IN- SERVICE MATHEMATICS TEACHER'S MATHEMATICAL KNOWLEDGE FOR TEACHING: A CASE OF VOLUME OF PRISM

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The purpose of this qualitative study is to investigate in-service mathematics teachers' knowledge of elementary students' solution strategies and students' difficulties in calculating the volume of a prism. Data were collected from four in-service elementary mathematics teachers in Turkey during 2011-2012 academic year. Two questions involving students' solutions on calculating the volume of a prism were served as the data source. The findings revealed that although teachers had knowledge of identifying students' incorrect solution strategy, their knowledge of explaining students' different solution strategies was limited. Moreover, teachers are able to determine students' difficulties which is account for their mistake.

Key words: in-service teachers, teachers' knowledge, the volume of prism

INTRODUCTION

Throughout many years, researchers gave special attention to investigate teachers' mathematical content knowledge. They agree that understanding of teachers' content knowledge has significant role for effective teaching and for students' achievement. However, many research studies concluded that mathematics teachers' content knowledge was not adequate for effective teaching (Ball, 1990; Hill, Rowan, & Ball, 2005). When the subjects of these studies were examined, it was realized that many researcher aimed to investigate teachers' knowledge related to subjects in mathematics. State differently, the number of studies which focused on exploring teachers' knowledge of geometry subjects is limited in the literature. Nevertheless, geometry has vital role in teaching and learning mathematics (The National Council of Teachers of Mathematics, [NCTM], 2000). Moreover, Maxedon (2003) stated that it is necessary to have powerful geometry content knowledge for effective teaching. For this reason, it will be significant to explore teachers' knowledge related to geometry concepts.

Literature reviewed revealed that teachers' knowledge concerning volume of 3D solids has not investigated yet. Many researchers investigated students' understanding regarding the volume of 3D solids (Battista & Clements, 1996; Ben-Chaim, Lappan & Houang, 1985). They concluded that students' performance on the volume of 3D figures is low and students have several difficulties in finding the volume of 3D figures. On that account, it will be significant to explore teachers' mathematical knowledge for teaching related to the volume of 3D figures. For this reason, the aim of this study is to investigate in-service mathematics teachers' knowledge for teaching related to volume of 3D solids. Particularly, the current study aims to investigate in-service mathematics teachers' knowledge regarding elementary students' solution strategies and their difficulties related to calculating the volume of a prism. Thus, research problems could be stated as follows:

1. How do in-service mathematics teachers describe elementary students' solution strategies related to calculating the volume of a prism?
2. What do in-service mathematics teachers know about elementary students' difficulties in calculating the volume of a prism?

THEORETICAL FRAMEWORK

In order to achieve the purpose of this research study, Mathematical Knowledge for Teaching (MKT) was used as a theoretical framework (Ball, Thames & Phelps, 2008). Researchers described content knowledge under two types of knowledge: common content knowledge (CCK) and specialized content knowledge (SCK). They asserted that every person has common content knowledge whether s/he is a mathematics teacher or not. On the contrary, they characterized specialized content knowledge as the knowledge that is unique to teacher who engages in teaching mathematics to children.

Moreover, Ball and her colleagues stated that Shulman's (1986) pedagogical content knowledge was divided into two subcategories: knowledge of content and students (KCS), and knowledge of content and teaching (KCT). The formed one (KCS) requires knowing the topics which the students find easy, difficult or confusing, knowing the students' preconceptions and misconception/difficulties, and knowing the way of responding students' misconceptions or wrong solutions. The latter one (KCT) involves determining the best teaching method and useful representations, choosing the examples which are appropriate for students to start with the topics and deciding the following examples to increase students' attention to the subject.

Another category is horizon knowledge which is certified as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball, Thames & Phelps, 2008, p.42).

In this research study, two dimensions of Ball and her colleagues' framework which were specialized content knowledge (SCK) and knowledge of content and students (KCS) were analyzed. While the aim of first research question was to investigate in- service mathematics teachers' specialized content knowledge, second research question was served to explore in- service mathematics teachers' knowledge of content and students.

RESEARCH STUDIES RELATED TO TEACHERS' CONTENT KNOWLEDGE

It is obvious that having sufficient content knowledge and being able to use it efficiently are at the heart of teaching mathematics. Several studies were conducted to investigate teachers' content knowledge related to variety of mathematics subjects all over the world. As it was stated, the number of research studies conducted to explore teachers' knowledge regarding geometry subjects are limited (Fujita, & Jones, 2006; Gomes, 2011).

Kellogg (2010) conducted a study to investigate 12 elementary pre-service teachers' content knowledge and knowledge of students' thinking with respect to principles, relationships, and misconceptions related to area and perimeter. The findings of the study let Kellogg to conclude that many pre-service teachers possessed procedural knowledge related to area and perimeter and they were not aware of students' difficulties/ misconceptions. In another study, Baturo and Nason (1996) aimed to investigate subject matter knowledge of first-year teacher education students' understanding about area measurement. Their findings indicated that first-year teacher education students' subject matter knowledge regarding area measurement was limited. In other words, their knowledge was incorrect, missing and unconcerned which is consistent with the result of Kellogg (2010). Besides, Fujita and Jones (2006) investigated primary trainee teachers' geometry content knowledge related to defining and classifying quadrilaterals. The results indicated that although trainee teachers could draw the figure of quadrilaterals, they could not provide their definitions. Besides, they did not have enough knowledge about hierarchical relationship between quadrilaterals. Consistent with the previous studies, Jones, Mooney and Harries (2002) reported that trainee primary teachers' personal confidence in geometry and their geometric vocabulary knowledge was poor. Particularly, they had difficulties in calculating the area and the volume of a geometric figures. Similarly, Gomes (2011) conducted an exploratory study to evaluate pre-service elementary teachers' content knowledge on geometric transformations. The findings revealed that pre-service teachers had knowledge on geometric transformations. However, their knowledge did not adequate to teach this subject and they had some difficulties regarding three geometric translations, translation, reflection and quarter turn rotation. Lastly, Aslan-Tutak (2009) carried out a study to understand three pre-service teachers' geometry learning and their geometry content knowledge for the case of quadrilaterals. Based on the qualitative investigation, pre-service teachers' geometry content knowledge was limited and they have problems of classification the quadrilaterals.

Literature review showed that much more emphasize is given to conduct studies with pre- service mathematics teachers. There is a gap in the literature in terms of investigating in- service mathematics teachers' knowledge of geometry subjects. In addition, teachers' knowledge related to calculating the volume of 3D figures has not been studied yet. For this reason, it will be significant to explore in- service mathematics teachers' knowledge related to calculating the volume of 3D figures.

METHODOLOGY

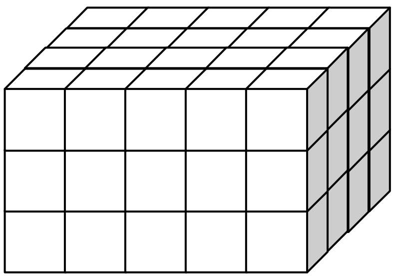
Participants

This study was carried out at the 2011-2012 academic year and it was designed as a qualitative case study. Four in- service mathematics teachers were the case of the study. All participants were graduated from one of the successful universities in Turkey. In addition, they teach 6^{th} to 8^{th} grade mathematics in the Turkish education system in different schools. Moreover, two participants had 6 year-teaching experience and two of them had 7 year-teaching experience. They had experiences in a real classroom and they taught calculating the volume of 3D figures many times. This was the reason for selecting them to participate in the study. The participants were called as Teacher 1, Teacher 2, Teacher 3, and Teacher 4.

Data Collection and Instruments

The original study involved eleven questions related to calculating the volume of prisms, pyramids, cone and sphere. Due to space constraints, it was decided to focus only on two questions pertaining to calculating the volume of prism. Two questions were given Figure 1.

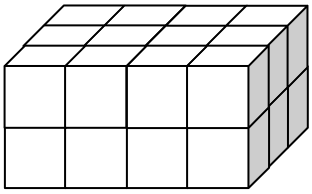
Question 1.



Ms. Güler asked her students to find the volume of a rectangular prism. Most of the students found its volume as 94 and Ms. Güler realized that these students made mistakes. She tried to understand how her students solved this question.

a. How did Ms. Güler's students solve this question?
b. What are the difficulties that Ms. Güler's students faced with while calculating the volume as 94?

Question 2.



Ela, Eren, Kuzey, and Berke developed different solution strategies which are presented below while calculating the volume of a rectangular prism.

Ela's solution: 26 x 2= 52 8 x 2= 16 52- 16= 36 36- 12= 24	***Eren's solution:*** 6 + 6= 12 4 + 4= 8 12 + 8 + 4= 24
Kuzey's solution: 4 x 3= 12 12 x 2= 24	***Berke's solution:*** 4 x 3 x 2= 24

a. Explain students' solution strategies.
b. If any student's solution is wrong, then explain what kind of difficulties do/does this/these students have?

The aims of asking these questions were to get knowledge about how participants describe students' incorrect and alternative solution strategies, and to investigate participants' knowledge regarding elementary students' difficulties in calculating the volume of a prism. Describing students' incorrect and alternative solution strategies to calculate the volume of the prism were served as a tool for exploring participants' SCK (Ball et al., 2008). In addition, explaining elementary students' difficulties in calculating the volume of the prism was asked for getting knowledge regarding participants' KCS (Ball et al., 2008).

The questions were given to all participants, then their answers analyzed to both questions. Later on, one-to–one semi-structured interview was applied to all participants for in-depth analysis. During interviews, participants explained their answers in detail and clarified elementary students' solutions and their difficulties/mistakes. All interviews were videotaped. To analyze data, content analysis technique was employed. Videotapes were transcribed and teachers' notes on the questionnaire were analyzed. Two coders evaluated data in order to ensure validity and reliability of the study.

FINDINGS

In this study, the aim was to examine in- service mathematics teachers' mathematical knowledge for teaching related to calculating the volume of a prism. Especially, the purpose of this study was to investigate in- service mathematics teachers' knowledge regarding students' solution strategies and students' difficulties in calculating the volume of a prism. In-service mathematics teachers' explanation of students' incorrect and alternative solution strategies and their knowledge about students' difficulties will be explained for each question.

First Question

In the first question, a case which includes students' incorrect solution was given to in- service mathematics teachers. Firstly, it was asked in- service mathematics teachers to describe how students found 94 while calculating the volume of a prism. Then it was expected them to determine students' difficulties which caused to make that mistake.

The analysis of the data revealed that although three in-service mathematics teachers (Teacher 2, Teacher 3, Teacher 4) were able to explain students' solution, one of them (Teacher 1) could not make any interpretation regarding students' solution. Teacher 1 clarified her thought as in the following:

> *Teacher 1: I am curious about how students found 94. When I calculated the volume of this rectangular prism, I found 60. If students' answer was smaller than 60, I might think that they forgot something or they counted the number of unit cubes in each edges incorrectly. Unfortunately, I do not know how they found 94. I could not make any interpretation regarding students' solution.*

Because of not being able to explain students' solution, Teacher 1 could not determine students' difficulties regarding the first question.

On the other hand, Teacher 4 had great difficulty in explaining students' solution strategy. After a while, he clarified students' solution strategy regarding first question while calculating the volume of a prism. Teacher 2 and Teacher 3 expressed students' solution strategies without thinking too much. Teacher 3 said that students calculated the area of a prism instead of calculating its volume. Moreover, Teacher 2 declared that students calculated the number of unit cubes on the visible faces of a prism and then they multiplied this number by 2. This teacher expressed that students have difficulties in this question and they found the area of a prism. In other words, Teacher 2 denoted that students confuse area and volume concepts.

Second Question

In the second question, it was given a case which includes four students' solutions regarding calculating the volume of a prism. In- service mathematics teachers were expected to explain how these students calculate the volume of a prism and to clarify students' difficulties if exists.

The analysis of data revealed that not all teachers could explain Eda's solution. Only Teacher 2 could interpret first operation as calculating the number of unit cubes on visible faces and then multiplying this number by 2 (26 x 2= 52). According to her, the reason for doing this operation is confusing area and volume concepts. However, she could not give meaning to the other operations in Eda's solution (8 x 2= 16; 52- 16= 36; 36- 12= 24). On the other hand, Teacher 1, Teacher 3 and Teacher 4 did not have any idea regarding each operation of Eda's solution. Owing to not explaining Eda's solution, they could not determine Eda's difficulty.

Besides, Teacher 3 and Teacher 4 could not explain Eren's solution. Teacher 4 thought that Eren counted the unit cubes systematically but he could not explain what Eren's strategy is while counting. On the other hand, Teacher 3 could not make any interpretation regarding Eren's solution.

Conversely, Teacher 1 and Teacher 2 explained Eren's solution correctly. They specified that Eren counted unit cubes systematically. According to them, Eren counted the unit cubes on the left and right faces firstly, then considered unit cubes which only belongs to front and back faces of the prism. Lastly, he counted unit cubes which do not belong to any faces of the prism, namely unit cubes at the middle of the prism. With regard to Teacher 1 and Teacher 2, Eren calculated the volume of the prism correctly.

Teacher 3 and Teacher 4 interpreted Kuzey's solution as using volume formula. However, Teacher 1 specified that Kuzey calculated base area and then multiplied it with height of the prism. On the other hand, Teacher 2 explained that Kuzey found the number of unit cubes in one layer, and then multiplied this number by this number of the layers, namely Teacher 2 thought that Kuzey took layers of the prism into consideration.

Last solution strategy is Berke's solution. All teachers could easily explain how Berke solved the question. They expressed that his solution was the easiest and widespread solution among elementary students. According to them, Berke multiplied three numbers. In other words, he used volume formula directly.

In addition to explanation of students' solution, it was expected teachers to determine students' difficulties if exists. All in-service mathematics teachers agreed that Kuzey and Berke solved the question correctly. Therefore, they did not have difficulty in calculating the volume of a prism. On the other hand, in-service mathematics teachers could not identify Ela's solution. Moreover, some teachers did not make any interpretation regarding Eren's solution. Because of not explaining Ela's and Eren's solution, they could not determine students' difficulties.

DISCUSSION

The findings of the study related to first question revealed that participants could identify how students find 94 while calculating its volume. For this reason, it could be concluded that in-service mathematics teachers who participated in the current study had strong knowledge regarding examining and understanding students' incorrect solution strategies. However, data analysis of the second question let us to conclude that participants did not have knowledge to explain students' alternative solution strategies to calculate the volume of the prism. Actually, there is an important issue which is being able to explain students' incorrect solution strategies and not being able to explain their alternative solution strategies. When the data was analyzed deeply, it could be realized that participants were familiar with such kind of students' incorrect solution strategy. All participants expressed in the interview that many students got in confusion between area and volume concepts. For this reason they calculated the area of 3D figures instead of calculating its volume. From their expression, it could be deduced that they had knowledge regarding students' solutions which they got experiences.

As it was stated, participants could not justify all students' alternative solution strategies which were presented in the question. Actually, this is important phenomenon since knowing and using mathematics require making sense of different solutions (Ball, Bass, &Hill, 2004). However, they only explained solution strategies which is the most widely- used by teachers and students and called it as volume formula strategy. Moreover, participants could not define multiplying width, depth and height of the prism as layer multiplying strategy as Battista et al. (1996) called. State differently, participants only focused on volume formula strategy. In addition, participants could not comprehend solution strategies which are based on layers and common unit cubes on the faces. For instance, participants indicated that Ela could not solve the question correctly even though it is correct. Moreover, Eren took the layers into consideration and he calculated the number of unit cubes on the layers systematically. Participants said that this solution was incorrect as Ela's solution. This finding let us to conclude that if teachers do not understand students' alternative solution strategies, they think that the solution is wrong. In this case, it could be concluded that participants' knowledge related to students' different solution strategies is limited. This result is consistent with Baturo and Nason's study (1996) and Kellogg's study (2010). They also concluded that mathematics teachers have inadequate knowledge about their students. Nevertheless, teachers should have deep content knowledge which has vital role for effective teaching and for students' achievement in order to understand students' solution strategy and determine their difficulties (Leinhardt & Smith, 1985).

Moreover, participants could be able to specify students' difficulties which caused students to develop incorrect solution strategy. As noted earlier, participants expressed that students had difficulty in calculating the area of rectangular prism instead of calculating its volume. This difficulty is also identified by other researchers who studied with elementary students in their study (Battista & Clements, 1996; Ben- Chaim, Lappan & Houang, 1985).

CONCLUSION and RECOMMENDATIONS

In conclusion, participants' SCK could be regarded as good in terms of explaining students' solution strategies if they had experiences in their lessons but their SCK could be regarded as low in terms of describing students' alternative solution strategies which they had no experiences in their lessons. On the other hand, if teachers could be able to explain students' incorrect or alternative solution strategies, they could be able to determine their difficulties. Otherwise, they might think that their solution is incorrect even if they are correct. This is important problem for effective teaching since teachers may prevent students from developing different strategies and they may solve the questions by using the strategies that their teachers teach. In other words, teachers' limited content knowledge may direct students to calculate the volume of prism by only volume formula. Therefore, it is recommended that teachers may develop different strategies apart from using volume formula to understand students' strategies and to determine their difficulties related to calculating the volume of a prism.

As a last concern, other components of mathematical knowledge for teaching may be the aims of further studies to investigate mathematics teachers' knowledge related to calculating the volume of prism. Moreover, calculating the volume and the area of other 3D figures might be investigated.

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