being obviously contradictory in respect to the intended meaning. In terms of mental model theory underlying the principles of our tasks' construction: For these students, the objects of the car task might be given (human impact: starter holding the car; non-human impact: car with a spiral spring that accelerates the car automatically) but their interplay might not be captured clearly enough by the description. For example, some students seem to overestimate the impact of the human object on the data generation: Some of the students’ answers give reason for assuming that these students might have thought that the displayed hand causes at least partly accelerating because they cannot see the accelerating spiral spring inside the car. Thus, those students were not able to make inference from the text to one object (the spiral spring) that is highly relevant for the situation’s structure and function.



Mark the position(s) where the next car (the next two cars) could stop. Give a rationale for your positions.

Figure 5: The car task

Based on these theoretical and empirical based considerations it is possible to change the situation’s representation according to the underlying theory of mental models. Since the disclosed, but relevant non-human object (i.e. the spiral spring) could be the main reason for students’ difficulties, we changed the situation’s structure concerning the acceleration of the car: Instead of using an automatically accelerating car we use curves as accelerating distance of a toy car racing track (figure 6).

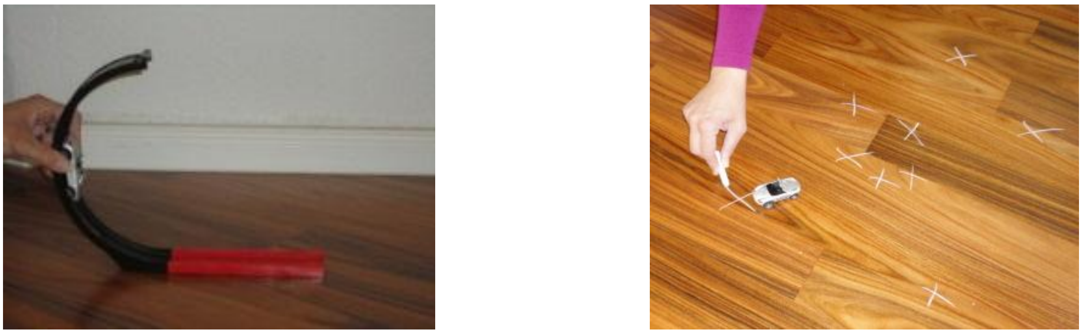
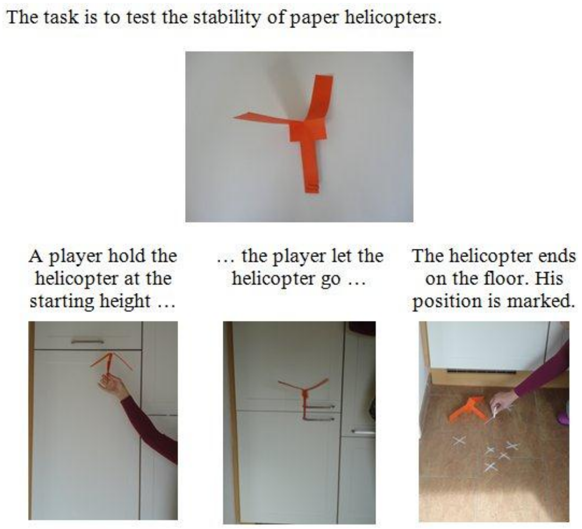


Figure 6: Use curves as accelerating distance of a toy car racing track in the car task

Thus, the assumed to be disturbing interplay between hand of the starter and car seems to be more clearly. Of course, this is a hypothesis which has to be proved.

According to our theoretical framework (founded by the theory of mental models and enriched by relevant aspects of theories of learning from multiple representations as well as of cognitive load theory) the principles of constructing new tasks in form of stochastics problem-storyboards could be applied concerning a considerable amount of existing tasks that potentially deal with centre and variation. Taking for example the question of design of paper helicopters (Ainley & Pratt 2010) we could design a task, represent it comparably to the other tasks in a stochastics problem-storyboard and characterize it along our principles of task construction (figure 7).



situation

A player starts a paper helicopter several times beginning from a fixed starting height. Each time the helicopters' end position is marked on the floor.

request

"Mark the position where the next helicopter could land. Give a rationale for the marks."

Figure 7: Paper helicopter task represented in a stochastics problem-storyboard

CONCLUSION

In this paper we proposed principles of diagnostic tasks' construction aiming to investigate non-schooled young students' thinking within statistical situations. The aim of the paper was to discuss the interplay between theoretical models and empirical findings as basis for the tasks' development. The development of valid diagnostic tasks representing statistical situations is a crucial point of our on-going research program into young students' mental modelling within simple situations of decision making under uncertainty.

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