GEOMETRICAL ASPECTS OF GENERALIZATION

Marta Pytlak

University of Rzeszow, Poland

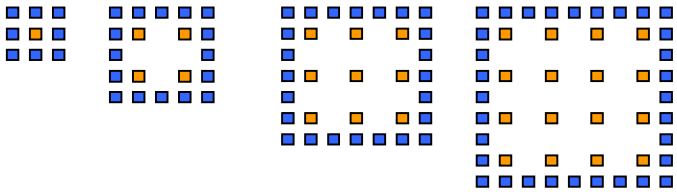
Generalization is considered one of the most import ant processes involved in mathematics. As Mason says: generalization is the heartbeat of mathematics (Mason, 1996). Whether it is viewed as part of higher level process, like abstraction (Dreyfus, 1991) or as the core process involved in a particular mathematics field, like algebra (Mason, 1996), there seems to be an agreement on its significant role in advanced mathematical thinking.

Discovering regularity and generalization are the kind of bridge between arithmetic and algebra. Looking on the historical development of algebra we do not omit the fact, that geometry have given many merits on this field. How could the geometrical aspects of generalization look like?

Methodology

The research, which was conducted among fifth-class students from primary school, aimed to find the answer to the following questions: How do the pupils form primary schools deal with a task involving discovering and spotting regularities? Are they able to make generalizations within some noticed rules?

The task that served as our research tool was an arithmetic-geometric and allowed a lot of different interpretations: there was given the sequence of four figures, each of them was built with two kind of elements. One is supposed to take how many elements we need for particular figures – not only those from the task, but also the next ones. The aim of the task constructed in this way was to check whether the students are able to discover the regularities occurring in the task in order to use them in their later work.



The research was conducted in school conditions, during normal classes. 38 students took part in this research. The students were working in pairs and their work was recorded. The atomic analysis of students' work and the atomic analysis of the video mentioned above were the research method that we used. Some additional information was obtained through the conversation with the teacher who was present during the whole experiment. The conversation took place not only after the finished work but as well during performing the task by the students.

Description of students' work

students started their work by counting the elements located in the pictures and then they filled in the table with the results of their work. Filling the first four lines took them comparatively little time. At this stage of solving the task any trials of

discovering regularities were spotted. They did not find any rules until they reached the fifth line of the table (to which no illustration was attached) which made them think about the task and looking for the proper solution. The quest followed different ways, and its analysis let us distinguish the following three types of reasoning:

- *arithmetic*: ignoring the pictures and paying attention to the numeric data taken from the table; finding and discovering arithmetic connections between number given in the table; students' work rely on an analysis the number column from the table filled to the fourth line
- *geometric*: paying attention mainly to the pictures, spotting geometric connections; strong visual aspect; students' work rely on an analysis figures' pictures and drawing the fifth figure and the next ones, work connected with drawing the next pictures of figures
- *arithmetic-geometric*: both the table and the pictures were taken into account at the same extent, finding geometric connections and combining them with arithmetic ones; replacing geometric connections with arithmetic ones; students' work rely on an analysis existing pictures of figures and then filling the table.

Results

The most frequently ways of work were generalizations within the limits of arithmetic dependences, which based on the number sequences. But some students started their work direct the figural property. Sometimes this approach did not end with success. In geometrical aspect student have to take into consideration much more factors. In this particular case the figural aspect have to be connected with the length of sides or an area.

The analysis of existing figures run in different ways: some students paid attention on the mutual position elements in the figure; some of them analyzed particular parts of the figures and the others analyzed connections between next figures Take as a whole. In poster I presented the examples of students work classified to emphasis category. Those example one could interpret in the light of geometrical paradigm or van Hiele levels of understanding geometry.

References

Dreyfus, T., (1991), Advanced mathematical thinking processes, in: D. Tall (Ed.) Advanced mathematical thinking, Dordrecht, Netherlands: Kluwer Academic Publisher, pp. 25-41

Houdement, C., Kuzniak, A. (2003). Elementary geometry split into different geometrical paradigms. *Proceedings of CERME-3*. Bellaria, Italy

Mason, J. (1996) Expressing generalaty androots of algebra, In: N.Bednarz, C.Kieran & L.Lee (Eds), *Approaches to algebra*, Dordrecht, Netherlands; Kluwer Academic Publisher, pp. 65-86