DEVELOPING MATHEMATICS TEACHER EDUCATION PRACTICE AS A CONSEQUENCE OF RESEARCH

Tim Rowland[a,b], Fay Turner[a] and Anne Thwaites[a]

[a]University of Cambridge, UK, and [b]University of East Anglia, UK

The authors of this paper are three members of the Cambridge-based research team who developed the Knowledge Quartet (KQ), a theory of mathematics teacher knowledge, with a focus on classroom situations in which this knowledge is applied. At the same time as being researchers, the authors were elementary mathematics teacher education instructors. Despite many years' experience of preparing trainee teachers in elementary mathematics, they found that the KQ research had brought about new awareness of the importance of some components of mathematics didactics, as well as providing new tools for undertaking some aspects of their teacher educator role. The paper explores some of these awarenesses and tools in detail.

Keywords: teacher knowledge, mathematics teacher educator, Knowledge Quartet

INTRODUCTION

This paper is a contribution to a young field of research, which seeks to understand the ways in which teacher educators (specifically, mathematics teacher educators) can gain in wisdom, competence and effectiveness in their work. The state of the art has similarities with the emergence of mathematics teaching as a research field, fuelled by action research, in the 1980s: until then, the research gaze was on students rather than teachers. Likewise, researchers into mathematics teaching, themselves typically mathematics teacher educators, have only recently viewed themselves (or their work) as suitable objects of research, having previously attended to the knowledge and performance of their own 'students'. Even the goals of what we are calling mathematics teacher educator 'development' are, as yet, unclear. In a Special Issue of the *Journal of Mathematics Teacher Education*, Brown and Coles (2010) address the topic in a neutral way, as "change", and ask what it might mean to say that a mathematics teacher educator has "changed". It is certainly indicative of the state of the art that none of the 12 contributions to the Special Issue explicitly addresses mathematics teacher educator development, or change. So how shall we theorise this development, and what means can we deploy it bring it about? At this point we should 'come clean'. The changes that we know to have come about in ourselves, as mathematics teacher educators, were not the goal of the research to be reported here, but a by-product, and one that we became aware of after the main phase of the research was completed. Our focus was on prospective teachers of school students, and on their mathematics-related knowledge in particular. We return to this point in the concluding section.

The work of University Departments of Education is typically distributed across diverse programs and agendas, including a leading role in the education and professional preparation of prospective teachers. There can be, in the UK at least, and probably elsewhere, a fuzzy divide between faculty engaged in teacher preparation and those engaged in research. Thus, while teacher education is expected to be research-informed, this basis in scholarship most often rests on the research of academics other than those doing the 'training'. This state of affairs comes about for a number of reasons, and many faculty on both sides of the divide are very content with it. However, the purpose of this paper is to exemplify how mathematics teacher educators can benefit and learn from *their own* research activity, with direct relevance to their teacher education role. In the paper we reflect upon our own experience as education department faculty who have endeavoured to straddle the research-practice divide. The paper begins with a brief account of a research project on mathematics teacher knowledge, to which each of us made a major contribution. The remainder of the paper is devoted to reflection on, and discussion of, some ways in which the research had a direct impact on our professional work with prospective teachers, thereby (we believe) making us 'better' teacher educators. Despite having, between us, over 70 years experience of preparing trainee teachers in elementary mathematics, we found that this particular research activity had given us new awarenesses of the importance of some components of mathematics didactics, as well as providing new tools for undertaking some aspects of our teacher educator role.

The Knowledge Quartet

In 2002-03, we undertook some empirical research into mathematics teachers' knowledge, in collaboration with two additional colleagues in Cambridge. Our approach to investigating the relationship between teacher knowledge and classroom practice was to observe and videotape novice teachers teaching. The participants were 12 graduate prospective ('trainee') elementary school teachers in our university faculty of education. We observed and videotaped two mathematics lessons taught by each participant. In the analysis of these videotaped lessons, we identified aspects of trainees' classroom actions that seemed to be informed by their *mathematics* subject matter knowledge or their *mathematical* pedagogical content knowledge (Shulman, 1986). We realised later that most of these related to choices made by the trainee, in their planning or more spontaneously. Each was provisionally assigned an 'invented' code, such as: 'choice of examples'; 'choice of representation'; 'adheres to textbook'; and 'decision about sequencing'. These were grounded in particular moments or episodes in the tapes. This provisional set of codes was rationalised and reduced (e.g. eliminating duplicate codes and marginal events) by negotiation and agreement in the research team. This inductive process generated 20[1] agreed codes, which were subsequently grouped into four broad, super-ordinate categories, or 'dimensions' – hence the 'Quartet'. The four dimensions and the corresponding contributory codes are shown in Table 1.

Dimension	Contributory codes
Foundation:	awareness of purpose; adheres to textbook; concentration on procedures; identifying errors; overt display of subject knowledge; theoretical underpinning of pedagogy; use of mathematical terminology.
Transformation:	choice and use of examples; choice and use of representation; use of instructional materials; teacher demonstration.

Connection:	anticipation of complexity; decisions about sequencing; making connections between procedures; making connections between concepts; recognition of conceptual appropriateness.
Contingency:	deviation from agenda; responding to students' ideas; use of opportunities; teacher insight during instruction.

Table 1: The Knowledge Quartet – dimensions and contributory codes

A brief conceptual outline of the KQ is as follows. The first dimension, *foundation*, consists of teachers' mathematics-related knowledge, beliefs and understanding, incorporating Shulman's (1986) classic taxonomy of *kinds* of knowledge without undue concern for the boundaries between them. The second dimension, *transformation*, concerns knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself. A central focus is on the representation of ideas to learners in the form of analogies, examples, explanations and demonstrations. The third dimension, *connection*, concerns ways that the teacher achieves coherence within and between lessons: it includes the sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks. Our final dimension, *contingency*, is witnessed in classroom events that were not planned for. In commonplace language, it is the ability to 'think on one's feet'. More detailed conceptual accounts can be found in Rowland, Huckstep & Thwaites (2005), and in the book Rowland, Turner, Thwaites & Huckstep (2009). Related reports at previous CERME conferences include Huckstep, Rowland & Thwaites (2006). In this paper we also draw upon the longitudinal doctoral research project of the second author (Turner, 2010), in which the findings of the 2002-03 KQ project were applied for the first time; and on continuation projects (e.g. see Rowland, Jared & Thwaites, 2011) in which the scope and methodology of the KQ were extended.

WHAT WE LEARNED FROM THE KQ RESEARCH

We now proceed to describe some of the ways in which the research outlined above brought about new awarenesses, and enabled new approaches, in our professional work as elementary mathematics educators. This will be organised into sections corresponding to specific issues, topics and approaches about which we became more sensitive and knowledgeable as a consequence of the research.

The role of 'theory' within pre-service mathematics teacher education

When using the KQ to analyse the practice of beginning teachers, it was salutary to find that they did not draw on what we thought they had learned from our methods courses in the university to the extent that we might have hoped. The mathematical knowledge for teaching (MKT: Ball, Thames and Phelps, 2008) of beginning teachers might be expected to be mainly *propositional* (Shulman, 1986), i.e. gained from their own mathematics education and from mathematics methods courses during teacher education programmes. Other forms of knowledge proposed by Shulman, i.e. *case study* or *strategic knowledge* are likely to be more limited, as these require experience, which by definition beginning teachers do not have. Therefore, we might expect the practice of beginning teachers to draw significantly on propositional knowledge addressed during university courses, and later, with experience, to draw more often on case study and strategic knowledge. However, analysis of teaching using the KQ indicated that the relationship between the three types of knowledge and experience is more complicated.

There were a number of instances where situations categorised under the *foundation* dimension indicated that, once in the classroom, trainees did not draw on propositional knowledge addressed during their graduate teacher education course. Although there was evidence that this was held as propositional knowledge, these beginning teachers were frequently unable to draw on this knowledge and activate it in their early teaching. Two examples will illustrate this observation.

Amy. During her final school placement, in a lesson about counting with 4–5 year-old children, Amy asked her pupils to write nineteen on their white boards. Several children wrote '1P', at least one wrote '99' and many wrote '91'. The trainee teacher focused on the reversal of the nine but did not address the problem of digit order. During the post-lesson interview the trainee teacher was asked why she thought children had reversed the digits:

> Because you say nine first, then you say the teen that's why often they write the nine first they often want to write nine first then write it from right to left instead of left to right. (Amy)

Amy clearly knew about the problems children encounter in writing teen numbers (Wigley, 1997; Anghileri, 2007), but did not apply this knowledge in her practice.

Kate used a number line to help children complete addition calculations such as '8 + 8' and '3 + 4' by beginning at one of the numbers and then counting on the second number. This pre-supposed that children had reached the 'count on' stage in addition. However, observation of the children's independent use of the number lines suggested that some were still at the 'count all' stage (Carpenter and Moser, 1984). Kate was asked if she remembered the stages children go through in learning addition:

> At first not knowing that you can just start at numbers, that you have to count the one, two, three … so you have to count three to get up to three before you can carry on. (Kate)

Although she knew that some children would not be able to understand the addition strategy of starting with one number and then counting on the second number, this propositional knowledge was not drawn on in Kate's teaching.

Analysis of further data using the KQ framework suggested that these beginning teachers became more able to draw on propositional MKT as they gained experience (Turner 2010, pp. 98ff). This suggested that teachers need experience, and focused reflection on their experience, in order to contextualise and make use of the propositional knowledge we present to them in the university. We should not be surprised or disappointed when we find beginning teachers not drawing on this knowledge. We learned that providing the KQ as a tool for reflecting on their teaching helps them to make links to this propositional knowledge and to apply it within the context of their practice.

The use of the KQ to structure review of, and reflection on, teaching

The KQ helped us to observe and to analyse the teaching of our elementary trainee teachers and to give detailed feedback which focused on the mathematical content of their lessons. This detailed analysis of teaching suggested the need to address more explicitly the importance of selecting appropriate examples and representations, as well as making connections and responding contingently to pupils, in our mathematics methods course. Guidelines based on the framework (Rowland et al, 2009, pp. 35-37) were also developed to support university and school-based colleagues working with elementary trainee teachers who were not mathematics education experts. These guidelines were presented and very well received during mentor training sessions at the university, and continue to be available to colleagues.

The usefulness of the framework for supporting observation of, and feedback on, mathematics teaching was explored in a study carried out between 2004 and 2008 (Turner, 2010). It was used as a tool to identify, analyse and chart developments in beginning teachers' MKT, and also as a tool to promote that development. As a tool for development, it was used to frame review discussions of mathematics teaching between teachers and the mathematics teacher educator (MTE) It was also used by the teachers to support individual reflection, helping them identify situations in which their MKT was revealed and to frame their written *reflective* accounts.

In the early phases of the study, the lesson review meetings were intensive and took the form of a stimulated recall interview. The researcher [the second author] used a KQ analysis of the lesson to determine questions to ask and comments to make when the teacher watched the videotape of their lesson. For example, a coding of *choice of examples* (CE) suggested stopping the videotape to ask whether the trainee teacher thought the examples they had used in their explanation of a mathematical procedure were the most appropriate, or whether they might have caused some confusion. The structure of these initial review meetings would be impossible to sustain across a large number of trainee teachers or with busy practicing teachers. The methods employed in the second stage of the study were therefore more appropriate as a model for scaling up the adoption of the KQ for structuring post-lesson review meetings. Lessons were again observed and videotaped however the review meeting was based on a 'broad sweep' KQ analysis of detailed field notes made while observing the lesson. The second author asked questions or commented on significant episodes which had been identified in the analysis and the teachers made observations in relation to the codes and dimensions of the framework with which they were now familiar.

The study also aimed to determine whether the KQ framework supported independent reflection on the mathematical content of teaching. Therefore, during the third phase teachers were not given feedback following their lessons, but were sent DVD copies of the lessons and asked to write reflective accounts independently, structured by the dimensions and codes of the KQ framework. A number of comments made by the teachers demonstrated that they found the framework useful when planning for, and reflecting on, their mathematics teaching. For example:

> I often find myself referring to it in my head when I am planning. …I think the most important effect is having the four headings, makes me more aware of what I am planning and teaching and why. You find yourself questioning yourself and justifying your decisions and choices, it makes you more purposeful in your choices, more precise. (Amy)

From this study we learned that the KQ can be used effectively to frame lesson reviews so that they focus on the MKT of teachers. We also learned that use of the KQ can help teachers to focus their independent reflection on the mathematical content of their teaching.

The role of representations and examples in mathematics teaching

Despite our experience as teacher educators, the KQ research gave us a new appreciation and understanding of the importance of *examples* in mathematics teaching. When teachers teach mathematics they choose and use examples all the time – the relevant code was present in our coding of every lesson. In fact our focus on examples built on earlier work by the first author (e.g. Rowland, 1998) and came at an interesting time from a national and international research perspective. While we were building an emergent theory of teacher-chosen examples (e.g. Rowland, Thwaites & Huckstep, 2003), Watson and Mason (2005) were developing a theory of learner-generated examples, applying and extending the ideas of Ference Marton on variation theory. Both of these perspectives were represented in a PME Research Forum (Bills et al., 2006) and in a special issue of *Educational Studies in Mathematics* (Bills & Watson, 2008).

As a consequence of our own research, we realised and understood better the different purposes for which examples are used, and that the choice of examples is far from arbitrary – some examples ‘work’ better than others. These insights have had a significant effect on our practice in our role as mathematics teacher educators. So whilst formerly we might have spoken in a general way about the importance of choosing examples with care, we are now able to offer our trainee teachers a more analytical account of the choice and use of examples in mathematics teaching and learning. In particular, we identify and exemplify three broad categories of examples that were commonplace in our data, but which, we argue, teachers would do well to avoid. We labelled these categories: examples which confuse the role of variables; examples intended to illustrate a particular procedure, for which another procedure would be more sensible; and randomly generated examples. For details, see e.g. Rowland et al. (2009).

By way of illustration, we exemplify the first of these categories (confusing the role of variables) here, with two excerpts from the classroom data.

Kirsty was reviewing the topic of Cartesian co-ordinates with a class of 10 to 11-year-old pupils. Kirsty began by asking the children for a definition of co-ordinates. One child volunteered that “the horizontal line is first and then the vertical line”. Kirsty confirmed that this was essentially correct. She then moved on to assessing the pupils’ understanding of this key convention by asking them to identify the co-ordinates of a number of points as she marked them on a co-ordinate grid, projected onto a screen at the front of the classroom. Before marking the first point, she reminded them that “the *x*-

axis goes first". Kirsty's first example was the point (1, 1). It is interesting to speculate reasons for Kirsty's choice of this example, recognising that these 'reasons' might be of different types – pragmatic, pedagogical, affective and so on. In any case, the example would seem to be entirely ineffective in assessing what Kirsty wanted to determine: the children's grasp of the significance of the order of the two elements of the ordered pair.

Michael's lesson with a Year 4 class was about telling the time with analogue and digital clocks. One group was having difficulty with analogue quarter past, half past and quarter to. Michael intervened with this group, showing them first an analogue clock set at six o'clock. He then showed them a quarter past six and half past six. When asked to show half past seven on their clocks, one child put both hands on the seven. We can't be sure, but the child's inference from Michael's demonstration example (half past six) seems reasonably clear. Of the twelve possible examples available to exemplify half-past, half past six is arguably the most unhelpful.

The role of *representations* in mathematics teaching has been extensively researched and theorised (e.g. Goldin, 2002). Nevertheless, our research yielded further insights that we were able to bring to our work with trainee teachers. These include the importance of the mathematical appropriateness of representations used for pedagogical purposes. We had observed the trainees' propensity to choose representations on the basis of their superficial attractiveness at the expense of their mathematical relevance (Turner, 2008). In addition, we are now better placed to emphasise the interplay between choice of representations and choice of examples (e.g. Huckstep et al., 2006).

New uses of classroom video data within initial teacher education

The use of video in mathematics teacher education is well-established (e.g. Borko et al., 2008), and articulates well with case method teacher education pedagogy (Merseth, 1996). In England, the video resources that have been most in evidence in primary teacher education are of the kind developed by a government agency for 'National Numeracy Strategy' training (Askew et al, 2004). These tend to feature 'best practice' examples of 'model' lessons given by experienced teachers, presumably with the intention that other teachers will emulate their example. With the permission of the participants in our research, we use video clips from their lessons in a somewhat different way, and with a rather different purpose. These clips feature novice teachers, not 'experts', and as we observe them it is not hard even for trainee teachers to identify things that could be done differently, and maybe should be. We have written about some of these episodes elsewhere (e.g. Huckstep et al., 2006; Rowland, 2010), and there is insufficient space to describe them here. These video stimuli promote lively and thoughtful discussions about what seemed to be successful and what 'went wrong', and why, and what these trainees would do themselves to avoid the errors made (in their judgement) so as to improve the instruction. By contrast, we propose that when an expert teacher's lesson 'goes well', the ingredients of its success can often be invisible to the novice trainee. Using our research video data, and in other ways, we now use these authentic classroom scenarios to pose challenging mathematical and didactical problems, and to raise awareness and insight, in our university-based sessions with trainees.

Cohesion and professional cooperation within and beyond the team

Engagement in collaborative research resulted in greater cohesion and professional cooperation within the mathematics teacher education team. From the beginning, we all engaged in background reading and were involved in discussions which contributed to developing the conceptual framework for the study. Members of the team played different roles according to their expertise, but regularly came together to discuss the progress of the research. The grounded theory approach meant that very intensive discussions were held in order to decide on the emergent codes, and later, on the categorisation of the codes into the four dimensions. These were lively discussions in which all team members suggested ideas based on their experience in the field and/or on their analysis and synthesis of the data. Such discussions also involved drawing on the research literature as well as on our own experience in classrooms. In this way we came to shared understandings of what our research data were indicating. We also came to respect, and to learn from, the different perspectives on mathematics teacher education which derived from our varied career trajectories.

Work using the KQ framework now involves colleagues from around the world. We continue to have intensive discussions about how we ‘understand’ individual codes, and this has contributed to further cohesion and cooperation within the team and within a much wider international KQ family.

CONCLUSION

Teachers and teacher educators often approach their professional development through action research. This entails investigating one’s own practice, adapting it, and looking for evidence of the impact of this change. The development in our professional practice brought about by our research was a consequence of a very different process. We did not set out with the primary aim of developing our own practice. Rather, our focus was on the practice of trainee teachers as we tried to understand how their MKT was revealed and applied in the act of teaching. However, in investigating the practice of trainee teachers, we developed a way of understanding mathematics teaching which supported our own professional development as teacher educators in a number of different ways.

Developments in our understanding of beginning teachers’ MKT, as revealed through KQ analysis of their practice, led to changes in the content of our methods courses, particularly in relation to the importance of examples and representations. We found that the MKT that was ‘learned’ by trainees in our methods courses was not always available to beginning teachers in their practice. However, we discovered that teachers can be supported in applying this knowledge by providing the KQ as a tool for focused reflection. We improved our teaching placement lesson reviews by using the KQ to focus discussion on the mathematical content of teaching, and began to induct school-based colleagues in the use of the KQ to support mentoring of trainees. We also presented the KQ framework to trainees themselves to support focused reflection on their mathematics teaching, so as to enable them to continue developing their MKT during school placements and after their mathematics methods courses were completed. We also developed new video resources for primary mathematics teacher education, and new ways of using them. Finally, a bonus in terms of professional development from participating in the KQ research related to the development of understanding and cohesion within the elementary mathematics teaching team.

These outcomes of our study illustrate the possibility of a symbiotic relationship between research into teaching and learning in classrooms and the professional development of teacher educators. The outcomes of this study show how the roles of researcher and of teacher educator can be complementary and mutually supportive.

Notes

1. In 2002 there were 18 codes in fact: two more were subsequently added in the light of new data.

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