ANALYSIS OF PRE-SERVICE ELEMENTARY TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE IN THE CONTEXT OF PROBLEM POSING

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In this study, it was aimed to study the level of pre-service elementary teachers' proficiencies in determining 5th grade students' problem posing errors about addition with fractions. This study was realized with 36 senior class pre-service elementary teachers in an eastern university during spring term in 2011-2012 academic year. Error Determination Test, which has six problem statements about addition with fractions, was applied to pre-service elementary teachers. Participants were asked to compare addition operations with the given problem statements and to clarify errors if there were any. Research findings indicate that participants had problems in determining errors and made different errors in their explanations about the students' errors.

Keywords: *Pre-service elementary teachers, problem posing, fractions, addition operation with fractions.*

INTRODUCTION

Problem posing takes increasing attention in recent years. Main reasons that lie behind this attention are establishing relationship between mathematical concepts, operations and daily life (Abu-Elwan, 2002; Dickerson, 1999; Knott, 2010), transitions between representations (English, 1998; Işık, Işık & Kar, 2011) and it contributes to these issues. This study draws attention to the analysis of pre-service teachers' pedagogical content knowledge which includes being aware of students' misconceptions and also errors. Particularly, the following aim was addressed: studying pre-service elementary teachers' proficiencies in determining errors in problem statements about addition with fractions.

THEORETICAL FRAMEWORK

There are some knowledge categories that professional teachers should have. One of these categories as determined by Shulman (1987) is pedagogical content knowledge. It is specifically about illustrations, explanations, and examples used in making a subject more comprehensible to learners. In detail, it is about knowing functional representations and illustrations of content and concepts, knowing what the issues that makes learning content easier or harder, knowing students' misconceptions and errors, knowing analogies, symbols, examples or explanations that helps overcoming misconceptions and understanding concepts, and lastly knowing different age group and level students' thoughts, perceptions, and previous knowledge about concepts (Shulman, 1987).

Problem posing is about generating new problems and reformulating problems from the given problems or situations (Duncker, 1945). As English (1998) indicated that students could improve defining symbolic mathematical expressions ability and relating them with daily life issues ability through problem posing. Besides, Crespo (2003) stated that problems posed by teachers give students chances to learn mathematics. Işık and Kar (2012) determined elementary school mathematics teachers realized more problem posing activities about fractions sub learning domain than other ones under numbers learning domain. Additionally, all of the teachers, who gave more place to problem posing activities about fraction sub learning domain, expressed that problem posing contributes students in relating conceptual understanding to symbolic expressions in daily life.

Moreover, Hill, Rowan and Ball (2005) stated that teachers' mathematical knowledge should give chance to students to explain and teachers to analyze their students' answers. On the other hand, there are limited studies studying teachers' and pre-service teachers' pedagogical content knowledge in operations with fractions. Toluk-Uçar (2009) found out that pre-service elementary teachers thought fractions represent pieces instead of amount and as well the solutions of the problem they posed necessitates addition in natural numbers instead of addition in fractions. The problem a participant posed for the operation $\frac{1}{3}+\frac{1}{2}=$ was "My mum gave me 1 of her 3 apples and my brother gave me 1 of his 2 apples. How many apples did I have altogether?'' and it exemplifies this situation. Ticha and Hošpesová (2009) asked pre-service teachers to pose a problem from $\frac{1}{4}\mathrm{x}\frac{2}{3}$ operation; afterwards, from the problems participants posed, they asked them to evaluate three of them. The researchers determined that pre-service teachers ignored the conceptual dimension of the operation, could not relate the given operation with the daily life issues, and some of them posed problems that necessitate multiplication instead of addition, and lastly students indicated that it was easy to formulate same type of problems but it was difficult to formulate problems of a growing difficulty. Işık (2011) concentrated on conceptual analysis of the problems about multiplication and division in fractions posed by pre-service elementary mathematics teachers. The results of study showed that pre-service teachers had difficulties in the conceptual dimension of fractions and operations in fractions. On the other hand, in the literature there is not any study about teachers' and pre-service teachers' pedagogical content knowledge in determining the errors in the problems posed about addition with fractions by students.

Problem posing is effective in both clarifying students' mathematical skills thoroughly and giving chance to assessing what students did (Whiten, 2004). In addition to this, problem posing informs teachers about students' skills, attitudes, and conceptual learning about a situation (Işık & Kar, 2012). As mentioned before, one of the categories of pedagogical content knowledge is being aware of students' misconceptions and errors. From this aspect, when pre-service teachers are

encountered with the students' errors in the problems posed, pre-service teachers' awareness could be enhanced.

The success of problem posing activities was based on the guidance of teachers to students about true problem posing and exploring. When the successes of instructional process and student were affected from teachers' knowledge (Dooren, Verschaffel & Onghena, 2002; Fennema & Franke, 2006; Shulman, 1987) issue is considered, it is significant to analyze teachers' and pre-service teachers' pedagogical content knowledge under different dimensions. Besides, as Crespo (2003) mentioned mathematical problem posing is one of the difficulties in learning mathematics, and as well it isn't clear when and how the pre-service elementary teachers could learn about this issue. In this regard, this study is realized with pre-service teachers and it was aimed to study pre-service elementary teachers' proficiencies in determining errors in problem statements about addition with fractions. Therefore, this study would contribute possible planning processes in teacher education.

METHOD

This study was realized with 36 senior class pre-service elementary teachers in a public university in eastern part of Turkey during spring term in 2011-2012 academic year. These participants took Basic Mathematics I and II (freshman year course) as well as Teaching Mathematics I-II (junior year course) during their instructional process. In addition, they had also chance to observe and practice in class instructional activities at schools. Pre-service teachers were coded with pseudo names like PT1, PT2,…, PT36.

Error Determination Test (EDT), which contains six problem statements about addition with fractions, was applied to pre-service teachers. Items in the test were given in Table 1 below.

Table 1: Problem Statements in EDT

	Problem Statements
1.	I ate $\frac{1}{3}$ of the oranges my mother bought. My brother ate $\frac{1}{2}$ pieces. How many oranges are left?
2.	We bought a cake for my birthday. My sister and I ate $\frac{1}{2}$ of the cake together. $\frac{1}{3}$ of the rest of the cake was eaten by my mother and father. Accordingly, what is the fraction of the rest of the cake?
3.	First day, Ali's classmates planted $\frac{1}{3}$ trees to the school garden. In the second day, they planted $\frac{1}{2}$. Hereunder, how many trees did Ali's classmates plant totally?
4.	Süleyman at first picked $\frac{1}{2}$ of the roses and then picked $\frac{3}{4}$ of the roses. How many roses did Süleyman have totally?

5.	Ahmet joined the game with $\frac{1}{2}$ of the marbles and Mehmet joined with $\frac{1}{3}$ of the marbles. So, what is the amount of marbles from the total did Ahmet and Mehmet join the game with?
6.	Ali participated in a penalty game two times. In the first game, he made one goal from two kicks, and in the second game, he made three goals from four kicks. Therefore, what is the fraction of goals did Ali do at the end of two games?

First four problem statements in Table 1 were selected from problem statements posed by 5th grade students from $\frac{1}{3}+\frac{1}{2}=\blacksquare$ and $\frac{1}{2}+\frac{3}{4}=\blacksquare$ operations. In the first item, the sum is a proper fraction, and in the second item, the sum is a mixed fraction. Işık and Kar (2012) asked 210 7th grade students to pose problems from the five items given about addition with proper and mixed fractions. Afterwards, the researchers analyzed the problems posed by the participant students and determined error types. In the first four problem statements in EDT, there are six error types determined by Işık and Kar (2012); *expressing the added second fraction over the remainder of whole (E1), failure in expressing the operation in the question root (E_2), attributing natural number meaning to the result of the operation (E_3), confusion about units (E_4), attributing natural number meaning to the added fractions (E_5),* and *failure in establishing part-whole relation (E_6).* Fifth and sixth problem statements were added to EDT by the researchers. In the literature, these types of problem statements were utilized by different researchers (Chick & Baker, 2005; Newton, 2008; Ward & Thomas, 2007) for determining students' conceptual knowledge about fractions. Explanations about the errors in problem statements were presented in the findings part.

In the implementation process of EDT, participants were told that *fifth grade students were asked to pose resolvable problems about addition with fractions based on only the given operations. All of the problem statements in the test were posed by students.* There is an explanation like *compare given addition operation with problem statement, and express error types if there is any;* in this process for participants not to lose their motivation and for them not to analyze with prejudice, fifth and sixth problem statements were also said as problem statements posed by students. EDT was applied to pre-service teachers in one class hour. Answers of pre-service teachers were analyzed with content analysis method.

Two different researchers analyzed pre-service teachers' answers about each item in the EDT concurrently and independent of each other. Based on the analysis of each item in EDT, the consistence was found as; 87,5%, 98,43%, 96,88%, 100%, 98,43% and 90,62%, respectively. In the comparison process, answers that were not appropriate to determined error types were presented under *other* category. This category contains some statements that do not express errors instead problem statements were written with little changes in the sequence of words (ex. *what is the fraction of money collected?* Instead it was written like *what is the fraction of money*

collected from the class did our teacher spend?), and expressions were not open enough to make error analysis.

FINDINGS

Conceptual Analysis of Problem Statements

In the first problem statement, it was mentioned that $\frac{1}{2}$ *of the pieces* were eaten by brother. With the *piece* word in a fraction form, it was tried to express quantities like in the natural number form. Therefore, there is an E_5 error type in the problem statement. Besides, *how many oranges are there left?* question has a subtraction meaning. From this aspect there is an E_2 error type in the problem statement.

In the second problem statement in EDT, addend $\frac{1}{3}$ fraction is expressed via rest of the cake. However, this necessitates $\frac{1}{2} \times \frac{1}{3}$ operation. On that sense there is an E_1 error type in the problem statement. In addition, in the problem statement rest of the cake is asked instead of the amount of that was eaten; so, it does not meet addition meaning. Consequently, there is also an E_2 error type in the problem statement.

Third problem statement in the EDT involves $\frac{1}{3}$ *trees* and *how many trees were planted totally?* question, with these expressions it was tried to give addend fraction and the result of the operation which were fraction the natural number meaning. From this aspect, there are E_3 and E_5 error types in the problem statement. Besides, with $\frac{1}{2}$ expression it is not clear if $\frac{1}{2}$ indicates the amount of trees or $\frac{1}{2}$ represents the area of school garden. On that sense, fraction does not represent an appropriate unit and there is an E_4 error type in the problem statement.

In the fourth problem statement of EDT it is mentioned that at first $\frac{1}{2}$ of the roses in the garden were collected. In this situation rest of the roses was the half of the total roses. On the other hand, student gave place to *later on* $\frac{3}{4}$ *of them were collected* expression. This situation is not logical from the point of part-whole relation. From this aspect there is an E_6 error type in the problem statement. In the question it is asked like how many roses did Süleyman have totally?; with this expression the result of the operation which was a fraction was given a natural number meaning. Therefore, there is an E_3 error type in the problem statement.

In the fifth problem statement in EDT, there is not any information that shows if the addend fractions were taken from the same whole or not. On that sense, although it looks like formal of problem statement meets the operation, in a conceptual sense it is not possible to add fractions from different wholes. Besides, although two wholes are similar the total of two fractions is not the result of the problem statement. In the sixth problem statement in EDT it was mentioned that at the end of the game only four kicks of the six kicks were turned into goal. On the other hand, the result of $\frac{1}{2} + \frac{3}{4}$

operation which was tried to be posed a problem based on this doesn't meet this result. Problem statement necessitates $\frac{1}{6}+\frac{3}{6}$ operation which accepts the total amount of kicks as a whole instead of $\frac{1}{2}+\frac{3}{4}$ operation. From this aspect, problem statement and the operation which was tried to be posed a problem based on this are not consistent.

Pre-service Teachers' Proficiencies in Determination of Error Types in Problems Posed

The distribution of what error types pre-service elementary teachers found in the first four items in EDT is presented in the following Table 2.

Table 2: The Distribution of Error Types Found in the First Four Items in EDT

Items	E_1	E_2	E_3	E_4	E_5	E_6	Errorless	Blank	Other
1		27(75)			18(50)		3(8,3)	0(0)	2(5,5)
2	18(50)	23(63,9)					5(13,9)	1(2,8)	1(2,8)
3			13(36,1)	8(22,2)	16(44,4)		8(22,2)	0(0)	2(5,5)
4			11(30,6)			3(8,3)	4(11,1)	3(8,3)	2(5,5)

*Values in table were calculated based on 36 participants and presented with frequency (percentage).

According to Table 2, in the first problem statement one fourth of the participants could not determine E_2 error type and half of them could not determine E_5 error type. From these proportions, it could be said that to determine E_5 error type is harder for participants. 13,9 % of the participants mentioned that there was not any error in the second problem statement. Moreover, half of them did not determine E_1 error type, almost 36 % of them could not determine E_2 error type. From these proportions, it is harder to determine E_1 error type. About the third problem statement 22,2 % of the participants mentioned there was not any error. Besides, 44,4 % of the participants could determine E_5 error type, meanwhile the proportions of determination of E_3 and E_4 error types are less. Specifically to determine E_4 error type is harder for participants. In the fourth problem statement, 11,1 % of the participants stated that there was not any error in the problem statement. Almost 70 % of them could not determine E_3 error type, and 92 % of them could not determine E_6 error type. From these proportions, it could be said it is harder to determine E_6 error type.

Besides, about the fourth problem statement some of the participants made different error types in their explanations about the errors in this statement. Five of the 11 participants who determined E_3 error type stated that the question should be like *what is the fraction of roses collected?*. PT23's explanation about this issue was;

> PT 23: It is a wrong way to ask how many roses did Süleyman have. If it is asked like what is the fraction of roses collected, it would be a true problem.

In the PT23's explanation the result must be fraction, so, it was emphasized with *what is the fraction of* expression. On the other hand, part-whole relation was

ignored; due to this, in the question *what is the fraction of* was used. This expression is not appropriate to logical aspect. Six of the 11 participants who determined E_3 error type stated that they did not know the beginning amount, so, they could not answer the issue that was asked in the question. PT4's explanation about this issue was that;

PT4: In here, the total number of roses in the garden is not mentioned, so, it is not possible to calculate how many roses were collected. Whereas if it is known given operation could be calculated.

Students were asked only to pose a resolvable problem with just $\frac{1}{2} + \frac{3}{4}$ operation. In the given application directive it was emphasized that pre-service teachers should analyze problems posed through this aspect. Therefore, explanations about the necessity of knowing the beginning amount are not appropriate approach when the given operation is considered. In addition, when it is thought if the beginning amount is known, in a logical sense this would not be an appropriate problem. Due to the result of the given operation being a mixed fraction, expressing addend fractions as a part of the whole would cause violation of part-whole relation.

The distribution of pre-service teachers' answers to fifth and sixth problem statements in EDT was presented in Table 3 below.

Table 3: The Distribution of Answers Given to Fifth and Sixth Items in EDT

	Errorless	**With Error**	**Blank**	**Other**
Fifth Problem	9(25)	24(66,7)	2(5,5)	1(2,8)
Sixth Problem	21(58,3)	6(16,7)	7(19,5)	2(5,5)

*Values in table were calculated based on 36 participants and presented with frequency (percentage).

According to Table 3, 25% of the participants stated that there was not any error in the fifth problem statement. On the other hand, two thirds of the participants mentioned that problem statement was not appropriate to given operation. 20 of the 24 participants, who mentioned the problem statement had an error, said that if the reference fractions indicating the number of marbles were equal or not was not known, so, these two fractions could not be added. PT27's explanation about this issue was as follows;

PT27: There is an error. Are the numbers of Ahmet's and Mehmet's marbles equal? This is not known, so, it is not possible to find the true answer with just addition.

In addition, four participants found the error in the way of how the question was asked. Participants mentioned that problem should be asked like *what is the fraction of total marbles that they joined to the game?*. PT5's expression about this issue was;

PT5: Problem should continue like: Accordingly, what is the fraction of total marbles that Ahmet and Mehmet joined the game with?

According to Table 3, more than the half of participants (58,3 %) stated that there was not an error in the sixth problem statement. On the other hand, six participants

determined the error in the problem statement, but they could not give a conceptual explanation to the reason of this error. Four of the participants said that there is not appropriate relationship between the result as $\frac{5}{4}$ to the operation and the result of the solution of the problem posed. Four of the participants mentioned that the result of the problem posed based on the given operation was not consistent with problem statement. PT1 mentioned about this issue as;

PT1: The result of given operation found as 5/4. The number of kicks that were done as goal was higher than the whole, so, the problem had an error.

Moreover, two participants saw the reason of the error as different number of kicks in each penalty kicking game. PT3 made an explanation about this issue as;

PT3: It was played two times. There were two different wholes, so, the sum could not give how many kicks were turned into goal.

RESULT AND DISCUSSION

In this study, pre-service teachers' proficiencies in determining the errors that 5th grade students done in problems posed about addition with fractions were studied. It was found that determining E_1 and E_2 error types in the problem statements was higher than other error types. In the focus of these errors, it could be said that there was problem of transferring fractions and addition to the problem statements. It could be said that these errors were related with formal aspect of the operation, but other error types were related with conceptual aspect of the fractions. Therefore, increased determination E_1 and E_2 error types could be thought that analyses were generally done on the formal aspect. It was found that pre-service teachers had more difficulty in attributing a natural number meaning to the result of the operation in the first four problem statements of EDT, not stating fractions with appropriate units and not associating fraction in the reference amount with whole. Specifically, not associating part-whole relation was the least found error type done by pre-service teachers. It could be mentioned that participants not being able to determine the sum as a mixed fraction caused them living more difficulty. Moreover, some participants made new errors in their explanations about their errors. These findings supported by different studies' results (Işık, 2011; Redmond & Utley, 2007; Rizvi, 2004; Toluk-Uçar, 2009; Zembat, 2007) that pre-service teachers had difficulties in problem posing about operations with factions.

25 % of pre-service teachers indicated that there was not an error in the fifth problem statement and only 55,6 % of them could make a conceptual explanation about the reason of the error. These findings were supported by Newton (2008) as pre-service teachers had difficulties in determining the impossibility of addition with fractions defined in different wholes. Only six pre-service teachers determined the error in the sixth problem statement in EDT. On the other hand, none of them could make a conceptual explanation to error. These findings were similar to the results of other studies realized by different researchers (Chick & Baker, 2005; Ward & Thomas, 2007).

Crespo (2003) mentioned that pre-service teachers posed problems without thinking mathematical and pedagogical aspects and did not study the resolvableness of them. Based on the findings of this study, they supported what Crespo mentioned, and besides they indicated that these skills like addition with fractions should be enhanced. Findings of this study were gathered from six problem statements about addition with fractions topic. This could be seen as a limitation of the study. In the future, this study could be extended with problem statements involving different fractional number and possible other error types and these studies could be done with pre-service teachers or in-service teachers. By qualitative studies, from participants' frames of mind that include the reasons behind different error types from some pre-service teachers' explanations it could be revealed. Experimental studies could be realized for removing the difficulties by considering the results of these like studies in the future.

REFERENCES

Abu-Elwan, R. (2002). Effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance. *Journal of Science and Mathematics Education, 25*(1), 56-69.

Chick, H. L., & Baker, M. K. (2005). Investigating teacher's responses to student misconceptions. In Chick, H. L., & Vincent, J. L. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol.2, pp. 249-256). Melbourne: PME.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in pre-service teachers' practices. *Educational Studies in Mathematics, 52,* 243–270.

Dickerson, V. M. (1999). *The impact of problem posing instruction on the mathematical problem solving achievement of seventh graders.* (Unpublished doctoral dissertation). University of Emory, Atlanta.

Dooren, W. V., Verschaffel, L., & Onghena, P. (2002). The impact of pre-service teachers' content knowledge on their evaluation of students' strategies for solving arithmetic and algebra words problems. *Journal for Research in Mathematics Education, 33*(5), 319- 351.

Duncker, K. (1945). On problem solving. *Psychological Monographs, 58,* (5, Whole No. 270): American Psychological Association.

English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education, 29*(1), 83-106.

Fennema, E., & Franke, M. (2006). Teachers' knowledge and its impact. In Grouws, D.A. (Ed). *Handbook of Research on Mathematics Teaching and Learning* (pp.147-164). Reston, VA: NCTM.

Işık, C., & Kar, T. (2012). Matematik dersinde problem kurmaya yönelik öğretmen görüşleri üzerine nitel bir çalışma. *Milli Eğitim Dergisi, 194,* 199-215.

Işık, C. (2011). İlköğretim matematik öğretmeni adaylarının kesirlerde çarpma ve bölmeye yönelik kurdukları problemlerin kavramsal analizi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 41*, 231-243.

Işık, C., & Kar, T. (2012). Analyzing problems posed by 7th grade students for addition operation with fractions. *İlköğretim Online, 11*(4). Retrieved May 30, 2012, from http://ilkogretim-online.org.tr

Işık, C., Işık, A., & Kar, T. (2011). Öğretmen adaylarının sözel ve görsel temsillere yönelik kurdukları problemlerin analizi. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 30,* 39-49.

Knott, L. (2010). Problem posing from the foundations of mathematics. *TMME, 7*, 413-432.

Newton, K. J. (2008). An extensive analysis of preservice elementary teachers' knowledge of fractions. *American Educational Research Journal, 45*(4), 1080-1110.

Redmond, A., & Utley, J. (2007). Prospective elementary teachers understanding of and attitudes towards the division of fractions. *Paper presented at the Research Council on Mathematics Learning Annual Convention,* Oklahoma City.

Rizvi, N. F. (2004). Prospective teachers' ability to pose word problems. Retrieved October 17, 2011, from http://www.cimt.plymouth.ac.uk/Journal/rizvi.pdf

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57(1),* 1-22.

Ticha, M., & Hošpesová, A. (2009). Problem posing and development of pedagogical content knowledge in pre-service teacher training. *Paper presented at the meeting of CERME 6*, Lyon.

Toluk-Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25*(1), 166-175.

Ward, J., & Thomas, G. (2007). What do teachers know about fractions? In *Findings from the New Zealand Numeracy Development Projects* (pp. 128-138). Wellington: Learning Media.

Whiten, D. J. (2004). Building a mathematical community through problem posing. In. Rubenstein, R. N. & Bright, G. W. (Eds.), *Perspectives on the teaching of mathematics: Sixty-sixth tearbook (pp. 129-140).* Reston, VA: NCTM.

Zembat, İ. Ö. (2007). Working on the same problem-Concepts; with the usual subjects- pre-service elementary teachers. *İlköğretim Online, 6*(2), 305-312. Retrieved June 25, 2011, from http://ilkogretim-online.org.tr/vol6say2/v6s2m22.pdf