DIDACTIC CODETERMINATION IN THE CREATION OF AN INTEGRATED MATH AND SCIENCE TEACHER EDUCATION: THE CASE OF MATHEMATICS AND GEOGRAPHY

Klaus Rasmussen and Carl Winsløw

Department of Science Education, University of Copenhagen

This paper presents an application of the Anthropological Theory of the Didactic to describe and analyse the genesis of an integrated mathematics and science pre-service teacher education. Reporting from the pre-experimentation phase, it is shown how the levels in the scale of didactic codetermination enable us to understand more clearly how integration is envisaged. We examine more closely the case of a bi-disciplinary teaching-module wherein math plays one part together with geography, and we demonstrate how the scale can be used to explore the precise nature of the intended interaction between the two disciplines.

INTRODUCTION TO THE AMBIGUITIES OF INTEGRATED EDUCATION

At the turn of the century, Czerniak, Weber, Sandmann, and Ahern (1999) made a literature review of science and mathematics integration. They concluded that a lot of "testimonials" existed for the positive benefits of integration, but few empirical studies actually supported this notion that an integrated curriculum is better than a well-designed traditional curriculum. They also emphasised that the term "integrated" was shrouded in ambiguity, with no clear distinction between the diverse labelling of the many-named phenomenon: interdisciplinary, multidisciplinary, cross-disciplinary, trans-disciplinary, thematic or blended, just to mention a few. Ten years later, Stinson, Harkness, Meyer, and Stallworth (2009) reported a similar lack of common characterizations when asking teachers to identify given scenarios as integrated or not. They further concluded that:

> "Potential gains from integration (i.e., time savings, improving on student achievement, improving student interest or motivation) are predicated on a common understanding for what integration means. At the very least, curricula or initiatives designed to foster integration must develop operational definitions for integration before laying claims to an integrated approach or product" (p.159)

The problems with specific labels like "multidisciplinary" or "interdisciplinary" are at least twofold: First, as indicated above, we have no commonly acknowledged definitions: In Andresen and Lindenskov (2009) p. 213-214, multidisciplinary was used to signify a cooperation with clear delimitation of the individual disciplines, proposedly in contrast to interdisciplinary where borders *between disciplines are claimed to be more or less cancelled*. In the same article, interdisciplinary is synonymous to cross-disciplinary, and trans-disciplinary is a radical form where no borders between disciplines are acknowledged. Completely opposite distinctions are found in Matthews, Adams, and Goos (2009), p.892, where interdisciplinary refers to curricula in which there is a mixture of science and mathematics although the *boundaries of the two disciplines remain visible*, and "integrated" is used to signify the lack thereof. A second, perhaps more profound problem is the principal inability of the labels to specify in any detail how the interaction between the disciplines is carried out, let alone what predicates the conditions of the interaction. For this reason too, we need a precise epistemological model of integrated mathematics and science education.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

In this paper we chose to use the term "integrated" to signify *any* educational setting where two or more institutionally established disciplines are intended to work together in order to bring about learning. It is thus used as an overarching name encompassing all the other labels which are usually employed to signify more specific ways of conducting integrated education. The Anthropological Theory of the Didactic (ATD) describes what happens in educational situations as situated in an institutional ecology. Such an ecology is described by a hierarchy of levels, cf. figure 1. This means that the conditions on one level depend on influences from other levels of the ecology. This interdependency is articulated using a scale of levels of determination (see Artigue & Winslow, 2010; or Chevallard, 2004 for more details). In integrated education we are considering at least two such ecologies where an explicit decision has been made to cooperate. We can then identify at which level the decision has been made, at which level the cooperation is meant to take place etc. At most times it will be understood that the cooperation is initially defined at the disciplinary level when considering integrated mathematics and science education, but note that our definition of "integrated" also encompasses cooperation initiated on other levels e.g. the math teacher and the science teacher could agree to use "cooperative learning" (Kagan, 1989) during their respective lessons in the same class, thereby situating the integration on the pedagogy level. Focusing on integration at the discipline level, we have the following framework for investigating the disciplinary cooperation, indicated in figure 2. Above the discipline level, the two ecologies are in principle the same, because we are considering the disciplinary interaction *at the*

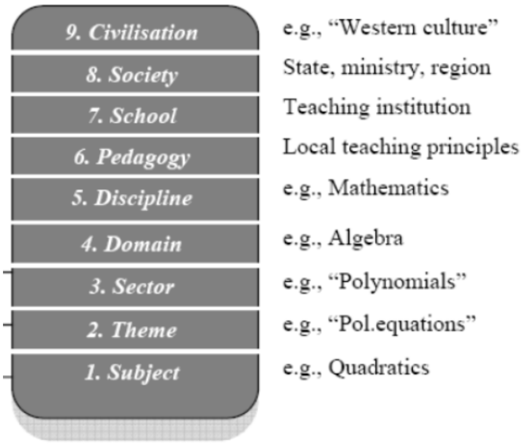

Figure 1: Mono-disciplinary ecology regarding quadratics. (Taken from Winsløw, 2011, p. 133)

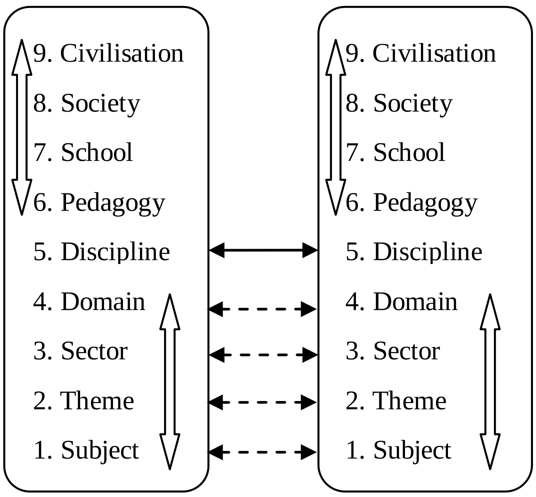

Figure 2: Levels of co-determination for bi-disciplinary ecology

same institution. It is important to note that “mathematics” and “geography” appears as disciplines in closely related, institutional contexts (such as lower secondary schools, universities, and “university colleges”) and while these appearances are indeed different and bound to the institutions, they are also strongly linked. In this paper we will limit ourselves to look at disciplines inside one institutional context (university college) only. This means that influences from same-name disciplines in another institutional context, like universities, comes into our model at a higher level (mainly society). This we will call *second order influence*. It is now possible to model some of the central questions associated with the two integrated disciplines. The first type of questions concerns the “knowledge to be taught” (Bosch & Gascón, 2006):

> “What bodies of knowledge are chosen? How are they named? Why these ones and why with this kind of organisations? What are the reasons to these choices?” (p.56)

The answers to these are determined at the level of the discipline, and the levels above, as indicated by the leftmost vertical arrows.

The second type of questions concerns the levels below the disciplines, where the interaction is realised. These are usually more controlled by the teacher, but still constrained from above (Bosch & Gascón, 2006):

> “Why are mathematical contents divided in these or those particular blocks? Which are the criteria for this division and what kind of restrictions on the concrete activity of teachers and students does it cause?” (p.61)

While both types of questions are phrased the same way for mono-disciplinary education, they take on special meaning when more than one discipline is involved. Decisions taken from the perspective of one ecology have to be informed by the other. This is indicated on figure 2 by the horizontal arrows, where the solid one indicates the level at which the cooperation is formally defined, and the dotted ones signify the possibilities of interaction, whose existence and character may be further specified in a particular context, e.g. as part of the planning of an intended curriculum. This leads us to summarise the following research questions: What are the main features of the interplay between institutionalized ecologies in the planning of integrated math and science education? How can we study the “integratedness” in an inductive way, starting with actual and concrete plans for interaction, rather than with general rhetoric that tends to blur the detailed features? What conditions the planning and cooperation in integrated approaches?

OUR CONTEXT AND METHOLOGY

Teacher education in Denmark is institutionally placed at so called “university colleges”, which are higher education institutions independent from research universities. A consortium between the University of Copenhagen, University College Copenhagen and the Metropolitan University College was formed to construct an experimental teacher education program (called ASTE, Advanced

Science Teacher Education). The goal was to investigate, among other things, the synergistic effects of a multi-disciplinary science teacher education. The students are to become teachers of math and science in the lower secondary school, and the design of ASTE has been developed jointly by participating college and university professors. One of the main characteristics of the program is that large proportions of ordinary curricular items have been placed in bi-disciplinary teaching modules. In this paper we will apply the theoretical framework to study the development of the module named: "Geographical Information Systems, data analysis and modelling in geography", where parts of the math and geography contents are to be taught together. It should be noted that the geography discipline in Danish lower secondary school and at Danish teacher education colleges covers both physical and human geography, and the module we consider here also reflects that.

To shed light on the planning of this interplay we have conducted qualitative interviews with five of the developers; one math and geography college-professor from each university college (below referred to as CM_1, CM_2, CG_1 and CG_2) and one university-professor from the geography discipline (called UG) and with special interests in education. The institutional affiliations are rather complex in the ASTE-collaboration, but we will present them here because the institutional setting is of importance to our model: The two math college-professors are women, and although at the time of interviews they represented two separate colleges, one of them had only recently changed from one to the other. The two geography college professors, male and female, come from another university-college than the two math professors. It should be noted that it is only the college professors who are expected to do the actual teaching, and the programme is implemented in the *physical* institutional setting of a branch of one of the participating university colleges, to which only one of the college professors (math) belong. The ASTE program comes with its own *formal* institutional settings that are written down in general sections of the curriculum, parts of which are tailored specifically, while others are adopted directly from the ordinary institutional framework.

The interviews were conducted in August 2012, beginning with a pilot interview of one geography college-professor. Informed thereby a scheme of questions were designed, and it was decided to ask the interviewees in advance to think of 1) a concrete activity to undertake in the module and 2) if possible, try to think in broad terms of an "entire" plan of action for the module. This aims to follow the inductive approach, starting with the levels below the discipline level. The interviews were semi-structured and lasted approx. 40 minutes each. They centered on two distinct parts: The design of the specific module as situated in the framework of ASTE and further thoughts on the realisation thereof. There was a focus on the individual respondent's perceptions and experiences from the curriculum drafting work, seen in relation to their stance in the existing education system.

DATA HANDLING AND RESULTS

All the interviews were recorded electronically and subsequently inventoried minute for minute. The quotations in the following subsections are translated and transcribed from oral Danish by the authors, and are indexed according to their temporal placement in the interviews

Determination of curricular items for bi-disciplinary integration.

Most of the curricular items chosen from each of the disciplines (see figure 3), are recognizable to both mathematics and geography teachers, that is, they have suggestions about what an item from the other discipline could contribute to their own, but generally they do explicitly acknowledge their lack of expertise in the other discipline e.g.:

> My challenge is that I do not know much about the geography discipline in teacher education …so although I do understand the words, I have difficulties knowing what contents they represent, because I do not know much about geography as a discipline in teacher education. But on the other hand, it is also what makes such cooperation enormously exciting. It is exactly to become knowledgeable about the other disciplines. Are the problems they work with similar to those of math, and how is it about methods? This, I think, could be enormously exciting to get insightful about. (CM_1; 16:11-16:42)

From Geography:

- Knowledge, theories and problem from physical and human geography
- Skills at using geographical sources and methods
- IT as integrated part of the discipline in university college and lower secondary school
- Knowledge to further students geographic language and "bildung"
- Skills to utilize informal arenas of learning and employ investigative methods of inquiry

From Math:

- Geometry, specifically analytic, parametrizisations and trigonometry
- Using and evaluating appropriate representations
- Defining, structuring, mathematizing, interpreting and critique of mathematical models
- Skills at planning, organizing and evaluating teaching.

Figure 3: Curricular items chosen for the module: "GIS, data analysis and modelling in geography"

This supports the notion that the intended integration, instigated at the discipline level, is indeed formal, but there is the desire to make it real by getting to know the other discipline through cooperation, and this is even seen as a separate advantage. It is also evident that curricular items, formulated along the lines of "the use of IT" are recognizable because they are determined at the school and pedagogy level, which are common to the two disciplines. Then there are some items, the determination of which, are situated exclusively at the level of the discipline: "Using and evaluating appropriate representations" and "skills at using geographical sources and methods" are intrinsic to the disciplines, but they pertain to similar disciplinary categories, such as the use of abstract symbols or graphics to represent data.

One could wonder why, or why only, the geometry domain is mentioned from the math discipline, and not e.g. statistics or functions, which could go well together with "data analysis". The interviewees agree that it is a practical and in a way arbitrary choice, because other domains could be made to work out just as well, but there is obviousness to the pairing of geometry and physical geography, which in our model can be seen as determined on the civilization level:

> "It is something about the measurement of the earth and maps, it all fits together very nicely, scales and similarity, it is nearly obvious! (CM_2 20:20-20:28)

The etymology of geo-metry and geo-graphy alludes to the kinship between the two and historically, one may contend that the choice of "maps" and "geometry" are the primary domains involved in the interaction, and is indeed natural and reflects culturally rooted views of the two disciplines and the links between them.

Co-determination of concrete activities in the module.

In fact, when it comes to respondents' ideas for teaching, they are all connected to the notion of maps: The reference to different kinds of maps, the making of maps, maps as a tool for investigation, the historical development of maps. This is determined both at the discipline level, but also conditioned by the requirements of the institution in which the pre-service teachers are going to teach (schools; this second order influence appears at the society level in our model):

> "Mathematics sometimes requires practical examples to illustrate what math is, and that is what you can do when working with mapmaking, ...because the math related to that, is also the math you use in lower secondary school" (CG_1 18:47-19:13)

In figure 4 we have shown an example of how to model the interdependencies surrounding the creation of a physical map, as the subject of a teaching activity. Using this model it is possible to identify possibilities and constraints that one discipline imposes on the other, to the enactment of such an activity: At the domain-level, the choice of geometry, applied to the measurement of the earth, precipitates the restriction to physical geography. This in turn determines what should be the types of maps to work with: e.g. physical or topographic instead of other more culturally oriented maps such as political maps. Then distances could be the information you would like to convey using the map, and the choice of plane geometry might be taken to avoid the time-consuming complication of earth curvature and the different heights of the landscape being drawn on the map, especially if triangles are to be the mathematical theme. Triangles are used when making the measurements of the land considered as a plane surface, and then, by similarity, transferred to the map:

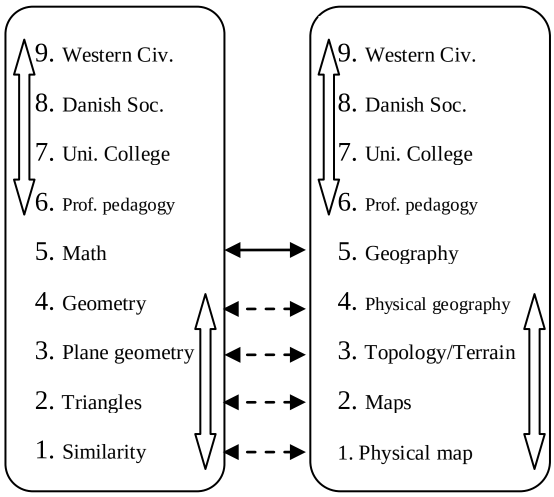

Figure 4 Example of bi-disciplinary ecology inferred from suggestions of interviewees

> "yes, that about surveying, at least triangulation and how maps has been made, and how maps look; I don't know how much surveying per se [they have in geography], when I

think about surveying, it is more from a mathematical point of view, when we go out and construct figures out there" (CM_2 14:31-15:50]

"It it clear that it would be tempting to measure on a sphere, meaning spherical geometry... but this [module] is not huge, so we have to be careful about how much we can achieve in the given time (CM_2 24:42-24:57)

It may appear curious that the math teachers so willingly lend their discipline to the making of maps and surveying, but it is a common feature of Danish teacher education that applied math is considered of quite high value, which is also directly reflected in the curriculum description of the mathematics discipline at university colleges:

"The history of the discipline, the discipline's function as a bearer of culture, and the application of the discipline, is an important part of its identity as a teaching discipline" Undervisningsministeriet (2011, appendix 2, section 3)

Looking at other suggestions to the possible contents of the module we find a lot of references to "problems of flooding and water flow" (CG_1 5:10, CM_1 19:20-19.37; 22:10, CM_2 15:20-16:00; CM_2 17:55-18:20). These are all strongly influenced by the society level:

"It would be really nice if it [the context for teaching] could be some kind of real problem" (CM_1 22:21-22:28)

"... but it [the problem of the activity] is strictly a concern for society, it is all concerns for society" (CM_2 18:30-18:35).

Let us now select two of the concrete examples: 1) Investigating the impact of a new national law, proscribing that no land may be farmed that is closer to a stream than 5 meters. 2) Investigating the flow of water through a lake or stream. Both open up a host of possible teaching avenues, both for math and geography. In figure 5 and 6 we have put this into our model to describe how decisions at higher levels of the scale will interact to produce the possible integrated practice at the subject level. In both examples we recognize the aim to use Geographical Information Systems as a tool to do investigations, but GIS does not appear as an object of knowledge in itself. Therefore it is not mentioned explicitly in the model. It is used at the praxis levels (thematic and subject), and it is conditioned at the discipline level:

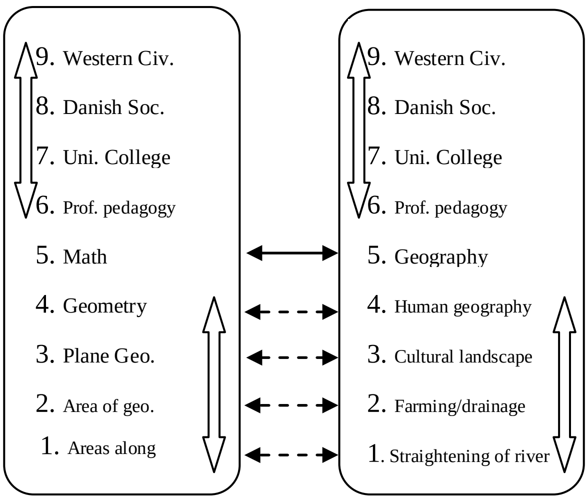

Figure 5 Example of bi-disciplinary ecology "Investigating impact of new law"

> “I think, it is because many who work with geographical information systems believe it is absolutely obvious that the whole world should know about it, because it is so incredibly smart and it appears in so many contexts” (CG_1 6:14-6:25)

It is worthwhile to notice that CG_1 expresses the desirability to have GIS included at the discipline level of math and geography, in the institutional ecology of the university college, whereas the reasons for the desire is said to come from parts of the society level, namely *those who work with GIS*. This is another example of a second order influence, which comes from an institutional context outside the university college (namely, from the same-name discipline in scientific institutions such as universities). Looking at the first avenue of teaching in figure 5 (example 1 above), we remark that it is directly determined on the society level as the idea originates from the consequences of a political decision. This clearly conditions the domain level to the geographical subfield of human geography, and the sector narrows it down to looking at culturally formed landscapes. Then what can be investigated mathematically is the area of farmland affected by the law, giving rise to the sector of plane geometry. One could hypothesize that the subsequent implications for the economy of the affected farmers could be mathematically considered, *but that would not be in strict accordance with the choice of geometry at the domain level*. As a consequence the theme and subject will, in regards to math, revolve around non-trivial calculations of area alongside curves, which could benefit from the aid of GIS. The geography part could draw on the area calculations and focus on issues of farming and the straightening of rivers (a classical subject of Danish human geography)

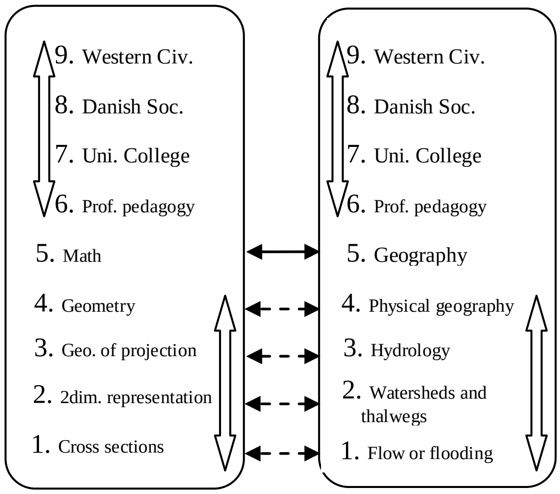

Figure 6 Example of bi-disciplinary ecology “Flow of water”

The second avenue of teaching (example 2, above) takes us into the “geometry of projections” sector of the chosen mathematical domain. (Figure 6) This is influenced by the choice of hydrology in the physical geographic domain, which has the study of moving water as an object. One theme could be thalwegs, in which two dimensional representations of river cross sections are extensively used. Also the cross section of the inlet and outlet of a lake, will determine the area of the lake surface. Horizontal cross sections of the lake landscape can be used to predict the extension of the lake for different flow rates:

> “We talked at some point about the flow through a lake, how the surface area, yeah, could be measured, but also how it changed in accordance with the flow in and out.” (CM_1 19:13-19:31)

To construct the two dimensional representation, it is suggested that students physically go into the geotope and make the measurements using a mobile phone application, that can transfer data to a GIS system. Doing investigations in the field is an important part of geography, valued at the discipline level, and the processing of data to make the graphical representations, in this case, of river cross sections, is firmly rooted in the mathematical ecology at university colleges (school and discipline levels). The above quote may also allude to finding some relationship or model of the changing lake surface area, expressed in terms of a function, as a pivot for the teaching activity. However, that would conflict with the choice of geometry at the domain level. This choice, if vigorously adhered to, seems indeed to impose rather strong restrictions on the lower levels:

> "You could easily get into functions here, and differential equations, if you look at the velocity with which the water runs from the lake. So that could be described in some dynamical systems, but we have nevertheless chosen that it [the module] should take another direction." [CM_1 29:15-29:39]

The two above analysed avenues of teaching both have common connection to the concept of "flow". This reminds us of what Wake (2011) calls a *bridging concept*, which "provide a driver to facilitate cross-disciplinary thinking" (p.1004). But we contend that the interaction among the different levels of didactic codetermination in the bi-disciplinary ecology provides a refined and more precise model of the idea reflected by the term "bridging concept".

CONCLUSION AND PERSPECTIVES

The way two disciplines, as situated in the institutional ecology of teacher education at university colleges, interact, when trying to establish integrated education, are determined by factors residing at levels above the one immediately considered. The interaction crosses the disciplinary boundaries, meaning e.g. that the domain level of one discipline will influence the theme level of the other. The route of influence, as expressed in interviews with the developers of the integrated math and geography teaching module, can be modelled by the levels of determination, and *it goes by way of the vertically and horizontally indicated directions*. That is, we have seen no determinations that appear to go, for example, directly from the pedagogy level of the math disciplinary ecology to the theme level of the geography ecology. But the possibility of such level crossing codetermination in integrated education needs to be further investigated. This question, and the more general one of seeing borders between disciplines as a criterion of demarcation, is by no means a trivial one when we look at the long ongoing debate about the nature of integrated education. In the illustrations used to represent our model we have what appear to be clear borders between the participating disciplines. This is to recognize that our model *does* operate with disciplines as distinct bodies of knowledge, and this also reflect evident conditions in the institutional context studied. Indeed, the curriculum construction in ASTE begins with the existing disciplines, which define relevant positions in the institutional context of the university college. In that fashion the disciplines come before "big questions" even in the early planning phases. The model is not to be taken normatively, and it does not say to which degrees borders do, or should be discernible, in order to represent "true" integration. It serves simply to organise our analysis of how integration of disciplines takes place, or is planned to take place. Finally it allows us to situate second order influences from same-name "scientific disciplines" and "secondary school disciplines" which co-determine the planning and cooperation in integrated approaches at the university colleges participating in this project. An extension of the model to study these influences further appears of great interest to understand more globally the interplay between participating institutions.

REFERENCES

Andresen, M., & Lindenskov, L. (2009). New roles for mathematics in multi-disciplinary, upper secondary school projects. *ZDM, 41*(1), 213-222.

Artigue, M., & Winsløw, C. (2010). International comparative studies on mathematics education : a viewpoint from the anthropological theory of didactics. *Recherche en Didactique des Mathématiques, 30*(1), 47-82.

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin, 58*, 51-63.

Chevallard, Y. (2004). La place des mathématiques vivantes dans l'éducation secondaire: transposition didactique des mathématiques et nouvelle épistémologie scolaire.In Ducourtioux, C. & Hennequin, P.-L. (eds) *La place des mathématiques vivantes dans l'éducation secondaire* (pp. 239-263). Paris: APMEP.

Czerniak, C. M., Weber, W. B., Sandmann, A., & Ahern, J. (1999). A Literature Review of Science and Mathematics Integration. *School Science and Mathematics, 99*(8), 421-430.

Kagan, S. (1989). The Structural Approach to Cooperative Learning. *Educational Leadership, 47*(4), 12-15.

Matthews, K. E., Adams, P., & Goos, M. (2009). Putting it into perspective: mathematics in the undergraduate science curriculum. *International Journal of Mathematical Education in Science and Technology, 40*(7), 891-902.

Stinson, K., Harkness, S. S., Meyer, H., & Stallworth, J. (2009). Mathematics and Science Integration: Models and Characterizations. *School Science and Mathematics, 109*(3), 153-161.

Undervisningsministeriet. (2011). *Bekendtgørelse om uddannelsen til professionsbachelor som lærer i folkeskolen.*

Wake, G. (2011). *Modelling In An Integrated Mathematics And Science Curriculum: Bridging The Divide.* Paper presented at the CERME 7, Rzeszow, Poland.

Winsløw, C. (2011). *Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education.* In M. Bosch et al. (Eds), Un panorama de la TAD (pp. 117-140). Barcelona: Centre de Ricerca Matemàtica.