# TEACHERS' BELIEFS ABOUT KNOWLEDGE FOR TEACHING – AN INDIRECT APPROACH

Jason Cooper, Israel Touitou

Weizmann Institute of Science, Israel

*In this paper we analyze teachers' beliefs about the knowledge needed for teaching elementary school mathematics. Eliciting such beliefs is important for designing and evaluating teacher education. We find indications of these beliefs in anonymous feedback questionnaires the teachers submitted in an in-service professional development course. This indirect approach avoids discrepancies between teachers' declared (conscious) beliefs and tacit beliefs that actually influence their learning. We found that beliefs changed during the course, at first favouring pedagogical content knowledge (PCK) as a learning goal, and shifting toward subject matter content knowledge (SMCK). The significance of this research is not only its findings but also its method, which avoids some issues inherent in traditional methods.*

**Keywords**: Teacher beliefs, knowledge for teaching, mathematics education, professional development, mixed methods

## INTRODUCTION

Teachers' beliefs are an important theoretical construct which has been receiving much attention in recent years. Teachers' beliefs about mathematics and about teaching and learning mathematics have an impact on their in-the-moment teaching decisions (Schoenfeld, 2010); however in this paper we are interested in teachers as *learners*. Teachers' beliefs about the knowledge they need for teaching influences what they learn and how they learn it. "*... teachers may be guided by their beliefs about teaching knowledge ... Such beliefs may lead them to question the value of information presented…*" (Fives & Buehl, 2008, p. 135). This idea is also supported by the theory of Adult Learning (Knowles, 1990), which states that adults tend to have a task-centred orientation to learning, i.e. are motivated to learn to the extent that they believe the learning will be instrumental in their professional practice.

There are at least two ways in which teachers' beliefs about knowledge for teaching might be considered when designing in-service professional development (PD[1]): taking their beliefs as given and aligning the PD teaching goals with them, or conversely, considering their beliefs as something that may change as a result of the PD and making beliefs an explicit teaching goal, thus aligning them with the intended PD content. In either approach, a reliable tool for revealing teachers' beliefs about knowledge for teaching is required.

The most direct method for revealing teachers' beliefs is asking the teachers about them, either through questionnaires or through interviews. A problem with such an approach is related to what Toerner (2002) calls *membership degree* attributes of beliefs, and more specifically, levels of *consciousness* and levels of *activation* of beliefs. Belief systems are complex; people may hold a variety of beliefs, more or less conscious, possibly conflicting, which are activated in specific situations. It is questionable whether beliefs that are activated when answering a questionnaire (necessarily conscious beliefs), or even during an interview, will be consistent with "situated" beliefs activated while teaching in a classroom or while learning (or refraining from learning) in a PD course. Beliefs activated in a "lab" environment may reflect an idealized version of teachers' beliefs, disregarding the complexity of natural situations, where unconscious or tacit beliefs may influence behaviour. Group interviews have been used to address this issue (Fauskange, 2012). A group discussion is more likely to elicit beliefs in all their complexity (Bryman, 2004). However, group dynamics are quite complex in their own right. It may be difficult to discern individuals' beliefs from a group interview, and furthermore, a group interview may actually influence the individual's beliefs - a fact which may serve the PD goals, but impairs the method's validity as a research tool.

In this paper we use an indirect approach to eliciting teachers' beliefs about the knowledge they need for teaching. We examine anonymous feedback forms, where the teachers reflect on more and less successful PD activities, and explain *why they were more or less successful in their opinion*. We will argue, after presenting our methodology, that this indirect approach to beliefs addresses the theoretical and methodological difficulties listed above; however, we justify this claim only from a theoretical perspective. Justifying this claim empirically is a goal for future research.

We aim to answer the following research questions:

- What beliefs about the nature of mathematical knowledge needed for teaching are tacitly implied in the teachers' feedback questionnaires?
- How did these beliefs change during the one-year PD course?

## RESEARCH SETTING

In our research project we observed a mathematics professional development course for in-service elementary school teachers. A unique aspect of this PD was the fact that it was conceived by a professor from the mathematics department of a leading university in Israel, and was taught by graduate research students, primarily mathematics Ph.D. students. In this setting, the teacher-students and the mathematician-instructors had quite different initial beliefs about the knowledge that should be taught in the PD. Roughly speaking, interviews with the instructors revealed a belief that the teachers' knowledge of mathematics should be deepened and broadened, whereas teacher expectation questionnaires revealed beliefs to the effect that what they need most is knowledge related directly to the teaching of mathematics, primarily teaching strategies and ready-made classroom activities. The course consisted of 10 3-hour sessions. *Learning episodes* typically began with a mathematical problem related to the teachers' grade-level content (i.e. multiplication properties, representations of fractions, etc.). Sometimes the problems were designed to challenge the teachers' understanding of these topics. The problems often led to open discussions where the teachers could raise pedagogical concerns. More details can be found in (Cooper & Arcavi, 2012).

In this paper we focus on one group of 19 elementary school teachers (grade 3), taught by two graduate research students in tandem[2]. The teachers were all general teachers, who teach a variety of elementary school subjects in addition to mathematics. In addition to expectation questionnaires, which revealed the teachers' *prospective* views, the teachers were invited to submit anonymous feedback questionnaires after each of the PD sessions. These questionnaires comprise the data for our research. We present here the questions, along with authentic sample answers:

1. Select an activity from today's PD which you consider particularly successful. Explain in what ways it was successful. *"...[because the activity] provided a look at what goes on in the children's heads."*[3]

2. Select an activity from today's PD which you consider less successful. Explain in what ways it was less successful. *"...[because the activity is] not suitable for my classroom."*

3. If you have any additional comments, write them here. *"...[the PD] is becoming more accurately aligned with needs from the field."*

Throughout the 10 PD sessions, a total of 69 feedback forms were submitted, comprising approximately 40% response rate. In the general research design, the main purpose of these questionnaires was as a means to identify more and less productive learning episodes as perceived by the teachers. However, in this paper we are not interested in the selected episodes themselves, only in the *reasons* the teachers brought to justify their selections. Our assumption is that this will elicit teachers' beliefs (possibly tacit) about the knowledge they need, and that eliciting these beliefs indirectly may alleviate many of the problems inherent in traditional methods. In referring to teaching episodes (and not to themselves), the teachers are more likely to reveal genuine beliefs. Furthermore, these beliefs are activated in the context of learning mathematics, which is exactly the situation in which these beliefs are relevant to mathematics education and research. Another theoretical issue that our approach addresses is related to the differences between teachers' prospective and retrospective views (Roesken, 2011). By gathering information throughout the PD, we are filling the gap between prospective views, as indicated at the outset, and retrospective views formed as a result of the PD.

## METHODOLOGY

In this research we mixed qualitative and quantitative methods. We analyzed the teachers' feedback in a qualitative manner – coding and categorizing the data – and proceed to analyze the coded data quantitatively to find patterns and trends.

Once the feedback forms were transcribed and fed into a technological tool (Atlas.ti), we segmented the data into phrases (*quotations* in the tool's terminology), and coded each data segment according to the knowledge for teaching that it implied. This is a crucial point: Teachers did not *tell* us what they believe, but rather they *made use* of their beliefs, perhaps tacitly, by selecting outstanding episodes, and revealed these beliefs indirectly by explaining their selections.

This initial coding was fine grained, resulting in 81 different codes. For example, the phrase "*... [the activity] provided a look at what goes on in the children's heads*" was coded as "*understanding students' thinking*", since this utterance, as a reason for selecting a particularly successful activity, indicates that for this teacher, understanding students' thinking is important. At this early stage, codes were **not** mapped to any particular categories. The fine-grained coding was intended to permit a variety of different categorizations. Although we had a theory-based coding scheme in mind (MKT, described below), we could later adopt a completely different categorization scheme very efficiently, without re-coding.

In explaining why a PD activity was particularly (un)successful, the teachers are implicitly answering the question "what kind of mathematical knowledge should be taught in the PD"? The theoretical framework of mathematical knowledge for teaching (MKT), as put forth by Hill, Ball, & Schilling (2008), was a natural framework for categorizing responses, since we expected the data to reflect the teachers' beliefs about the types of mathematical knowledge that they need in order to teach, and therefore hoped would be taught in the PD. This framework, inspired by Shulman (1986), differentiates between subject matter content knowledge (SMCK) and pedagogical content knowledge (PCK), and further refines each of these categories. SMCK is sub-categorized into common content knowledge (CCK - mathematical knowledge that is used in teaching in ways similar to the ways in which it is used in other occupations that use mathematics), specialized content knowledge (SCK – mathematical knowledge that specifically serves teachers when engaging in teaching tasks), and horizon knowledge (HK - an awareness of how mathematical topics are related over the span of mathematics included in the curriculum). PCK is sub-categorized into knowledge of content and teaching (KCT – knowledge that combines knowing about teaching and knowing about mathematics), knowledge of content and students (KCS – content knowledge intertwined with knowledge of how students think about, know, or learn this particular content) and knowledge of curriculum (KC – a familiarity with text books and other teaching resources).

Codes were categorized into one of the six categories of MKT. For example, the code *understanding students' thinking* was categorized as KCS. Some teachers were more verbose than others, and indicated a variety of reasons for selecting activities. In such cases more than one code was assigned to a single quotation, but each code was mapped to one MKT category at most. Some codes (for example *I enjoyed the activity*) were not mapped to any MKT category.

All the coding was carried out by the two researchers, working together. Some quotations were difficult to code and to categorize, due to ambiguity in the teachers' words. These and all other points of disagreement were debated until unanimous agreement was achieved. As a rule, when teachers indicated satisfaction or dissatisfaction with the subject matter, we tended to assume they were referring to

SCK, since the **common** content (CCK) for lower elementary school (the mathematical content that the children should acquire) is quite straightforward.

In our analysis we checked the prominence of the various categories, and looked for trends over time.

## ANALYSIS

We begin with some descriptive statistics. In 67 out of the 69 forms the teachers indicated a particularly **successful activity**. A total of 32 different activities were selected as particularly successful, and were indicated a total of 80 times (clearly, some teachers listed more than one successful activity in a single form). Nearly all of the teachers explained their selection. These explanations generated 27 codes in 82 quotations, for example: the quotation "*... use this method to uncover student errors*" was coded as *discovering student errors*. In 24 out of the 69 forms a particularly **unsuccessful activity** was indicated. A total of 12 different activities were selected as particularly unsuccessful and were indicated a total of 24 times. The teachers' explanations for their choice generated 16 codes in 23 quotations, for example: the quotation "*the movie clip was tiresome*" was coded as *tiresome*.

28 out of 69 the forms included **general comments**, which generated 38 codes in 62 quotations. Teachers' general comments, e.g. "*please teach us fractions*", "*I like the PD because it provides tools I can use*", were analyzed separately from teachers' reasons for choosing particular activities. Although general comments do elicit beliefs about knowledge for teaching, we felt that this data suffers from many of the methodological problems we described in the introduction, since they are not situated in learning activities. We analyzed these data, and found that the picture they reveal is similar to the one drawn by the other data, but this analysis is not included here.

The number of quotations associated with each category is summarized in table 1:

| SMCK | | | PCK | | |
|---|---|---|---|---|---|
| Common Content Knowledge | Specialized Content Knowledge | Horizon Knowledge | Knowledge of Content and Teaching | Knowledge of Content and Students | Knowledge of Curriculum |
| 2 | 18 | 0 | 32 | 10 | 0 |

**Table 1: Number of quotations assigned to each knowledge category**

We see that reasons related to PCK (42 quotations) were provided twice as many times as reasons related to SMCK (20 quotations). Within SMCK, specialized knowledge is more prominent than common knowledge, and within PCK, KCT is more prominent than KCS. We note that HK and KC were not indicated at all.

We now take a temporal view, presented in figure 1. In order to compare PD sessions, where varying numbers of teachers submitted feedback questionnaires, we normalized the quotation count by dividing by the number of submitted forms. At the beginning of the PD aspects of PCK were prominent, and this prominence tended to decrease as the PD progressed. Specifically - the average number of PCK-related comments per feedback in the first three sessions was 3 times greater than the average in the other session. On the other hand, the prominence of SMCK started out low and tended to increase over time.

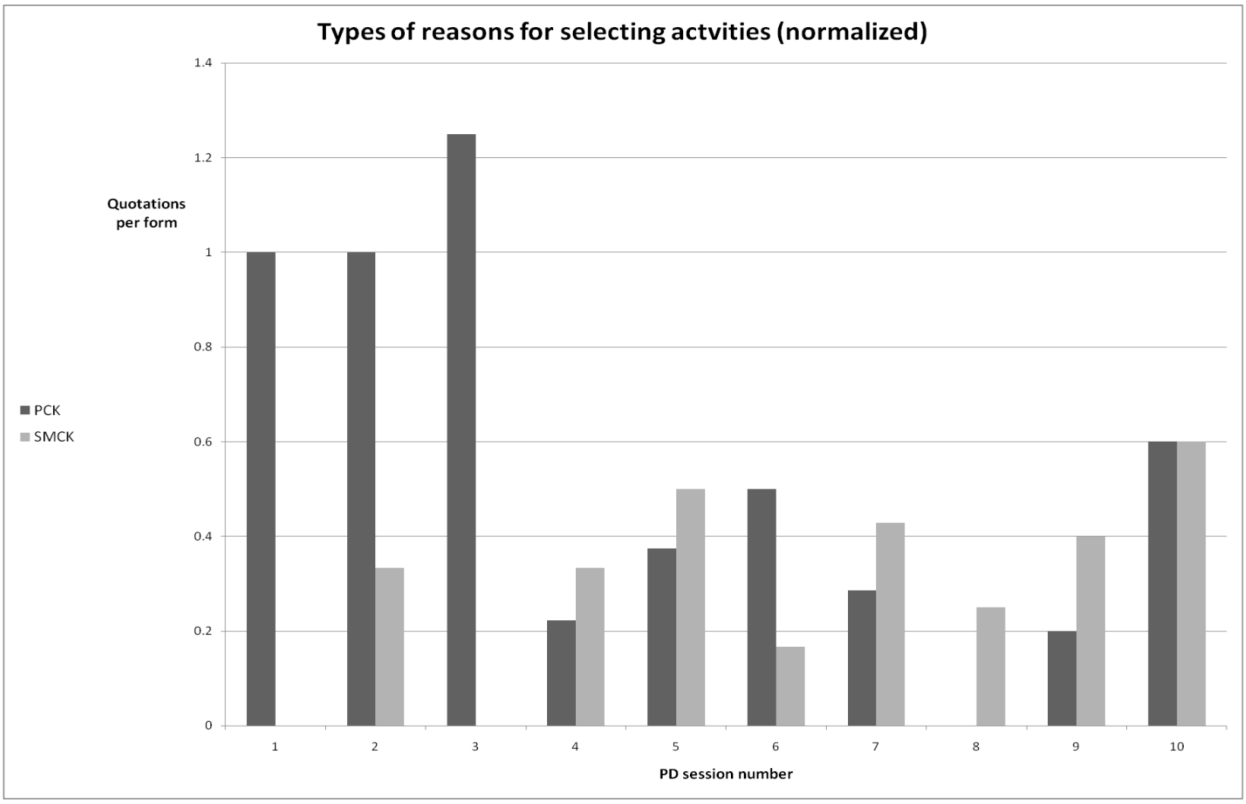


**Figure1: SMCK/PCK reasons for selecting activities (normalized)**

We see in these findings an indication of a shift in the teachers' beliefs. In this shift, the teachers are moving closer to beliefs that are consistent with the instructors' goals for the PD, focusing on SMCK. We cannot say with certainty what caused this shift, but we speculate that it is connected to the nature of the content that the instructors brought to the course. Initially, the teachers did not believe there was anything new for them to learn about grade 3 mathematics, but the mathematicians managed to bring a new depth to this content, as described in (Cooper & Arcavi, 2012) and in (Cooper & Pinto, 2012). In a broad sense of the term, this can be considered an indication of the teachers *learning* in the PD.

## METHODOLOGY AND ANALYSIS – A DIFFERENT CATEGORIZATION

Our initial analysis provided some interesting results, but we were not totally satisfied with the coding scheme. Ten of our codes, representing 33 quotations (30% of the total), did not map to any MKT category, suggesting that there may have been an important message in the data which we were missing. We decided to look for an alternate categorization scheme, based on the data. Upon re-reading the feedback forms, we realized that the teachers' reasons for selecting particular activities said something not only about *what* they would like to learn, but also about the role of PD in their eyes, and how it should relate to their teaching. At one end we found quotations implying that the PD should contribute to the teachers' practice in a direct way, mainly by providing classroom activities and teaching tips, for example: "*…[the activity] teaches how to teach in the classroom*". We call this category *classroom focus*. At the other end we found indication of a loose connection, where the teachers' comments were aimed directly at what occurred in the PD, with little or no reference to teaching practices, for example: "*I really enjoyed that activity*". We call this category *PD focus*. In between were quotations that indicated a connection between the PD and teaching practice, but not by contributing directly to classroom teaching, for example: "*…[the activity] got me thinking about … how to teach for understanding*". This intermediate attitude seems to see the teacher as having a role in incorporating knowledge learned in the PD, and adapting it for her classroom practice. We call this category *teaching focus*. In this new categorization, <u>all</u> codes were assigned a category, suggesting that this categorization has better grounding in the data. This new category scheme is very different from the first. Its categories cut through the various MKT categories, for example, a focus on *classroom* teaching is related to SMCK in some quotations (e.g. "*content was not appropriate for my classroom*") and to PCK in others (e.g. "*...ways for dealing with students' difficulties*"). Furthermore, in this new scheme some codes which appeared indistinguishable in terms of MKT were seen to be quite different. For example, *ways for dealing with students' difficulties* is a case of *classroom focus*, whereas the apparently similar *understanding students' thinking* is a case of *teaching focus*. Using the MKT categories, they both mapped to KCS.

We see (table 2) that the quotations are distributed among all three categories, with more prominence for *PD focus*, but again the temporal picture is more interesting (figure 2). The focus on *classroom* practices started high and tended to gradually decline over the course of the PD, whereas the focus on the *PD* itself started out low and increased quite steadily. The prominence of the *teaching* category changed very little over the course of the PD. It appears that teachers started out with beliefs indicating that the PD should contribute to teaching practices in a direct manner, and gradually accepted the possibility of less direct contributions, even to the extent where they consider the PD on its own terms, setting aside the question of how it will contribute to their teaching. This shift in beliefs, like the shift described in the previous section, can be attributed to the nature of the content that the instructors brought to the course. If the teachers accept goals of deepening their understanding of the elementary content as worthwhile, it is natural that they should not see this as contributing *directly* to their teaching practices, since this is not content they will bring to their own classrooms.

| Classroom focus | Teaching focus | PD focus |
|---|---|---|
| 33 | 20 | 43 |

**Table 2: What teachers focus on in their reasons for selecting activities**

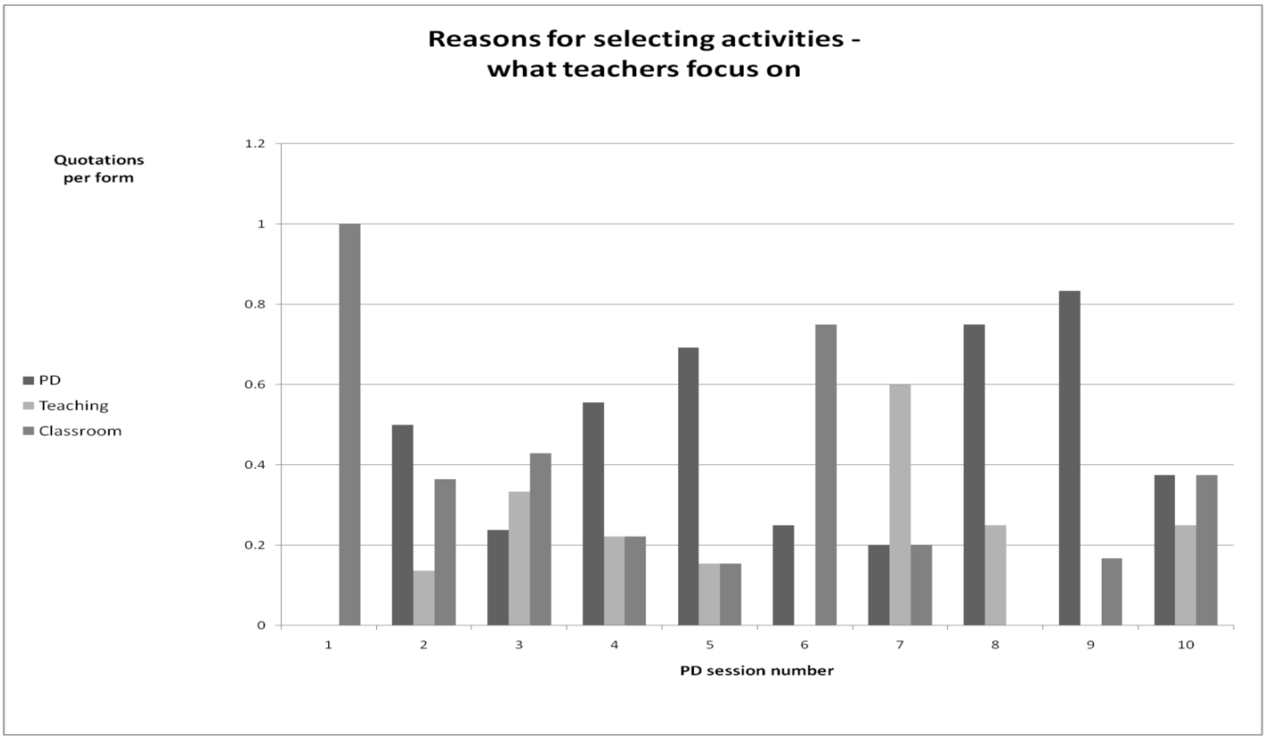


**Figure 2: Reasons for selecting activities - normalized**

## DISCUSSION

We have shown trends in the teachers' attitude towards the PD, both in the knowledge it should focus on, and in the nature of its role as a means for improving teaching practice. The teachers moved from beliefs whereby the PD should enhance mainly PCK in a manner that can be utilized in classrooms, to beliefs whereby the PD may also enhance SMCK, and that the PD need not focus on classroom teaching directly. Although the change is modest, this is encouraging news for teacher educators, since it is usually difficult to show evidence of changes in teacher beliefs as a result of PD. But beyond these results, we would like to reflect on the research *method*.

### Validity and reliability

Our research makes use of qualitative methods. Validity should be considered in this context. It is impossible to ascertain that our findings reveal the teachers' *real* beliefs, even if we were to accept that such beliefs have an objective reality. However, we do have a kind of triangulation. The two very different frameworks we adopted – the well accepted MKT and our own bottom-up framework, both revealed patterns of a shift in the teachers' beliefs over time. This lends a degree of validity to our findings.

There remains one important issue of reliability. The scope of this paper does not permit us to provide much detail about the content of the particular activities that the teachers referred to in their feedback. Such data will be presented at the conference; yet we wish to claim that this data is of limited relevance. When a teacher says that an activity was **successful** because it *suggested ways for dealing with students'* *difficulties*, we are not concerned with what actually transpired in the activity. We are concerned only with the implication that knowing how to deal with student difficulties is important knowledge for this teacher. The question of reliability is thus whether each PD session provided activities that were rich and varied enough to elicit the teachers' beliefs reliably. Consider, for example, what would happen if all the activities in a particular session were to focus exclusively on SMCK. There would be no opportunities for the teachers to select a successful activity based on its PCK nature. This is a serious concern, which is addressed as follows: If there is a lack of activities which focus on PCK, teachers will still be able to indicate their beliefs about the importance of PCK through *unsuccessful* activities ("*this activity was unsuccessful because it did* ***not*** *address PCK*"). In such a case, teachers may leave the question regarding successful activities unanswered. The converse also holds – teachers may leave the question regarding unsuccessful activities unanswered and select only successful activities. If the feedback questionnaires had asked only about successful (or only about unsuccessful) activities, reliability would have relied strongly on the variety of the PD activities. The coupling of the two questions makes its reliability independent of the activities.

## CONCLUSION AND IMPLICATIONS

This paper has both practical and methodological implications. On the practical side, it highlights an unusual PD course, conceived by a mathematics professor and taught by mathematics Ph.D. students, where there is an explicit focus on SMCK and on teachers' attitudes to mathematics. We have shown a change in teachers' beliefs during the course, and there is reason to believe that this change is a result of the PD. This suggests that the PD is worthy of the careful scrutiny it is receiving in the first author's Ph.D. dissertation. On the methodological side, we presented an indirect approach to eliciting teachers' beliefs about knowledge for teaching. This method was shown to be sensitive enough to reveal a change in teachers' beliefs. Although the change was modest, we have more faith in these results than we would have had in results obtained directly, for all the reasons listed in the introduction to this paper. We believe our method is less susceptible to teachers' tendency to gratify researchers, and can reveal tacit beliefs which may even contradict their declared beliefs. Whether or not this method really is an improvement over traditional methods is an important question left for future research.

## REFERENCES


Bryman, A. (2004). *Social Research Methods* (2 ed.). New York: Oxford University Press.

Cooper, J., & Arcavi, A. (2012). Mathematicians and elementary school mathematics teachers – meetings and bridges. Accepted for publication

Cooper, J., & Pinto, A. (2012). *Mathematicians as designers of tasks for in-service professional development - opportunities, attitudes and constraints.* Submitted.

Fauskange, J. (2012). ”For norske lærere har stort sett en algoritme” - om undervisningskunnskap i matematikk. [English: “Norwegian teachers usually have an algorithm” - about mathematical knowledge for teaching]. In F. Rønning, R. Diesen, H. Hoveid, & I. Pareliussen (Eds.), *FoU i Praksis 2011. Rapport fra konferanse om praksisrettet FoU i lærerutdanning [English: R & D in Practice 2011. Report of the Conference on Practice-based research in teacher education]* (pp. 129-141). Trondheim: Tapir Akademisk Forlag.

Fives, H., & Buehl, M. M. (2008). What do teachers believe? Developing a framework for examining beliefs about teachers’ knowledge and ability. *Contemporary Educational Psychology , 33* (2).

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers' Topic-Specific Knowledge of Studnets. *Journal for Research in Mathematic Education , 39* (4), pp. 372-400.

Knowles, M. S. (1990). *The Adult Learner.* Gulf Publishing Co.

Roesken, B. (2011). *Hidden Dimensions in the Professional Development of Mathematics Teachers: In-Service Education for and With Teachers.* Rotterdam: Sense Publishers.

Schoenfeld, A. H. (2010). *How We Think: A Theory of Goal-Oriented Decision Making and Its Educational Applications.* New York: Routledge.

Shulman, L. S. (1986, February). Those who understand: Knowledge growth in teaching. *Educational Researcher* , 4-14.

Toerner, G. (2002). Mathematical beliefs - a search for a common ground: Some theoretical considerations on structuring beliefs, some research questions, and some phenomenological observations. In G. C. Leder, E. Pehkonen, & G. Toerner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (Vol. 31, pp. 73-94). Dordrecht: Kluwer Academic Publishers.

---

**NOTES**

[1] PD is used throughout the paper for "professional development".

[2] One of the instructors was a mathematics Ph.D. student, the other was a computer science M.Sc. student.

[3] The scope of this paper does not permit us to describe the activities to which the teachers referred. Data of this nature will be presented at the conference.