PRACTICES TO ENHANCE PRESERVICE SECONDARY TEACHERS' SPECIALIZED CONTENT KNOWLEDGE

Fatma Aslan-Tutak, F. Gunes Ertas

Bogazici University

In this presentation, the authors will discuss practices to enhance preservice secondary mathematics teachers' subject matter knowledge, specifically specialized content knowledge. The mathematical knowledge for teaching model (Ball et al., 2008) and Usiskin's perspective of teachers' mathematics (2001) were integrated to develop a content course for secondary mathematics teachers in Turkey. The focus of this presentation will be the nature of mathematical tasks used during the course. Those tasks will be discussed from four approaches; unpacking concept definitions, applications and modelling, procedures and generalizations, and historical perspective of concepts. Pre- and post-test results and the participants' opinions showed that these practices were influential on their SCK.

INTRODUCTION

Shulman's model of teacher knowledge (1986) consists of three types of subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curriculum knowledge; SMK has an important role in this model. Brown and Borko (1992) asserted that preservice teachers' limited mathematical content knowledge is an obstacle for their training on pedagogical knowledge. Furthermore, according to the mathematical knowledge for teaching (MKT) model, developed from observations of elementary school teachers' classroom teaching, (Ball, 2000) there are six domains of teacher's content knowledge which can be categorized under Shulman's different types of knowledge (Ball et al., 2008). There are three domains under SMK: common content knowledge (CCK, mathematics knowledge not unique to teaching), specialized content knowledge (SCK, mathematics knowledge unique to teaching), and horizon content knowledge (knowledge of mathematics throughout the curriculum). Also, there are three domains under pedagogical content knowledge: knowledge of content and students (KCS, interaction of knowledge of mathematics and students' mathematical conceptions), knowledge of content and teaching (KCT, interaction of knowledge of mathematics and teaching methods), and knowledge of content and curriculum (interaction of knowledge of mathematics and mathematics curriculum). Among the three domains of SMK, specialized content knowledge stands out as teachers possessing deep understanding of the mathematics they will teach.

Furthermore, one of the most important features of this model is its emphasis on interweaving content and pedagogy for mathematics teaching (Ball, 2000). This model also describes kinds of practices which may be helpful to improve teachers' content knowledge. Ball (2000) stressed that it is crucial for teachers to experience mathematics from varied perspectives and in different contexts. Even though MKT provides a general framework for designing learning tasks for preservice teachers, it has limitations for secondary school mathematics teaching because it was developed as a result of research studies with elementary school teachers. In this paper, the authors will discuss using MKT framework and adapting it for secondary school teacher learning to design practices of a content course for secondary school teachers. Furthermore, Usiskin (2001) provides an approach, *teachers' mathematics,* to develop mathematical practices to enhance secondary school teachers SMK. It should be noted that, Usiskin is not proposing a model for teacher knowledge research studies, but provides an approach to guide practices to study mathematics with teachers.

Moreover, Usiskin and his colleagues (2003) explained three features of the advanced perspective; concept analysis, problem analysis and mathematical connections. They discussed that even though these three kinds of mathematics are essential for teachers, they are not addressed in a typical college mathematics course. Furthermore, he states that "Often the more mathematics courses a teacher takes, the wider the gap between the mathematics the teacher studies and the mathematics the teacher teaches" (p. 86). Among these three features of studying advanced mathematics, concept analysis addresses especially the SCK type of knowledge that a teacher needs. In concept analysis, a secondary school teacher is expected to be able to discuss *alternate definitions*, *instances and applications*, *generalizations*, and *the history* of the concept. Therefore, the idea of teachers' mathematics may be used to study the SCK aspect of secondary school mathematics teachers. In this study these two perspectives for teachers' mathematics knowledge were integrated to answer the following research question: What kind of instructional practices enhance preservice secondary teachers' specialized content knowledge during a content course?

METHODS

This study took place at a public university in western Turkey. The target population of this research is secondary school preservice teachers in Turkey. There were 28 students enrolled to an elective content course for secondary school mathematics teachers. All of the students were asked to participate in the study voluntarily. Among them sixteen of those students were chosen to investigate research question. It was not practical to choose all of the students for the study so the participants were chosen to be representatives of the whole group. They were chosen according to their pre-test scores (high, median and low scores) and their grade level in the program (senior or junior). There were only four junior students and we chose them all but data were missing from one of them, so in total data from fifteen participants were used for this study.

Furthermore, it is important to portray students' mathematical background before discussing findings. The participants had been prepared in advanced mathematics topics such as group and ring theory, complex analysis, linear algebra in addition to calculus topics and differential equations. The curriculum for teacher education departments is provided by the Higher Education Institute (government) with some slight changes in it according to the university. There are some universities where students take mathematics courses from faculty members of college of education. There are also universities where students take advanced mathematics courses from faculty members of mathematics department. The study was conducted at a university of the latter case. When preservice teachers take many advanced level mathematics courses, it is believed that they will be ready to teach high school topics. Even though SMK is a prerequisite for PCK, it is important to address SMK from a different perspective, SCK, rather than common content knowledge. In other words, preservice teachers will benefit greatly from a content course in which they will examine, relearn and reflect on mathematics topics that they will teach (Ball et al., 2008; Usiskin, 2001). For this study we used *teachers' mathematics* practices (Usiskin et al., 2003) in the content course for secondary school preservice teachers. In this course, real numbers, complex numbers, equations, and functions (definition, exponential and logarithmic function) topics were studied.

The classroom environment for this course was community of learners such that future mathematics teachers explore mathematical ideas that they would be teaching. We would like to note that since participants studied mathematics as teachers, it was inevitable for them to learn SCK type of mathematics without thinking of teaching practice. Indeed, SCK was defined by Ball et al. (2008) as knowing mathematics for teaching. From this point of view, there were two main components of the content course: mathematical explorations and the context of teaching for mathematics. The mathematical explorations were addressed by mathematical tasks prepared by the researchers by adapting resources (e.g. Usiskin, et al., 2003). Furthermore, the second component of the course, the context of teaching component, was addressed as participants collect and analyze student work for three topics (rational numbers, complex numbers, and logarithmic function) during the semester. The protocol for analyzing the student work was developed by the researcher for a previous study (Aslan-Tutak, 2009). For this paper, the focus will be the nature of mathematical tasks used during the course. We would like to provide further discussion on the practices which were utilized to study SMK, specifically SCK of preservice secondary school mathematics teachers. It should be stressed that the authors also worked with a mathematician in order to strengthen validity of the mathematical tasks. Mathematical explorations did during the course will be discussed from four approaches of the concept analysis (Usiskin et al., 2003); *unpacking concept definitions*, *applications and modelling*, *procedures and generalizations*, and *historical perspective of concepts*.

DATA SOURCE AND ANALYSIS

There were various data sources; mathematics knowledge pre-test, video records of the classroom instruction, two individual interviews during the semester, and mathematics knowledge post-test. All of the video records of the classroom instructions and individual interviews were transcribed.

The participants' written mathematical work (pre-test and post-tests) were analyzed according to content; their correctness and their ability to provide valid mathematical proofs. The test was developed by the researchers. There were definition and proof questions in the pre- and post-tests such as definition of commensurability, proof of .99999... = 0 equality.

Individual interviews were used for two purposes; participants' perception of effective practices for their learning and their answers to given mathematics questions (e.g. the rule for converting infinite repeating decimals to rational numbers). We will report both uses of the interviews in this paper. There were two interviews conducted with fifteen participants. The first interview was about in the middle of the semester while the second interview took place at the end of the semester. Furthermore, the video-records of the classroom instructions were used to support other data sources as they were also used to study the practices of the course.

FINDINGS

The findings section will be organized according to the four aspects of concept analysis in order to provide information about practices used in the content course, and discuss their effect. It should be noted that all four of these approaches were used for all topics. In this paper, we tried to choose examples which demonstrate the instructional approaches.

(a) Unpacking Concept Definitions

Concept analysis includes alternate definitions of mathematical concepts (Usiskin et al., 2003), how and why concepts arose mathematically would provide experiences to unpack definitions for concepts. In the MKT model, Ball et al. (2008) defined SCK as a skill unique to teaching and as it required a special work of unpacking of mathematics concepts. It is not necessary for others but it is a requirement for effective teaching. In Turkish setting, preservice teachers' earlier experiences with secondary school mathematics concepts were limited to short textbook definitions to memorize, as participants stated in interviews. During their education they built further mathematics procedures or concepts without unpacking. However, these preservice teachers are asked to teach mathematics conceptually. So they had to look at these well-known concepts and definitions with the eyes of a teacher which requires to ask how and why questions. A primary purpose of the content course was to provide mathematical explorations to encourage preservice teachers to unpack the concepts that they thought they know very well.

During classroom discussions, it was the primary practice to raise questions as a whole group to understand the origin of the mathematical concepts. All of the participants stated that they would not think of questioning already known mathematics concepts. In the individual interviews, most of the participants mentioned about the discussions of complex numbers and the geometric definition of number *i* as one of the most eye-opening experience for them.

From the video-records of the class discussions on complex numbers, we saw that all participants were aware of the rationale of the complex number i. They knew that when mathematicians were solving quadratic equations there were negative numbers inside square root. So they said that mathematicians came up with a method to represent -1 inside the square root. However pretest results showed that none of the participants were aware of geometric definition of number *i* before the content course. They defined the number *i* either as $i^2 = -1$ or as $i = \sqrt{-1}$ (Aslan-Tutak, 2012).

In order to explore number *i*, instruction started with discussion on the roots of an equation as its constant terms changes (Usiskin, et al., 2003). Then graphs of the equations were plotted on Cartesian plane (geometric representation of the equations) and their roots showed on a number line (geometric representation of real numbers). The number line was sufficient for the roots of first two equations but participants realized that they could not show the roots of third equation on number line. Then, the definition of the imaginary number *i* was discussed by rotation of 90^o perspective (Lakoff & Núñez, 2000; Trudgian, 2009; and Usiskin et al., 2003) which provides a geometrical understanding of the imaginary number. At that point, the participants realized the need of extending real numbers by discussing what does it really mean having i^2=-1. During the classroom discussions and individual interviews they stated that none of them had known geometrical definition of the imaginary number *i* before this course. After the discussion on the geometric definition of complex number, *i* as a rotation of 90^o from real number line, during the class discussion and individual interviews, they pointed out that understanding operations with complex numbers, and the connection between rectangular and polar form became easier. Furthermore, according to posttest results, six of the participants defined the imaginary number *i* as a rotation by 90^o (Aslan-Tutak, 2012).

> Participant 1: I consider the origin of the imaginary number *i* from a different perspective and I have learned from where it emerges [...] in the course everyone was able to aware and learn something that are substantially unknown and unnoticed before.

It is important to note that almost all of the participants were able to carry on simple to advanced calculations with complex numbers. However, being able to do some mathematical calculations is not enough for teaching mathematics meaningfully. Teachers SMK is not just merely doing calculations (CCK) but also knowing the concept definitions in-depth. Since SMK affects PCK, even if it was not the intention of the course for participants to learn how to teach complex numbers, they stated they would use geometric definition while teaching complex numbers.

(b) Applications and Modelling

During the content course, the students had the chance to investigate some topics from applied examples. For instance, participants discussed the alternative definitions of function and modelling activities or real life application word problems (e.g. water lily problem for exponential growth). When we analyzed the interview data, it was surprising to see participants' emphasis on modelling activities. Participants especially expressed that they did not use idea of function from modelling perspective; rather they solved computational problems by using functions.

> Participants 2: I didn't think of modelling for functions before. When I first looked at problems in the book I was very surprised. We never learned such modelling for functions.

Furthermore, it is interesting to note that for her internship, this student used a logarithmic modelling example (logarithmic functions for modelling earthquakes) which was discussed in the content course. Therefore, we would like to present our approach to implement modelling examples or real word examples in the content course.

First of all, it was interesting for the authors to realize that preservice secondary school teachers' perception of the concept of function was limited to the definition of ordered pairs between two sets. They could not think of function as an operation to explain how variables change according to each other.

> Instructor: What is a function?
>
> Participant 3: a set which contains ordered pairs
>
> Participant 4: according to a certain rule, it is an operation which brings an element of a set to an element of another set.

By the end of the class hour, examining examples and classroom discussion yielded the definition of a function as "If a v and a y can be related through an equation or graph, they are called 'variables': that is, one changes in value as the other changes in value. The two have what is known as a functional relationship; the variable whose change of value comes about as a result of the other variable's change of value is called a 'function' of that other variable" (Rao & Latha, 1995, p. 32). Later, two examples from Bremigan et al. (2011) were discussed whether if they were an example of a function. The purpose of the discussion in this course was to make preservice teachers to relearn concept of function by studying alternative definitions for a very important concept of high school mathematics.

As the concept of function was introduced from covariation perspective, during the following parts of instruction preservice teachers experienced modelling examples for exponential and logarithm functions. In addition to growth problems, participants were given a problem about oil spill and the cleaning procedure of such an environmental issue. The problem starts with some information about the latest oil spill incidents in the Gulf of Mexico and France. We discussed briefly about the enormous destruction of oil spill in even one day. So we decided that it is important to clean oil in the fastest way and to know when all oil would be cleaned up. Then preservice teachers were given the problem: *If there was 8000 gallon oil due to a spill, and the crew can clean only 80% of oil for a week. So how much oil would* *remain after a week? And how long will it take to clean until there is 10 gallon of oil is left?* In this problem, one can start solving by using simple arithmetic calculations for percent. However especially for the second question one should think of exponential function and its inverse logarithm function in order to find required time to clean oil.

During the individual interviews, participants stated that they were surprised not being able to think of covariation definition of function, and not being introduced to modelling practices for functions until the content course. As we have explained before these preservice teachers had a high school education focused on a high stake test, university entrance exam, and an advanced college level mathematics perspective.

(c) Procedures and Generalizations

Besides mathematical concepts, the procedures, rules and formulas are important elements in mathematics. The theorems including mathematical rules and formulas are deduced from both definitions and procedures. Deductions can be made from mathematical definitions (Usiskin et al., 2003), and the rules and formulas may seen as generalizations of procedures. Beyond knowing procedures and rules teachers needs to know why a certain procedure works (SCK). Ball and her colleagues gave an example from elementary school years. Division of fraction procedure (invert and multiply) is commonly used but one may not necessarily need to know why the algorithm works. However, a teacher should know why it works, and this knowledge is defined as SCK. Therefore, in this content course, we also had mathematical tasks to explore underlying mathematical reasons of mathematical procedures and rules. For instance, for a high school teaching context, a teacher should be able to prove why $\sqrt{n}$ is irrational when n is a positive real number but not a perfect square.

According to participants' responses to the first interviews, the most impressive experience in terms of unpacking procedures for them was the rule from the topic of rational numbers and their decimal representations. Furthermore, the pre-test included the question which asked for the proof of the equality 0.999…=1. Seven participants tried to prove the equality and other eight did not answer the question. Among the seven answers, three of them just used the method of converting a repeating decimal into a rational number and four of them used different arguments. Also, at one point of the instruction on decimal representations there were discussions on the rationality of infinite repeating decimals. Students had homework from previous lesson and they were asked to give justifications of each step of arguments which are given in the textbook for the equality of 0.999…=1. During the classroom discussion, infinity concept was also discussed with this question; however, the focus of the classroom discussions was to study 0.999... as an infinitely repeating decimal. The purpose was to study the procedure of converting infinitely repeating decimals to rational numbers.

Individual interviews show that most of the participants (14 of all fifteen participants) did not know why the procedure worked. The connection between the procedure (the method of converting a repeating decimal into a fraction) and the argumentation was provided in class by extending one of the arguments for the 0.999...=1 equality. The discussion was broken into proofs of three theorems for terminating, simple-periodic and delayed-periodic decimal representations respectively.

Moreover, participants' answers to mathematics questions asked in the individual interviews showed that for many of them, this was the first experience of elaborating the rule of converting an infinitely repeating decimal into a fraction. Until the content course they have known the rule but they didn't know why the methods work. After the course, according to post-test results, all of the participants can explain the rationale of the rule. This kind of knowledge which belongs to SCK is a special knowledge for teachers, knowing why a rule works but when teachers decide how to use this kind of knowledge in their practice that would be knowledge of content and teaching (KCT).

(d) Historical Perspective of Concepts

Another aspect of concept analysis of mathematics is the historical perspective of concepts. It is important to note here that it is not the story part of mathematics history but the historical aspect of mathematics problems and concepts. For example, while studying number *e*, it was important to study Euler and his contributions to mathematics. Throughout the semester for each topic, there were discussions about historical perspectives in order to study why some rules work and how some mathematics concepts developed. Furthermore, almost all of the participants stated that addressing historical mathematics problems and how mathematicians handled them was a helpful approach for them to be able to see concept formation. Even though we cannot support their perceptions with pre- and post-test results, we take their feedback in consideration for developing practices to enhance their learning in the content course. We will report about using the history of logarithm because in individual interviews all of the participants stated that this was an approach that helped them to conceptualize logarithm.

First, logarithm was studied as the inverse of exponential function in the course. Then when we told preservice teachers that logarithm was developed before exponential function, they did not believe it. Then the instructor introduced Napier and his problem which led to invention of logarithm function. Preservice teachers were given the problem and asked to work on Napier's problem. Their work and Napier's solution yielded into the Napier's definition of logarithm. Later, there were discussions on Euler's definition of logarithm.

According to their feedback in the classroom and their responses during the second interview, we were able to conclude that even though this kind of knowledge of logarithm was available in resources, it was never introduced to preservice teachers, neither in high school nor in college level courses. They have been using logarithm since high school but none of them ever heard of Napier's problem or his definition of the logarithm function. Pre-service teachers were able to relearn logarithm by studying the historical perspective of the topic. In other words, we may say that they possessed CCK for logarithm in terms of carrying out calculations but they were lacking the in-depth definition of logarithm. On the other hand, they thought that this was a kind of knowledge that a teacher should have. Therefore, we may again conclude that this kind of practice, studying the origins of mathematics concepts allows preservice teachers to unpack and relearn some mathematics concepts.

DISCUSSION

The purpose of this paper was to report findings for the research question of practices which can be used in a content course to enhance preservice secondary teachers' SMK, specifically SCK. While studying the research question, we used the MKT model (Ball et al., 2008) as the theoretical framework, and teachers' mathematics (Usiskin et al., 2003) approach to construct the practices for the content course. Learning tasks for secondary school teachers would not merely advance mathematical tasks. Rather the emphasis should be on looking deeper in mathematics concepts, unpacking and relearning them. Therefore, concept analysis practice as suggested by Usiskin and his colleagues (2003) was our primary approach to design learning tasks to enhance preservice secondary teachers' SCK. Concept analysis is consisted of four elements; unpacking concept definitions, applications and modelling, procedures and generalizations, and historical perspective of concepts. It should be noted that during the content course all four of these approaches were blended in a way to allow preservice teachers to study mathematics concepts. Since research questions were about the practices, we reported findings according to these four elements of concept analysis. As, participants' knowledge from pre- and post-tests, and their perceptions from individual interviews and classroom discourse showed that all four of these elements of concept analysis helped participants to examine and study mathematical concepts which in turn helped them to enhance their SCK.

Furthermore, all participants stated their purpose of relearning mathematics was to be more knowledgeable teachers. They were not engaging in those tasks not just as a student but as a future teacher. Even though there were no discussion on how to teach topics in this course, some findings suggested that preservice teachers were thinking of using what they learned for their teaching practice. So, we cannot propose that these practices affected only their SCK but not their KCT, because SMK is a necessary but not sufficient condition for PCK. Here, the authors might conclude that as participants experienced learning mathematics for conceptual understanding, they valued teaching in such a way. One of the participants stated that the course was like a laboratory class for them to experience this kind of teaching in first hand. Further research may focus on including more topics to this study with a larger sample. The authors also plan to study effect of the content course on preservice teachers' belief about mathematics and teaching mathematics. Lastly, another study may focus on effect of this course on their teaching (PCK) by observing their instruction.

Acknowledgement: We appreciate contributions of Müge Kanuni, an Associate Professor of Mathematics at Bogazici University, to the content course discussed in this paper.

REFERENCES

Aslan-Tutak, F. (2009). *A Study of Geometry Content Knowledge of Elementary Preservice Teachers: The Case of Quadrilaterals*. Unpublished Doctoral dissertation, University of Florida, Gainesville, Florida.

Aslan-Tutak, F. (2012). *Preservice Secondary School Mathematics Teachers' Specialized Content Knowledge of Complex Numbers*. Paper presented at 12th conference of International Congress on Mathematics Education (ICME-12), 2012, Seoul, Korea.

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51(3),* 241.

Ball, D. L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59 (5),* 389-407.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23(3),* 194-222.

Bremigan, E. G., Bremigan, R. J. & Lorch, J. D. (2011). *Mathematics for Secondary School Teachers*. The Mathematical Association of America.

Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning (pp. 209–239).* New York: Macmillan.

Lakoff, G., & Núñez, R. E. (2000). *Where Mathematics Come From: How the Embodied Mind Brings Mathematics Into Being.* New York: Basic Books.

Rao, D. P. & Latha, D. B. (1995). *Achievement in Mathematics*. Delhi, India: Discovery Publishing House.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15(2),* 4-14.

Trudgian, T. (2009). Introducing complex numbers. *Australian Senior Mathematics Journal, 23(2),* 59-62.

Usiskin, Z. (2001). Teachers' mathematics: A collection of content deserving to be a field. *The Mathematics Educator, 6 (1),* 86-98.

Usiskin, Z., Peressini, A., Marchisotto, E. A., & Stanley, D. (2003). *Mathematics for High School Teachers: An Advanced Perspective*. New Jersey: Prentice Hall.