# INVESTIGATING MANIPULATIONS IN THE COURSE OF CREATING SYMMETRICAL PATTERN BY 4-6 YEAR OLD CHILDREN

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*Summary. In this paper the role of gestures and manipulation in solving geometrical problems is investigated. Children aged 4–6 were subject to a series of observations during an experiment, aimed at finding a special placement for the figures in the symmetrical pattern. Results show, that rotation was taken as the first, most intuitive movement for them. Manipulation with rotation was taken independently on visual recognition of the relation of axis symmetry. It suggests that such approach can have a great impact on „tacit knowledge" used in further learning about geometrical transformations, and as consequence the dynamic imagination of rotation could be closer to acquaintance than other rigid movements on the plane.*

Key words: rotation, geometrical intuition, tacit knowledge, isometric transformations

## IMPORTANCE OF GESTURES AND MANIPULATION IN EDUCATIONAL RESEARCH

In recent years, there have been a number of works dealing with the role of gestures and manipulation in the process of solving mathematical problems (Edwards, 2005; Radford, 2005; Freitas and Sinclar, 2012). Gradually the whole theory for this type of research has been created. Generally, the relationship between the language, gestures and mathematical reasoning is considered. Sometimes, the distinction between different kind of gestures is made, for example between pointing and movements along the axis of the coordinate system (Bjuland at el 2005) or gestures indicating the relations between some parameters (Steinbring, 2005) According to some opinion, the gestures related to linguistic expressions stimulate dynamic thinking in real time among his subjects (Nunes 2004, quoted by Bjuland at el 2005).

The examples of research, mentioned above, are related to reasoning within the arithmetic or early algebra. But what about geometry? It seems that gestures, movements and any other manipulations can play a crucial role in geometrical thinking. They can replace a verbal utterance (using language at the lower level of geometrical reasoning is quite difficult). But first of all they can represent various geometrical relations and transformations. For this reason, investigating gestures in the course of solving geometric problems can have a high educational value.

## THE ROLE OF MANIPULATIONS IN THE DEVELOPMENT OF GEOMETRICAL THINKING

Theories that describe the development of geometrical concepts indicate how the process of geometrical reasoning functions. There is not much said about the very beginnings of the geometrical cognition. A Czech mathematician and teaching

specialist M.Hejny and philosopher of mathematics – Petr Vopenka are the few ones who I know among those who examine the beginnings of the development of geometrical terms and include it into the whole theory. According to their views, geometrical concepts ”emerge” from the surrounding world through a specific “geometrical sensitivity”, a kind of a sixth sense. “To notice something” is he first condition for the consciousness to focus on the geometrical phenomena. This first cognition is passive and static. Such an attitude is a mathematical specification of this, which developmental psychology defines as a place of visual thinking in the development of intelligence:
The use of the images is considered to be one of the basic characteristics of the thinking of preschool children (Jagodzińska, 1991). Epistemology of geometrical reasoning requires the transition to further levels, in which an imagining of dynamic changes is desirable. Geometrical reasoning is consistent with operational thinking: while solving problems, we create a new reality; passage to the new reality requires the use of dynamic images of changes. Thus, finding and describing the mechanism of creation of operational geometrical concepts, based on dynamic transformation of imaginations, becomes an important research issue.

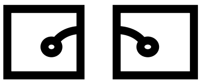
The relation between this research problem and Piagetian conception of development of thinking is especially important. According to well-known Piaget’s theory *child’s mathematical concepts emerge by operations and interactions with the real world. An action on the object leads to creation of schemata. As the results, through the process of reflective abstraction, actions can be replaced by symbols and words* (Inhelder and Piaget, 1958).

My previous investigations lead to the conclusion that 4- 6 year old children are able to act in the geometrical pattern environment (Swoboda, 2006). Children spontaneously arrange the plane, creating such relations between figures which may be described with the language of geometric isometric transformations. However, it is an activity which take place on the visual level, and child is interested only in the final arrangement of objects and the results of the action is verified visually.

It is worth to investigate very young (up to 7-year-old) children’s dynamic actions in such geometrical environment. Piaget is very skeptical about their ability to internalize their actions as operations. I accept this point of view. At this level we couldn’t expect the reflection on movement used but intuitive movements and gesture can create a solid base for the further, more conscious activities.

## RESEARCH – AIMS, RESEARCH TOOL

The experiment took place in March and April 2008. Children from a typical Polish kindergarten, aged 4 – 6, were subject to a series of observations (altogether – 60 children). This experiment was part of a broader study (Swoboda, Synoś & Pluta, 2008; Swoboda, 2009; Swoboda & Tatsis, 2010), but for the purpose of this paper I will focus only on the aims described below. Children were tested individually and all session were videotapes and transcribed afterwards. As a research tool two types of tiles were used (Figure 1). The tiles were arranged separately on the table.



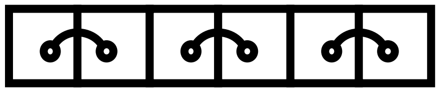


**Fig. 1 – research tool** **Fig.2 – a segment of the pattern prepared by a teacher**

At the beginning a child was asked to continue the pattern prepared by the teacher (fig.2). When a child was not able to create the regularity, the teacher helps him in this.

Later on the teacher replaced one tile from a pattern by a different one in such a way that the regularity was distorted. The child was asked to show where a change was done and "to repair" the regularity [1].

The basic research aim was to investigate movements used by children. The research questions were as following:

1. How did the children recognize the symmetry in the tiles (visually, or by using any action)?
2. What kind of placement of the two congruent figures provoked the children to make any movements?
3. What kind of movement played the most important role in children's actions?

For creating a pattern, it was necessary to use two kinds of tiles - the motifs on the tiles were either "left" or "right". The left motif presented was a mirror reflection of the other one, thus, none of the types could have been obtained by rotating the other tile. Additionally, there was no motif on the back of the tiles, which made it impossible to correct the distorted pattern by making movement out of plane.

Building and correcting pattern required manipulations. Some of them were not interesting for me (browsing the tiles on the table, pointing the "wrong" place in the distorted pattern). I was interested in movements related to the verification of tiles placements. Such movements were observed in both parts of the experiment: during creating the pattern and during correcting it.

## MOVEMENTS DURING CONSTRUCTING PATTERN

Two different strategies were observed in all children groups: 1. after visual recognition of difference between the tiles (placed on two separated piles) a child created pattern taking consciously two tile types, 2. a child starts by "blind searching", to connect tiles in order to obtain symmetrical configuration.

Example 1. Kacper (4-year-old boy).

Teacher: *Kacper, look at this pattern and try to continue it*.

Kacper: Takes one tile (the correct one) form the left pile, attaches it to the pattern but after a moment he starts to rotate it and later puts it back. Now takes a tile from the right pile, looks at this, says – no – and puts it back to the left pile. Takes another "right" one, visually compares it with the tiles at the table, put it over piles, but afterwards slides it under the right pile. After a moment he decides to take the right tile again and now he puts it in the pattern line, however without connecting it (keeping an empty place for one tile). In the second hand he takes another right tile and rotates it, trying to adjust to the gap between the pattern and non-connected tile. Next, he exchanges the right tile with the left one and constructs the whole symmetrical motif. He connects it with the pattern.

This boy stared from “blind searching”. The first movement, important for him, was the rotation. Although the first position he chose (and the tile) was correct, he felt the need to investigate its different placements.  It is clear that he didn’t know how to use the visual information in a constructive way, he was only able to state, that some of his choices were not proper. In spite of this, he was very persevering in the work. Thanks to manipulation he gains some experiences, useful for solving his task.

Older children worked on the visual level. They successfully utilized the information, that two piles contains different types of tile. But also in such a situation some of them felt the need to investigate whether it was possible to obtain a symmetrical position, by using two tiles of the same type. It is visible in the work described below:

Example 2 . Martynka (6-year-old girl) .

At the beginning she takes one tile from the right pile, later on a second tile from the left pile. She joins them and puts together as the pattern’ continuation. After that, she takes simultaneously two tiles from two different piles and using them creates another motif. From that moment she works very fast, sometimes taking simultaneously tiles, sometimes – taking one tile at time, but even in that situation she is aware where the tile should be put (fig.3)  She uses almost all the tiles form the table. Two last tile were of the same type. She takes them into two hands and starts to manipulate – for a long time (23 sec.). She rotated them, trying to connect (fig.4). After that she looks at the teacher, by this informing that it is impossible to use these tiles for the pattern.

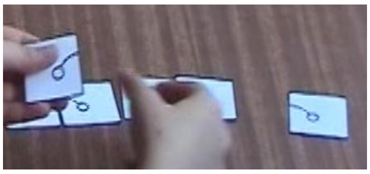


**Fig. 3**

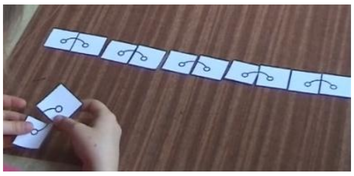


**Fig.4**

Martynka’s work, described above, was not typical for older children. It seems that for the 6-years-old children the visual identification of one type of tile stopped any actions on them. It is clearly visible in the Example 3, described below:

Example 3. Karolek (6-year-old boy).

K: At the beginning of his work he makes some trials and afterwards he works very fast, taking successively the correct tiles form the table without any doubt. In this way he builds a very long pattern, extending it on the right and left side. At the end, there are three identical tiles on the table left (fig.5).

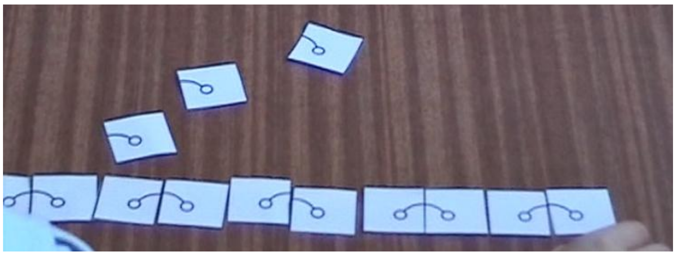


**Fig 5**

The boy sits still (18 sec.), looks at the tiles.

T: *Do you still want to work*?

K: *no*.

This boy preferred to make a visual analysis than a manipulation – supposedly, the visual information was more important to him. In addition – he knew how to use these information. Perception was the foundation for each of his decisions, manipulations only supported and verified the undertaken actions. At the end of his work no action was needed – it was clear for him, that tiles are ‘the same”. Those tiles were placed “almost” parallel, the child didn’t feel obligate to make movements to check anything.

## MOVEMENTS DURING CORRECTING PATTERN

Again, two different strategies emerged here. The first one – “replaced strategy”, when the child exchanged the tiles, taking the proper one from the table. The other one – “manipulative strategy”, when child tried to obtain a correction by manipulation of tiles lying in the pattern.

Almost all 4-year-old children started their work from manipulation, making rotations. This way could be independent from the previous stage (creating pattern), where they differentiated “right” and “left” tiles.

Example 4. Zuzia (4-year-old girl)

Teacher: *You built a very long pattern, it is enough for us. Now, please close your eyes, I will change something* (she distorted the regularity). *Open the eyes and say if there is something wrong.*

Zuzia: (7s.) *here* (she shows by her hand, pointing a place in the pattern)

T: *why?*

Z: *Because here is in this direction and here in this one* (showing).

T: *so, please, correct it.*

Z: immediately starts to rotate - firstly by one tile, than by both tiles. Later she moves two tiles close to her, still making rotations. Movements starts to be slower and slower. Finally she takes one tile from the table and finalizes her work.

T: *Perfect! But – did you notice, that the tiles were different in two piles?*

Z: *Yes*.

At the first part of the experiment (creating the pattern) Zuzia muddled up all the tiles on the table. Teacher, wanted to help her, decided to tidy up and put tiles into the proper piles. After that the girl benefits from this. We may make a conclusion, that this “teaching episode” was too weak for Zuzia to take an advantage of in the next stage of the experiment. Therefore it seems that maybe we should focus on children who in own way distinguished two types of tiles. Ola (4-year-old), described below, is one of these children. But, while correcting the pattern, she started from rotations, too.

Example 5. Ola (4-year-old).

O: (2s.) she takes one “left” tile from the pile, puts it in some distance from the patter (to keep a place for the “right” tile). She moves her hand to the same pile, but immediately recognizes, that this is not what she needed, than takes the “right” tile and makes the whole motif. Next motives she builds very fast, creating long patterns (through the whole table).

T: *Fantastic! And now I will give you a riddle (*she changes the tile*).Tell me where something is wrong?*

O: (immediately) *here* (she removes one of the double tiles, takes it in the hand – fig. 6).

T: *any why*?

O: *because it should be differently*

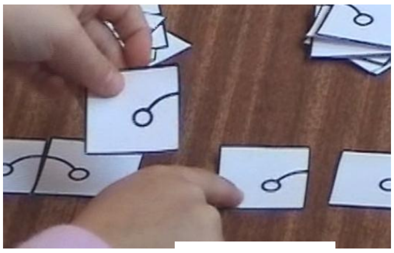


**Fig 6**

She moves one of the ”left” tiles to the right place in the pattern, the second one rotates by $180^0$. Comes back to the first tile and rotates it many times (fig. 7). After some time she changes the action – she starts to manipulate with the second tile (fig.8), and in a moment she turns it on the blank side (fig. 9). She makes some other rotations, one by one with different files. Sometimes she changes an order of tiles (making shifts). Finally she stops to manipulate, keeping the tiles in her hands.

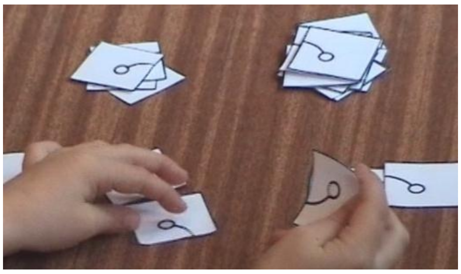


**Fig 7**

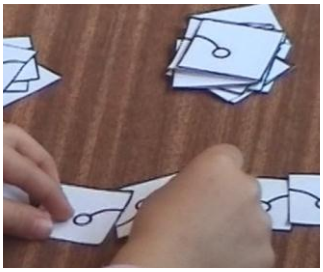


**Fig 8**

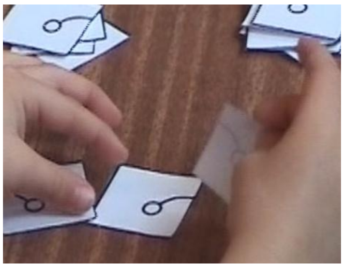


**Fig 9**

T*: is it possible to correct the pattern using those tiles?*

O: *no (with determination).*

T: *Do you know what I did? I replaced one tile by the other, from the table.*

O: she immediately takes the correct tile from the table and finalizes the work.

Ola showed a great awareness of how to build pattern. In spite of this, she starts its correctness from manipulations. She makes lot of movements, which have different meaning. First movement – parallel shift – is used for the convenience only. Movements used for searching for solution are rotations. She started from rotations, after that an idea of the mirror reflection emerged (when she turned a tile to the back side). If the tile would have been printed on both sides, she would have been successful. In the present situation Ola came back to the rotation, trying to compose the rotation with translation. After an investigation she states that it is impossible to solve this task.

Approach to the ways of repairing patterns vary in subsequent research groups – older children used replaced strategy more often. It is visible in the Table 1

| Age | Numer of children | Manipulative strategy | Replaced strategy | Helpless |
|---|---|---|---|---|
| 4 | 18 | 13 (72%) | 4 | 1 |
| 5 | 25 | 9 (36%) | 14 | 2 |
| 6 | 17 | 6 (35%) | 10 | 1 |

**Table 1: Approach to the ways of repairing patterns**

## DOES A LACK OF SUCCESS IN REPAIRING PATTERN BY USING ROTATION FORCES A REFLECTION ON PERFORMING MOVEMENT?

It seems that the answer for this question should be positive. It is supported by children's reactions, described in examples presented in this paper: after some trials with manipulations, children draw conclusion, that such actions are pointless. Although their attention was focused on the visual recognition of the motif, performing many repetitive rotations without obtaining expected result caused some awareness about its features. Children were conscious that it is impossible to achieve some placements of the figure by using it.

In order to illustrate the opinion that children can differentiate various types of movements, I use an example taken from the following stages of this experiment. The session took place some days later. In this session, the child who participated in the first session took a role of a teacher and tried to lead an experiment with his/her colleague from kindergarten [2].

Example 6. Nikodem (5-year-old boy).

Nikodem's task is to repair the pattern distorted by his colleague, Paweł. (fig. 10)

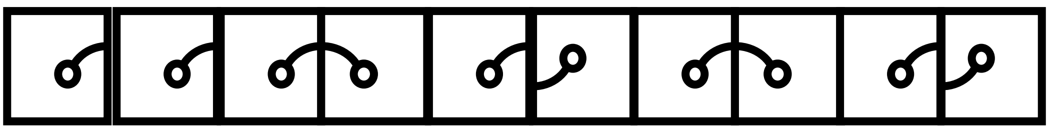


**Fig 10**

Ni: *Here the turnings are upside down* (he shows 6th tile). He rotates the tile over the table by $180^0$ , looks at it, puts into a gap in a “normal” position. He changes the last tile in the same way. He stops to – to think. After while he changes the order. Stops again, than he looks at the pattern.

Teacher: *So, Paweł gave you a very difficult riddle*.

Ni: *Can I take from here?* (he shows at the tiles on the table)

T: *You can use whatever you want.*

Ni: Immediately he takes the proper tiles and corrects pattern.

Diverse manipulations were aimed at obtaining different aims. The first rotations were done in the clear situation – tiles were put upside down. Then, looking for the solving strategy, Nikodem uses translation – not rotation anymore. When also by this he didn’t get expected results – he stops any further action.

**SUMMARY**

Observing children’s work it was clear that the possibility of manipulation played a great role for them. Rotation was used as the main movement for investigating of changes in figure’s placement. The reader can rightly state that children’s behavior was provoked by the research tools. It doesn’t change the fact that the children presented many behaviors (mainly during repairing pattern) that require deeper interpretation. Manipulation indicate for a need and a manner of such discovering, going beyond on visual recognition of geometrical phenomena.

Referring the results of observations to basic research questions, I conclude that:

1. Deciphering visual information concerning mutual position of two congruent figures clearly falls into two levels. The first one can be defined as ‘I know that’ and the second as ‘I know how’ On the first level, each possible arrangement is perceived on the level of impression and some arrangements are aesthetically preferred. Axis symmetry and parallel shift belonged to these preferred ones. But it does not give a sufficient basis to understand what actions can lead to an expected position of one figure in relation to another. Such understanding, knowledge ‘know how’ develops with age, what should be associated with gaining more and more experiences (both visual and manipulative). Younger children started from ‘blind’ discoveries, using manipulations and checking their effects visually. For older children, visual information was frequently sufficient enough.
2. Former information (visual and manipulative), that for axis-symmetrical position two different tiles are needed, does not have to prevent further study of this position through movement. It could be observed in children’s behaviour while arranging the pattern and it was even more explicitly marked while improving the band. The fact that children from their own wouldn’t

decide to exchange (replace) the tile can be interpreted twofold. One of the possible explanation is “didactical contract”, evoking the tacit conviction that if the teacher asks to repair the pattern that it is possible to do with the use of such tiles which are already used only. Even if so, children showed what are the sources of their actions which appear to be effective for such tasks. Other explanation – (quite obvious) is the lack of knowledge of the properties of isometric transformations. They could have known, that the tiles are right and left, but not, that by using “right” tile they are not able to receive the “left” one by making the physical manipulation on a plane. In mathematical terminology children could assume that the composition of rotation and translation would lead to a mirror symmetry.

3. When a child *searches for* an appropriate arrangement of two congruent figures relative to each other it starts from rotation. Rotations as treated as the basic way of object transforming. They are like an elementary tool used in solving problems with placement of figures on plane.

Children's behaviours related to the use of rotations are easy to explain: In the rotation and mirror reflection pieces looks differently than the model, unlike in the parallel transformation where pieces looks the same as model. In translation an identification of figure and its image is immediate and doesn’t require any conscious action. In rotation and mirror reflection the identification is easier after manipulation. Therefore such placements forces making manipulations. The manipulation of flat figures by rotations is more natural (and therefore more spontaneous) than mirror reflection.

However, that is not the explanation that I consider as the most important effect of the carried out research. Much more crucial to me is the statement that children *need* manipulations even in a situation where the former visual cognition suggests senselessness of such actions. This indicates child’s need to examine various solutions through movement, the visual information proves to be insufficient.

In psychology, an archetype is defined as reflection or instinctive reaction to the particular situation. Intuition can be treated as thinking on visual level. The results, presented here, clearly show that when child look how to compare one figure with another one placed differently, he/she starts from rotation. The child's attention in such action is still directed by the arrangement of two pieces in relation to each other and not to the movement as such, but these results can suggests that rotation can be used as the first tool for turning children’s attention on movements on the plane.

## NOTES

1. Here, only one part of the research’ scenario is presented. More detailed description can be found in Swoboda, E; *Natural differentiation in a pattern environment (4 year old children make patterns),* Proceedings of CERME6,

2. In Poland this research method is quite popular. It is taken for observing the resistance of children’s behaviors.

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