THE TRIGONOMETRIC FUNCTIONS - CONCEPT IMAGES OF PRE-SERVICE MATHEMATICS TEACHERS

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Abstract. According to the present Croatian curriculum for secondary education, substantial time is allocated to the trigonometric functions of real argument. However, results of the Croatian State Matura show that students' performance on tasks involving this mathematical content is unsatisfactory. In order to gain a deeper insight into causes of this discrepancy in the context of the Croatian educational tradition, we have conducted a survey of basic concepts related to trigonometric functions of university mathematics education students in Croatia.

Key words: pre-service teachers, concept image, trigonometric functions

THE STUDY AND RESULTS

In our study, we were focused on concepts such as the radian measure, as well as sine, cosine and tangent function and the way they are used in solving simple trigonometric equations and inequalities (e.g. $\sin x < \cos x$, $\tan x > -1$). In order to describe dominant cognitive structure associated with the concept, the results obtained are analyzed against the theoretical framework of concept image vs. concept definition (Tall & Vinner, 1981) and the notion of procept (Gray & Tall, 1994) which is as well concerned with predominance of procedural knowledge over conceptual. Our research questions were: What kinds of concept images of radian and of trigonometric functions do pre-service teachers have? What kind of understanding of radian and trigonometric functions do pre-service teachers have: dominantly procedural or dominantly conceptual? In our study, we have collected data over entire populations of 79 students of the 3rd year and 26 students of the 5th year of university mathematics education programmes at the largest Croatian university. They completed a one-hour questionnaire with 13 open-ended questions addressing their understanding of the concepts mentioned above. Students' responses to each question were classified and the code plan has been developed accordingly. Based on the most typical concept images detected and strategies used to solve given problems, 8 students (4 from each group) were selected for semi-structure interviews on the same and some additional questions. Interviews were audio recorded and then analyzed.

In our poster, we present findings with major contribution to the research questions posed. Results suggest that pre-service teachers relate radian measure dominantly to rectangular coordinate system, circle trigonometry, as well as to trigonometric functions, whereas right triangle trigonometry and degree measure make significant part of their concept images of sine and cosine functions. On the contrary, tangent

function is seen dominantly only as a ratio of sine and cosine, without referring to its geometric interpretation in circle trigonometry. Pre-service teachers refer to π as the unit for radian measurements, while real numbers not being of the form $q\pi, q \in \mathbb{Q}$, are not recognized as the radian measures of an angle. Moreover, degree angle measure dominates students' conceptions of angle measure. For example, to find a length of an arc subtending central angle given in radians, they preferably convert radians into degrees, neglecting the definition of a radian measure. This reveals also a students' need to use conversion formulas, emphasizing their procedural knowledge. Dominance of procedural knowledge over conceptual is also seen regarding basic trigonometric inequalities. Some of these findings on Croatian dataset confirm earlier findings in Fi, 2006, Topçu, Akkoç, Yılmaz & Önder, 2006, and Chin & Tall, 2012. Our results evidently show that a change in Croatian initial mathematics teacher education should be made to promote pre-service teachers' conceptual knowledge on radian measure and trigonometry functions of real argument. Radian measure should be more explicitly related to angles and arc length, while values of basic trigonometric functions (especially of tangent function) should be recognized as coordinates of points in rectangular coordinate system. Our study is the first research on students' understanding of trigonometry concepts in Croatia.

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