Preservice Teachers' Knowledge and Beliefs: Their Association to Practice in the Context of Teaching Function with Analogies

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The present study concentrates on pre-service teachers' knowledge and beliefs and their association to teaching practice in the context of teaching function with analogies. During the first phase of data collection, pre-service teachers completed a knowledge test concerning the definition of function, its types, and analogy. In the second phase, pre-service teachers were observed and videotaped during their teaching practice. In the last phase, interviews were conducted upon the completion of pre-service teachers' teaching practice to yield data about their beliefs about using analogies. Results revealed that pre-service teacher knowledge and beliefs about function and analogy strongly associated with the nature and the extent of analogy use in teaching the function concept.

Keywords: Function, analogy, pre-service teachers, teacher beliefs

INTRODUCTION

There is an agreement among mathematics educators on the importance of analogies or situations from everyday life within the verbal definitions of the function (Elia, Panaoura, Eracleous, & Gagatsis, 2007), the examples of functions from their applications in real life (Elia & Spyrou, 2006) or the tasks that aim to use the definitions of functions (Christou et al., 2005) because of their valuable opportunities for students to gain understanding in functional thinking (Cooney & Wilson, 1993). An analogy is drawn by transferring ideas from a *familiar* concept to an *unfamiliar* one (Thiele & Treagust, 1995). The familiar concept that provides basis features to interpret the unfamiliar concept is often called the source or analog; whereas the unfamiliar concept to which the information is transferred is called the target (Gentner, 1983). Researchers have basically emphasized a correspondence in some respects from source to target, thus underlined the "*relation*" and "*similarity*" in which the same system of relations holds across different objects (Eid, 2007; Heywood, 2002). Matching the familiar domain to the unfamiliar domain is achieved by accessing the similarities and differences of the domains, and then mapping similar characteristics between the two domains by indicating the breakdown points. Analogy generation is crucial for teaching, the origin of which grounds to the connections between the real world and the target concept that establishes as real world linkage, and which, in general terms, could be said to prompt visualization of abstract concepts that facilitate understanding (Thiele & Treagust, 1992). The sense of this perspective is clearly articulated in recent years, such as that of several mathematics educators (Fast, 1996; Richland, Holyoak, & Stigler, 2004). Analogical

representations such as function machines attempt to move students from an action to a process conception (Selden and Selden 1992). To be able to manage this move, the quantity and quality of teachers' mathematical knowledge on functions and analogy are important aspects. The collection of articles written by Even (1990, 1993, 1998) and by other researchers (Hitt, 1998; Lloyd & Wilson, 1998; Sanchez & Llinares, 2003) pointed out subject matter knowledge and pedagogical content knowledge, as defined by Shulman (1987), essential for teaching the function concept. Moreover, characteristics of teachers' belief systems have also been linked to various components of these knowledge aspects.

Although a few studies investigated pre-service teachers' use of analogies in teaching function concepts (e.g., Ubuz et al., 2009), not much is known about teachers' knowledge, beliefs, and ability to use analogies in teaching functions that are fundamental components of their pedagogical content knowledge. Considering these facts, we aimed to investigate how pre-service teachers' knowledge and belief are associated with how and when they use analogies to teach the idea of functions. Response to this question can provide insight as to possible linkages between teachers' knowledge and beliefs about functions and analogy, and their teaching of functions with analogy.

METHODS

Context and Participants

The participants were all seven preservice secondary mathematics teachers (PT1, PT2, PT3, PT4, PT5, PT6, and PT7) attending to Master of Science without Thesis Program at Middle East Technical University. The first three were male and the rest were female. All the students were the total number of the students in their last term. Participants were graduated from the Department of Mathematics and were attending to this program. They all had some previous teaching experience through their participation to the private tutoring programs. Master of Science without Thesis Program is a three semester certificate program to teach mathematics at secondary school level (grades 9-12). Data were collected during *Practice Teaching in Secondary Education course* provided at the last semester including 14 weeks. This course involves practice teaching in classroom environment for acquiring the required skills to become an effective mathematics teacher. Each week PTs spend their six class hours in a classroom environment at an arranged public secondary school, and two class hours at the university. In that two hours period at the university, PTs presented sample lessons one by one to their peers and the instructor. They were required to present the function concept and its types by generating analogies. At this stage, their knowledge on functions and analogy, and their images resulted from their previous experiences in school and university mathematics as well as the method courses offered at the Master of Science without Thesis Program. With regard to functions and analogy, method courses involved history of function, misconceptions about functions, definition of analogy, and importance of analogy for learning and teaching. At the public school they taught two lessons with presence of

the instructor (the first researcher) and the classroom teacher. At other times they presented lessons whenever the classroom teacher allowed them to do. Teaching at the university and the school constituted 30 percent of the course grade. Lesson plans constituted 15 percent of the course grade. While preparing the lesson plans, they mainly focused on objectives, materials, teaching techniques and the development process in the lesson.

Data Collection and Analysis

Knowledge Test: Knowledge test assessed the PTs in two major strands of knowledge prior to their teaching practice: subject matter knowledge and pedagogical knowledge. The item on subject matter knowledge was chosen to test their factual knowledge about functions, particularly the definitions of function and its types (one-to-one, onto, into, and one-to-one and into). The item on pedagogical knowledge was prepared to assess their knowledge on analogy, particularly the definition of it and its characteristics. They were required to complete the test in the presence of the research assistant of the course during the first week of their course and no time limit was imposed. At this week, all PTs, except PT7, were registered on the course, and ultimately the knowledge test data were driven from them. At the add-drop week PT7 registered to the course and were involved in the present study. Themes on PTs' definitions of functions were explored by considering the historical development of functions (Cooney & Wilson, 1993). Reviewing the development, it becomes obvious that development has moved in the direction of including various elements of functions, that is, of the concept of set, arbitrary correspondence, and the requirement that each value of the independent variable has a unique image. These elements on functions provided us to keep track of PTs' knowledge of the function concept and its types. In line with this, the correctness of their definitions was also explored. The underlying ideas of analogy contributed significantly to our analysis and interpretations of the responses given to "*What is analogy? Define and determine the main characteristics of it*". The responses were categorized under the following descriptors: (1) transferring the familiar domain to the unfamiliar domain, (2) accessing the similarities and differences of the domains, and (3) mapping similar characteristics between the two domains by indicating the breakdown points.

Teaching Practice: At the beginning of the Practice Teaching in Secondary Education course, function topics covered at the 9th grade were assigned to each participant to be presented using analogies, to provide an effective flow of lesson and to cover all topics relevant to functions. Each participant prepared two lesson plans about assigned topics to be presented at the classroom in the university. One of the lesson plans was on function concept and the other on its types. The first author (instructor of the course) and the third author (assistant of the course) observed and videotaped the PTs during their teaching practice experiences to be consulted in further analysis. The teaching practice data were collected from the observations of all the seven PTs. All PTs except PT7, were observed on two different occasions in terms of teaching function concept and its types within an average duration of 30

minutes. PT7 was observed on a single occasion involving both teaching function concept and its types within 40 minutes period.

Content analysis (Philips & Hardy, 2002) was conducted to discern meaning in teachers' written and spoken expressions. Videotapes of 13 sessions were fully transcribed and considered line by line whilst annotated field notes were used as supplementary sources. The first phase of data analysis included detecting analogy-based teaching instances and identifying source analogies and the target concepts. A portion of the course was considered to be analogical if it was aligned with the working definition stated above and/or it was stated in the lesson as being analogical. Then the spotted cases was scrutinized concerning nature and extent of analogy use, considering analogical relationship, presentational format, level of enrichment, position, and limitations. The framework suggested by Thiele and Treagust (1994) served as a tool for analyzing the spotted cases. In addition, analogies were analyzed through the presence of any limitation and categorized as applicable or not applicable. The ones that are applicable were analyzed in terms of the presence of any stated warnings which highlights to the students where possible attribute mismatches may occur and categorized as specified or not specified. Furthermore, analogical instances were analyzed whether they are generated by listeners (as student) or presenters (as teacher).

Interview: Interviews were conducted by the third author upon the completion of PTs' teaching practice to yield data about their beliefs about using analogies. They were required to respond to three questions: At which stage should analogies be used in the teaching process? and why?; who should construct the analogies? and why?; and is it appropriate to use analogies in teaching the function concept? and why?. Each interview was approximately 17 minutes. Interviews were recorded and transcribed. Only PT4 was not interviewed because of her personal excuses.

While analyzing the responses given for "At which stage should analogies be used in the teaching process?", we delineated the position aspect of the theoretical framework that was used in analyzing the observations. The analysis brought forth three descriptors: (a) advance organizer, (b) embedded activator, and (c) post-synthesizer. The focus of the analysis on the responses given for "Who should construct the analogies?" was partially attributable to teachers' conceptions on the pedagogy of the analogy-generated lessons. Thus, our analysis of the interview responses focused on finding useful ways to think about the perspectives held by the PTs. This analysis traced perspective characteristic of the construction of analogies within two descriptors (Harrison & Coll, 2008): (a) student-generated analogies, and (b) teacher-generated analogies. The responses given "*Is it appropriate to use analogies in teaching the function concept?"* were categorized in two descriptors: (a) appropriate and (b) inappropriate. The PTs' value of using analogies was pivotal in our analyses to categorize the responses because it signaled the insight into their conceptions about analogies and provided an opportunity to inquire into those conceptions in relation to the appropriateness of teaching functions via analogies. While analyzing the

responses given to why questions subsequent to each question we did not rely solely on categorizing the aforementioned descriptors rather we sought for the essence of PTs' reasons of their beliefs.

Reliability of Coding

The initial classification of the knowledge test, teaching practice, and interview data were undertaken by the last two researchers after repeated reading of the knowledge test responses, and teaching and interview transcriptions. Headings were created in relation to the theoretical framework. These analyzed data were then evaluated by the first author who was an expert in teaching functions and analogy. The analyzed data were then subject to discussion by the three members of the research team to further refine the headings. At this meeting, it was decided that all the data will be analyzed by the aid of the matrices comprising the PTs, headings, and descriptors. Subsequently, the independent analyses were carried out by the last two researchers using the matrices for the knowledge test, teaching practice, and interview data in conjunction with the first author's comments. Analyses of the analogies in teaching practice in relation to the two headings - position and level of enrichment - were germane to disagreement. The conflicts were driven from the different conceptualizations of embedded activator and post-synthesizer descriptors in the position heading and the extended descriptor of level of enrichment heading. After consulting with the first author and utilizing the review of literature, embedded activator was restricted to the analog domain be presented after the introduction of the target domain; post-synthesizer was restricted to the analog domain be presented following a complete treatment of the target; and level of enrichment was restricted to the detail of mapping (e.g., expressing the domain, range, process, and the univalence feature) rather than the degree of mapping (e.g., using one analog to express multiple targets and/or multiple analogs to express a single target).

RESULTS

Knowledge of Functions and Analogy

Definition of function and its types. The notion of function as a dependence relation between elements became dominant in the definitions provided for all six PTs. According to their definitions, relations pairing elements of the second set with one or several elements of the first set and each element in the first set has a unique image were considered functions. Analysis of the responses revealed that all of the six PTs approached the types of function from a modern perspective. Their definitions endeavored to reflect function's correspondence perspective referred to as relation between sets. Apparently, the concept of set becomes a fundamental element in the definitions. Further, it was noted in the definitions that the relation or correspondence need not involve numbers but could also involve relationships or correspondences between other elements that vary. Mostly definitions did not illustrate the requirement of a definite "law" correspondence. To summarize,

Bourbaki's definition has remained dominant. The types of functions were also defined correctly referring to their specific characteristics.

Description of analogy. The underlying ideas of analogy under the following descriptors: (1) transferring the familiar domain to the unfamiliar domain, (2) accessing the similarities and differences of the domains, and (3) mapping similar characteristics between the two domains by indicating the breakdown points were emphasized by all PTs in different words. They underlined the fact that an analogy cannot hold all the shared attributes, rather the similarities can be built in terms of particular features of a concept.

Teaching Practice

Amongst the 45 analogies, 26 of them were teacher-generated and the remaining 19 were student-generated. Teacher-generated analogies refer to analogies that are constructed by the presenter either during or before the lesson. Student-generated analogies were developed by the listeners/participants either during or after the lesson mostly with the presenters' initiation during lesson.

Analogical Relationship. Results revealed that the vast majority of analogical relationships were functional (41 of 45, 91%) as they include the behavior of the source shared by the target concept. Only four (9%) analogies shared both functional and structural relationship. Structural-functional analogies were generated considering the spelling and meaning similarities of the terms while teaching types of functions.

Presentation Format. Results indicated that 31 of the 45 identified analogies (69%) were verbal, and only 14 (31%) had a pictorial representation together with the verbal representation. The 10 of the 14 pictorial-verbal analogies (71%) and 16 of the 31 verbal analogies (51%) were generated by the presenters of the lesson. This might be due to the fact that presenters tend to support their teaching to enhance the understanding of the function concept via pictorial-verbal analogies.

Position. Most of the analogies (23 of 45, 51%) were generated prior to the investigation of target concept that referred to as advanced organizer. Twelve of the 45 analogies (27%) acted as post-synthesizers while the remaining 10 (22%) were generated as embedded activators. The 20 of the 23 advance organizers (87%) and two of the 12 post-synthesizer (17%) were generated by the presenters of the lesson.

The Level of Enrichment. According to the "level of enrichment" criteria, it was observed that most of the analogies (20 of 45, 45%) generated were enriched analogies following extended analogies (15 of 45, 33%) and simple analogies (10 of 45, 22%). Further, three of the 10 simple analogies that primitively state that the target is like the source, 10 of the 15 (67%) extended analogies that indicate several shared attributes of a single source used to teach a variety of targets, or a variety of sources used to teach a single target, and 13 of the 20 (65%) enriched analogies in which some shared attributes between the source and target concepts for the

analogical relations are stated, were generated by the presenters themselves. That is, simple analogies were generated mostly by the PTs who were student participants in the class, and enriched and extended analogies were generated mostly by the PTs who are the teachers of the class.

Limitation. Limitations play a central role in the teaching and learning with analogies by contributing to the conceptualization of the links between analog and target. Four of the 45 analogies were discarded from the analyses of this heading since these analogies were mathematically incorrect (see aforementioned section). Of the 41 analogies 24 (59%) were classified as not applicable due to having no limitation; 17 (41%) were reserved for having a limitation among which nine were not specified and eight were specified.

Epistemological Appropriateness. The epistemological appropriateness of the analogies was classified in terms of whether the domain, range, process, univalence, and other features of the target concept correctly mapped to the analog or not. Results documented that 37 of the 45 analogies (82%) were correct and four of the 45 (9%) analogies were incorrect. The rest four (9%) structural-functional analogies cannot be analyzed in terms of epistemological appropriateness as they corresponded to the target concept in terms of their spelling and verbal meaning. As expected, the most of the incorrect analogies (3 of 4, 75 %) were formed by the PTs as student participant with the presenters' initiation during lesson.

Beliefs

When to use analogy? PTs appeared to have an insight into the use of analogy from the introducing of a new concept to the developing an understanding for this new concept. All six PTs indicated that analogies could be used as an *embedded activator* since analogies can be presented as an example at a point after the definition of a concept or at a time when the mathematical content is becoming more abstract or difficult to the students.

Who should construct the analogies? PTs' preference for the teacher-generated analogies was evident during the interviews. The reasons can be illustrated with two underpinnings. In generating the analogy the PTs put emphasis on the importance of the subject matter knowledge and pedagogical knowledge. As teachers have knowledge both on subject and analogies, they can direct students more efficiently, emphasizing similarities and differences between the analog and the target. They deliberately viewed analogy generation as a difficult process since the analogy generation grounds explicitly on the connectedness between the source and target concepts.

The Use of Analogies in Teaching Functions. The use of analogies in teaching functions was greatly valued and instigated in the comments of all PTs except PT3. They clearly found them appropriate as functions are difficult for students and engenders anxiety among them as it was introduced for the first time at the 9th grade. They articulated that analogies help conceptualization because correspondence

between analogy or familiar concept and function concept requires students to extend their understandings in a meaningful way.

DISCUSSION

This study's foregoing findings lend credence to the consistent associations between PTs' subject matter knowledge, pedagogical knowledge, beliefs, and their practices advanced by Fennema and Franke (1992). In terms of practical implications, the findings of this study clearly support the need for developing PT knowledge and beliefs as they have an association on PTs' teaching practice.

Knowledge test and teaching practice data revealed that all PTs tended to use similar definitions in both. This implies that PTs were consistent in their knowledge and teaching practice. PTs' understandings clearly articulated that they were able to reconcile the analogical approach to functions with the prominent features of their own knowledge on functions. This study thereby illustrates that PTs' knowledge of functions acted as a filter for the interpretation and a springboard for epistemologically appropriate analogies.

PTs tend to think of analogy as a tool with the descriptors of transferring the familiar domain to the unfamiliar domain, accessing the similarities and differences of the domains, and mapping similar characteristics between the two domains by indicating the breakdown points. The level of enrichment of analogies in their teaching of functions was associated with the first two descriptors of analogy. Within these considerations they were expected to generate extended analogies, however, most frequent use of analogies were enriched due to their overlook to state the univalence requirement and the process. This study supports our previous findings (Ubuz et al.,2009) that PTs were in favor of applying a real-life context to extract meaning for the function concept. In particular, their focus on patterns of real-life connections empowered them to utilize the interdisciplinary aspects to facilitate analogies that established links between mathematics and other sciences. However, they were limited in their attempts to generate analogies in scientific contexts. This supplied evidence that generating an effective analogy requires not only knowledge of mathematics but also the knowledge of interdisciplinary subjects that can furthermore support the development of new mathematics concepts in real-life contexts. This study's foregoing findings lend credence to the consistent associations between PTs' beliefs and their practices advanced by Fennema and Franke (1992). More use of analogies was associated with more temperate beliefs about the use of analogies in teaching practices. Thus, for example, the more teachers valued and instigated the use of analogies in teaching functions in their comments, the higher number of analogies they generated in their teaching practices. Although association was evident between teachers' knowledge, beliefs, and teaching practice, what teachers teach is mediated by the external factors such as textbooks since teachers' thinking about functions is based on how functions are presented in school textbooks (Cooney & Wilson, 1993). Thus, future research can explore the textbooks and to what extent teachers can

enhance the treatment presented by a textbook in order to accommodate particular objectives.

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