# THE MATHEMATICAL KNOWLEDGE FOR TEACHING

## A view from the Onto-Semiotic Approach to Mathematical Knowledge and Instruction

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One of the problematics that has drawn a lot of attention from both researches community and policy makers alike, is the identification and characterization of the knowledge web that a mathematics teacher should have in order to teach effectively and to facilitate their students' learning on specific mathematics topics. One proposal on the teachers' knowledge that is widely accepted, is the model called "Mathematical Knowledge for Teaching (MKT)", developed by Ball and colleagues (Ball, Lubienski & Mewborn, 2001; Hill, Ball y Schilling, 2008; Ball, Thames & Phelps, 2008). This proposal is a remarkable advancement for describing the complex of knowledge that a teacher should have to teach mathematics. Nonetheless, despite the advances that the MKT model represents, there are still questions to be addressed, such as: how to identify the teachers' didactic-mathematical knowledge when the teachers' knowledge models include categories too wide? Specifically, under what criteria can the MKT be evaluated? How can the teachers be supported to acquire or to develop the MKT components? In general, as Godino (2009) points out, both the MKT model and the others various models on the mathematical knowledge for teaching, informed by the researches in mathematics education, include categories too "wide" and disjoint, that call for models that allow conducting a more precise analysis of each knowledge component that are put into effect in an effective teaching of mathematics. The latter will allow orienting to the design of formative actions and the elaboration of tools to assess the mathematics teachers' knowledge.

Thus, in this work, based on both the Onto-Semiotic Approach to Mathematical Knowledge and Instruction (OSA) (Godino, Batanero & Font, 2007) and the categories of didactic-mathematical teacher's knowledge (Godino, 2009), we propose a model called "Didactic-Mathematical Knowledge (DMK)". This model proposes six facets or dimensions to analyse the teacher's didactic-mathematical knowledge about a specific mathematical topic (Godino et al, 2011, p. 278-279): 1) *Epistemic Facet:* The intended and implemented institutional meaning for a given mathematical content, that is, the set of problems, procedures, concepts, properties, language, and arguments included in the teaching and its distribution over the time; 2) *Cognitive facet:* Students' levels of development and understanding of the topic, and students' strategies, difficulties, and errors as regards the intended content (personal meaning); 3) *Affective facet:* Students' attitudes, emotions, and motivations regarding the content and the study process; 4) *Mediational facet:* Didactic and technological resources available for teaching and the possible ways to use and distribute these

resources over time; 5) *Interactional facet:* Possible organisations of the classroom discourse and the interactions between the teacher and the students that help solve the students' difficulties and conflicts; 6) *Ecological facet:* Relationships of the topic with the official curriculum, other mathematical themes and with the social, political, and economical settings that support and condition the teaching and learning. These facets reinterpret and organize the different components of the MKT. Furthermore, to each of the said facets the OSA provides theoretical and methodological tools that allow more detailed analysis. For example, for the epistemic and cognitive facets the tool "objects and processes configuration" is proposed, which refers to the detailed and systematic description of the linguistic elements, concepts, propositions, procedures and arguments, involved in the mathematical activity. An example of application of this tool can be seen in Pino, et al. (2012). The relationships between the components of MKT and the facets and levels of analysis of the DMK will be graphically illustrated in the poster.

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## REFERENCES

Ball, D. L., Lubienski, S. T. & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.

Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching. What makes it special? *Journal of Teacher Education*, 59(5), 389-407.

Godino, J. D. (2009). Categorías de análisis de los conocimientos del profesor de matemáticas [Categories to analyze the mathematics teacher's knowledge]. *Unión, Revista Iberoamericana de Educación Matemática*, 20, 13-31.

Godino, J. D., Batanero, C., y Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education*, 39(1), 127-135.

Godino, J.D., Ortiz, J.J., Roa, R. & Wilhelmi, R. (2011). Models for statistical pedagogical knowledge. In C. Batanero, G. Burril & C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study* (pp. 271-282). Berlin: Springer.

Hill, H. C., Ball, D. L., y Schlling, S. G. (2008). Unpacking pedagogical content knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400.

Pino-Fan, L., Godino, J.D., Font, V. & Castro, W.F. (2012). Key epistemic features of mathematical knowledge for teaching the derivative. In Tso, T.Y. (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 297-304). Taipei, Taiwan: PME.