LEARNING AND TEACHING GEOMETRY AT THE TRANSITION FROM PRIMARY TO SECONDARY SCHOOL IN FRANCE: THE CASES OF AXIAL SYMMETRY AND ANGLE

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We are currently running a three year research project focusing on the teaching and learning of geometry at the transition from primary to secondary school in France. In a global approach, we take into account teaching contents, pupils and teaching practices at the same time. This research aims at identifying continuities and gaps in curriculum, textbooks but also in teachers' practices, in order to get a better understanding of the difficulties related to this transition. In this communication, we present the first results of the part of this project dedicated to axial symmetry and angle. Our analysis shows a large variety of choices concerning crucial aspects of the concepts, likely to facilitate the transition or make it more difficult.

INTRODUCTION

This paper presents the first results of a three year research project that started in January 2012. This study focuses on the teaching and learning of geometry at the transition from primary to secondary school in France. This transition is decisive for pupils future education but difficult for some of them, and may even lead to academic failure, in France as in most of the industrialized countries. Previous studies on mathematics teaching dealing with the questions of transition generally approach these questions from the angle of pupils' difficulties, or they compare the intended curricula showing for example the conceptual changes (see for example Salin, 2003). Colomb *et al*. (1987) take into account teachers too, but they are mostly focused on the teachers' representations of mathematics and mathematics teaching; however, later research (Robert, 2007) showed that representations are not the only factors which influence teachers' practices: Robert distinguishes institutional, social and individual determinants. More recently, Bednarz *et al*. (2009), aiming at developing connections between both orders of education (elementary school and middle school), also point out the necessity of considering teachers' point of view. Given that the involved processes are very complex, we choose to tackle the question with a global approach. We take into account several aspects of the problem (institutional, teachers' and pupils' points of view) and their connections. We thus study the following research questions: what is the nature of pupils' difficulties? In what extent are they connected to the notions? Can we identify, in curricula, textbooks and teaching practices of both levels, elements which can explain these difficulties or, on the contrary, facilitate the transition? Another specificity of this project is to focus on some particular concepts. Eventually, our aim is also to develop resources for teachers and devices for teachers' training in order to facilitate the transition. First datas were collected between January and June 2012. They include videos of class sessions, textbooks and the results of some tests pupils were submitted to.

In this communication we only expose the first results of the project part related to axial symmetry and angle. Those two mathematical subjects are chosen because of their ability to reveal various phenomena concerning the transition, as we will develop in the first part through a curriculum study. Using literature and our previous research, we then point out some crucial aspects of these two concepts as subjects of teaching and learning (including conceptions, difficulties and recommendations for teaching). In the third part, we study textbooks from both levels of education: we consider them both as resources for teachers to help them interpret the official instructions and as examples of classical tasks pupils are exposed to in classrooms. Finally, in the fourth part, we show some examples of ways teachers deal with these crucial points. Let us add that in the two last parts, we complete the analyses with the results of the tests.

CURRICULUM STUDY

In France, the official instructions for 5th grade (last grade in primary school, 9-10 y. o.) and 6th grade (first grade in secondary school, 10-11 y. o.) show few differences on the subject of geometry. However, a change of status of the objects, from drawings to figural concepts (Fishbein, 1993) has to be initiated in 6th grade in order to prepare a transition from the paradigm of geometry 1 (G1) to geometry 2 (G2) (Houdement & Kuzniak, 1999). This change of status is explicited in the introduction of the official instructions (MEN, 2008b) but the way it has to be adapted for each subject is not detailed. This transition is supposed to be complete in 8th grade (Houdement, 2007).

Axial symmetry is one of the subjects usually chosen by textbooks and teachers to initiate this change (Chesnais, 2012). In primary school, pupils learn how to draw mirror images on graph paper, how to draw them on plain paper using tracing-paper and folding and how to identify axes of symmetry on a figure. The work is validated by perception or using tools such as tracing-paper. In 6th grade, pupils are supposed to extend their knowledge to the case where the axis crosses the figure and to learn how to construct mirror images on plain paper using geometrical tools (ruler, set square and compass). The validity of the constructions is related to mathematical definitions and properties of the objects.

Concerning the angle concept, elementary school instructions (MEN, 2008a) introduce the right angle in grades 1-2 (cycle 2) when pupils learn to distinguish between different geometric shapes (triangle, square …), but the general angle concept is introduced in grades 3-5 (cycle 3). In both primary and secondary school, this concept is present in two parts of the instructions: “geometry”, where pupils study plane figures, and “attribute and measurement”. In grades 3-5, pupils learn how to compare angles and how to reproduce a given angle. They discover the different angles, and they must use a set square to validate their estimation of the acute, right or obtuse character of an angle, tasks that relate to paradigm G1. In 6th grade (MEN, 2008b) in the "attribute and measurement" part of the instructions, pupils are firstly supposed to compare angles without measuring them (using templates or tracing-paper); the main teaching objective is then learning how to measure angles with a protractor (using degrees), and how to construct an angle with a given measure, which are tasks related to G1. However the teaching of angle concept in 6th grade can also contribute to the transition from G1 to G2: some tasks may involve deductive reasoning based on the properties of the figures and not only measuring.

Some researchers suggest ways to deal with this transition problem. For example, Perrin-Glorian (2003) suggests a way to get primary school pupils used to considering figures according to their properties; she notably created a new kind of construction tasks called "figure restoration": pupils are given a figure and the beginning of a reproduction of it; they have to complete it using geometrical tools. In another perspective, Houdement and Kuzniak (2003) question the relevance of teaching Geometry 2 within compulsory education. Without entering this debate, we consider that, if Geometry 2 is part of the curricula, besides special tasks, a specific work has to be done expliciting the "rules of the mathematical (geometrical) game".

What we will seek for in textbooks and teacher's practices are evidence of how they deal with this change of status of objects. In 5th grade, how do they prepare pupils to this change? In 6th grade, how do they deal for example with the apparent contradiction which is inherent to requiring measurement of angles in some tasks and considering it as non relevant when the measure of an angle has to be determined knowing the measure of its mirror image and the property claiming that symmetry preserves angle measures? We expect, for these two subjects (axial symmetry and angle), to observe phenomena of different nature, since they play different roles with regard to the transition from G1 to G2.

CRUCIAL ASPECTS OF AXIAL SYMMETRY AND ANGLE CONCEPTS

Mitchelmore and White (1998) underline that angle is a highly complex and multifaceted concept, which is constructed slowly and progressively. The construction process runs into numerous obstacles (see for example Lehrer *et al.*, 1998). The major one is that many pupils think that an angle's size depends on the length of its sides (Wilson & Adams, 1992; Berthelot & Salin, 1994-95; Mitchelmore & White, 1998). Another element likely to hinder the conceptualization of angles and reported in several experimental studies (Baldy *et al.*, 2005, Lehrer *et al.*, 1998) is the prototypical conception of right angle (a right angle that opens on the right with arms parallel to the edges of the paper). The connection between those two obstacles was even recently pointed by Devichi & Munier (2013). According to van Hiele (1986), learning can only take place if pupils actively manipulate and experiment with geometric objects in relevant, suitable contexts. In the same way, Mitchelmore and White (1998) point out the necessity of drawing upon children's informal knowledge to teach them geometric concepts like “angle” (White & Mitchelmore, 2010). In France, Berthelot and Salin (1998) stress a similar idea in saying that pupils should be taught geometric concepts using concrete activities, and they propose an adidactic introduction of angle, entitled Geometriscrabble (Berthelot & Salin, 1994-95). These authors recommend also that pupils experiment in meso-space. Taking this approach, we proved the interest of using physical situations involving angles that pupils had to model (Munier & Merle, 2009). Mitchelmore and White (1998) suggest another way of invalidating side length: using situations involving both static and dynamic angles, which can be done using technology (Clements & Sarama, 1995). Other studies suggest using body movements to apprehend the angle (Wilson & Adams, 1992; Fyhn, 2008).

Using Vygotski’s terms (Vygotski, 1962), we claim that axial symmetry is not only a mathematical concept but also an everyday concept (Chesnais, 2012). Moreover, we distinguish symmetry as an innate property of a figure and as a geometric transformation: we respectively name it the static and the dynamic aspects of symmetry (Chesnais, 2012). Mathematically speaking, the transformation precedes the property, since the last one results from the invariance of the figure under the transformation. In the everyday concept, the transformation is almost absent, except in the folding movement, but in this case a symmetrical figure is a figure such that half of it is the image of the other half: the symmetrical character of the figure is not associated to global invariance. We claim that linking both aspects is necessary in order to achieve the conceptualization of symmetry. This led us to point out one of the main misconceptions of symmetry, which is partly related to folding: symmetry as a transformation moving from one half-plane onto the other one. Overcoming this conception is necessary to conceptualize the fact of being symmetrical as global invariance. This misconception and the fact of not being able to link the static and dynamic aspects of symmetry don’t prevent pupils to successfully perform classical tasks like constructing the mirror image of a figure located on one side of the axis or identifying axes of symmetry on a single figure. Some tasks may allow children’s conceptions to change. For example, constructing the image of a figure crossed by the axis requires considering the symmetry not only as a one-way transformation. Also, having to complete a figure for it to become symmetric (which is a task explicitly recommended in the official instructions from 4th grade) requires to link the static and dynamic aspects of symmetry (Chesnais, 2012).

Curricula remain vague about what should be aimed at in terms of level of conceptualization concerning symmetry. In a G1 perspective, the static aspect of axial symmetry could be taught in a perceptive way or related to folding but it could not then be related to global invariance. However, global invariance can be considered in a G1 perspective, when associated to flipping tracing-paper over instead of folding it. In the perspective of the transition to G2, one must find a way to validate constructions or the existence of axes of symmetry without using instruments. It requires considering axial symmetry as a plane transformation acting on points. Let us add that previous instructions (until 2005) explicitly required teachers to distinguish between the transformation and the innate property and to define the last one as global invariance.

The above analysis shows the complexity of the concepts of angle and symmetry. Given that curricula don't contain details about how to handle these difficulties for each teaching level and how to handle the transition (particularly the change of status of geometrical objects), we can expect difficulties for both pupils and teachers. For instance, 6th grade teachers might overestimate pupils' knowledge about the status of the geometrical objects and figures or the meaning of measurement. They might also underestimate the difficulties like the importance and persistence of misconceptions. In the following parts we analyze textbooks and teaching practices to identify how they deal with these questions.

TEXTBOOKS STUDY

In this part, we study lessons and exercises proposed by textbooks in order to analyze the way they deal with angle and symmetry; in particular, we will look for tasks designed to help pupils to overcome the above-mentioned misconceptions.

For axial symmetry, we studied eight elementary school's textbooks: *Cap maths* (Hatier), *Outils pour les maths* (Magnard), *Petit Phare* (Hachette), *La clé des maths* (Belin), *Au rythme des maths* (Bordas), *Euromaths* (Hatier), *J'apprends les maths* (Retz), *La tribu des maths* (Magnard). We also studied eight textbooks of 6th grade: *Multimath* (Hatier), *Dimathème* (Didier), *Bréal*, *Transmath* (Nathan), *Magnard*, *Diabolo* (Hachette éducation), *Triangle* (Hatier) and *Phare* (Hachette éducation). Concerning the angle concept, only a few of them were chosen as being representative of the variability of approaches: *Euromaths*, *La tribu des maths* and *J'apprends les maths* for elementary school and *Triangle*, *Phare* and *Sésamaths* (Generation 5) for 6th grade.

At both 5th and 6th grades levels, we could find some textbooks proposing exercises designed to make pupils overcome the sides' length misconception but also some which don't include any exercise of this type, the teacher's handbook sometimes not even mentioning this difficulty. One of the 5th grade textbooks (*Euromaths*) introduces angles using the Geometriscrabble situation designed by Berthelot and Salin.

At elementary school level, concerning the introduction of angle measurement, we could find one textbook which introduces angle measurement using 1° templates, anticipating the 6th grade official instructions: it even includes tasks in which pupils have to construct figures knowing the measure (in degrees) of some angles. Making other choices, *Euromaths* proposes tasks using fractions of a right angle (in compliance with the 5th grade instructions) which can help pupils to construct the meaning of angle measurement unit.

In 6th grade, the way the different textbooks introduce angle measurement is variable, from textbooks taking explicitly into account the introduction of measurement (using arbitrary units before degrees), to textbooks introducing it very quickly, directly using degrees (supposing obvious for pupils that the measure of the angle formed by two adjacent angles is the sum of their two measures).

Tasks related to the paradigm G2 are present in all the textbooks: for example, exercises where pupils have to calculate measures of angles, reasoning on freehand drawings. The particularities of this kind of tasks remain often implicit but we could find one textbook proposing an exercise and its solution which clarifies explicitly the status of observation, measurement and demonstration.

Concerning axial symmetry and elementary school, about the link between the static and dynamic aspects, only five out of eight textbooks contain at least one task consisting in completing a figure for it to become symmetric, although it is recommended in the official instructions; five of them contain a task consisting in identifying axes of symmetry on figures constituted of two different parts. Only two of them (*Petit Phare* and *Euromaths*) do both. These two textbooks are also the only ones trying to make pupils overcome the misconception of transformation from one half-plane onto another: the first one by adding elements on both sides of the axis when completing a figure for it to become symmetric; the second one proposes an exercise where the number of axes of symmetry of a figure is related to the number of ways it can be positioned to match its outline after flipping it over. The *Euromaths* teacher's handbook mentions the objective of enriching pupils' mental pictures. It also suggests teachers to experimentally bring up some properties of symmetric figures like the invariance of the axis' points or the fact that a segment joining a point and its image is perpendicular to the axis. In another perspective, *La Tribu des maths* mixes perceptive tasks and constructions of mirror images on plain paper using instruments, which should be dealt with only in 6th grade.

Half of the 6th grade textbooks separate what corresponds to the static and dynamic aspects of symmetry in two different chapters. Three of them show double figures in the chapter devoted to the axes of symmetry: this facilitates links between the two aspects. About the static aspect, three of them restrict the work to perceptive tasks; half of them mention global invariance but sometimes in an illogical way: *Transmath* introduces global invariance in the lesson right after exercises on folding and before bringing up the transformation; *Phare*, *Multimath* and *Bréal* link folding and global invariance but only *Multimath* mentions it in the lesson. All the textbooks contain constructions of figures crossed by the axis of symmetry.

Finally, we can say that the textbooks' approaches are very different. What we note is that textbooks handle the transition problem in various ways. Namely, some of 5th grade textbooks choose to introduce notions or tasks which correspond to 6th grade's curricula expectations whereas other ones make different choices: for example, *Euromaths* tries to prepare pupils to 6th grade precisely by working on the difficulties, the misconceptions and specifically on the meaning of the concepts (dynamic and static aspects of symmetry, meaning of measurement unit for angle etc.). Depending on the textbooks, it then appears that misconceptions are not necessarily handled in 5th grade. On the other hand, most of 6th grade textbooks consider that some aspects of the two concepts have already been grasped by pupils. Hence, the responsibility of dealing with certain aspects of the concepts and enabling pupils to overcome their misconceptions is devoted to teachers.

TEACHING PRACTICES

The methodology we use to study teaching practices focuses on the mathematical activity the teacher organizes for students during classroom sessions and the way he manages the relationship between students and mathematical tasks in two approaches: a didactical one and a psychological one (Robert & Rogalski, 2005). However we'll only mention here the didactical analysis. We worked on videos made during 'ordinary' lessons on axial symmetry and angle in a six 6th grade classrooms. In each class, pupils were submitted to tests designed by the research team. For each teacher, we collected all the videos of the sessions concerning these notions, even if we will only mention in this paper some short extracts from four classrooms (the four teachers are named Fabien, Marianne, Maryse and Sébastien). Our examples aim at showing how some 6th grade teachers deal with the characteristics of the concepts we mentioned in the second part of this paper.

Example 1: Misconceptions about angles (Fabien, Marianne, Maryse)

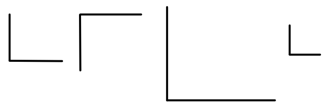
Concerning the sides' length misconception we have seen that instructions recommend starting with comparison activities independently of measurement, and that some textbooks propose such exercises which may facilitate the overcoming of this misconception. Yet the analyses of teaching practices show that some teachers use these exercises (for example Marianne and Maryse), without seeming fully aware of what is at stake. Some of them seem to consider that it is not necessary to work further on this misconception. However, the misconception of the sides' length still appears frequently in both grades when pupils are asked to compare pairs of angles (for one of these pairs, the rate of success in the tests varies between 14 and 67 % in 5th grade and between 43 and 86 % for 6th grade).

As a second example, the following extract of the transcript shows that this misconception and the salience of the prototypical right angle are underestimated by Fabien:

P: What is an angle for you? […] Océane?
Océane: An angle it is like a right angle
P: a right angle, OK, a right angle is an angle.

> Teacher then asks Océane to draw it and she draws a right angle. Then he asks her to draw another angle which should not be a right one. She draws again a right angle, modifying only orientation and sides' length [making it less recognizable]. The teacher agrees and says that this angle is too small to be seen from the back of the classroom, then he removes all the drawings and draws two obtuse angles, totally different, with arbitrary orientation of the sides and he goes on with the lesson.

The results of the tests we ran show the salience of the prototypical right angle in both levels: for example when pupils are asked to draw an angle, then a different, a smaller and a larger one, numerous pupils are unable to produce a different angle or to change its size (see example below).



The rate of this answer varies between 0 to 57 % in 5th grade, and between 0 and 28 % in 6th grade).

Example 2: Misconceptions about symmetry (Sébastien and Marianne)

Both Sébastien and Marianne propose to their pupils a task where they have to overcome the misconception of symmetry as a transformation from one half-plane onto another: Sébastien's pupils have to construct the mirror image of a polygon crossed by the axis on graph paper; Marianne's pupils have to identify if two triangles crossed by a straight line are symmetrical of each other with regards to this line or not. In both classes numerous pupils seem unhinged by the task and ask questions to the teachers; for example, in Marianne's class: "Can we still talk about symmetry when the two figures cross each other?"

Sébastien's reaction reveals that he underestimates the difficulty, ("nothing makes it impossible", "don't forget to construct point C's image"). He mentions the rigor when using the techniques, instead of using a conceptual argument. Marianne uses the folding of a piece of tracing-paper to make pupils check that the two parts of the figures coincide one-by-one and conclude that the two triangles coincide: she uses the fact that the transformation is working in the two directions.

Note that the results of Marianne's class are much better than Sébastien's one for the task of the test where pupils have to construct the mirror image of a figure crossed by the axis.

Example 3: Transition from G1 to G2

Some items of the tests were designed to determine whether 6th grade pupils are able to mobilize the paradigm G2 or not. We ask them to draw an angle measuring 89°, then to say if this angle is a right one or not and to justify their answer. In the 6th grade classes a large part of the pupils (between 11 and 57 %) base their estimation on the validation with the instruments, protractor or set square. We identified moments of classroom sessions when problems related to this question arise. We are currently analyzing these extracts to seek an understanding of how teaching practices affect pupils' activity.

CONCLUSION

Our analyses point that some of the choices done by textbooks' authors and teachers in 6th grade may cause difficulties for pupils during the transition from primary to secondary school. Some difficulties could also be related to what happened before in pupils' schooling. Indeed the results of the tests at both levels show that pupils haven't overcome the main misconceptions. We have namely highlighted very important gaps in textbooks.

This research project has just started but the first results we expose here seem to confirm the validity of a global approach in order to understand pupils' difficulties in the transition from primary to secondary school. However, more data has to be collected and the analyses have to be completed in order to confirm our hypotheses and especially to study to what extent the observed teaching practices are representative of ordinary practices. Another objective is also to try to evaluate their effects on pupils' learning.

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