THE IMPACT OF THE DYNAMIC GEOMETRY RESOURCE QUALITY EVALUATION ON TEACHERS' PRACTICES

Jana Trgalová*, Ana Paula Jahn**

*Claude Bernard University, Lyon, France, **UNESP, Guaratinguetá, Brasil

Since four years, a group of seven secondary school mathematics teachers and teacher educators has been involved in a research project dealing with the issue of dynamic geometry resource quality. The aim of this paper is to examine the impact of this involvement on their practices both as teachers and teacher educators. Based on the analysis of various resources produced by the group members before their participation in the research and nowadays, as well as on the group's auto-analysis of the evolution of their own practices, we could highlight a significant evolution in their way of using dynamic geometry in a classroom, as well as in their teacher training offer and content.

In this contribution, we report about an experience of a research group composed of two mathematics education researchers and seven secondary school math teachers (called DG-group in the sequel) working on dynamic geometry (DG) resource quality issues. We attempt to highlight the impact of the teachers' involvement into this research on their practices both as teachers and teacher educators.

The paper is organised as follows. First, we briefly describe the research project on DG resource quality that gathered together researchers and teachers. This project will be referred to as I2G project. Next, we present the theoretical framework and the methodology we used to examine the impact of the teachers' involvement in the I2G project on their practices. Finally, we discuss the most significant findings and propose some concluding remarks.

I2G PROJECT

The I2G project, which ran between September 2008 and June 2012, was conducted in the framework of the Intergeo European project [1], aiming at developing a community of DG users all over Europe around an open web-based repository [2] specifically designed for sharing resources and practices related to the use of DG in mathematics teaching. In order to help platform users identify suitable resources regarding their instructional aim and context of use, as well as to allow the available resources to be improved, two main tools have been developed and implemented into the repository: a search engine based on mathematical notions and competencies ontology to help searching for relevant resources, and a resource quality review questionnaire helping the users analyse available resources and highlight their weaknesses so that they can be improved. Two mathematics education researchers and the DG-group were in charge of designing and testing the questionnaire. In the next section, we present briefly the questionnaire and its design process, focusing on the roles of the researchers and the teachers involved in the I2G project.

Design of the questionnaire for DG resource analysis

The questionnaire, which is the main tool for the resource quality assessment in the repository (Fig. 1), was designed in a cyclical process consisting in the elaboration of its successive versions, followed by their tests and subsequent improvements.

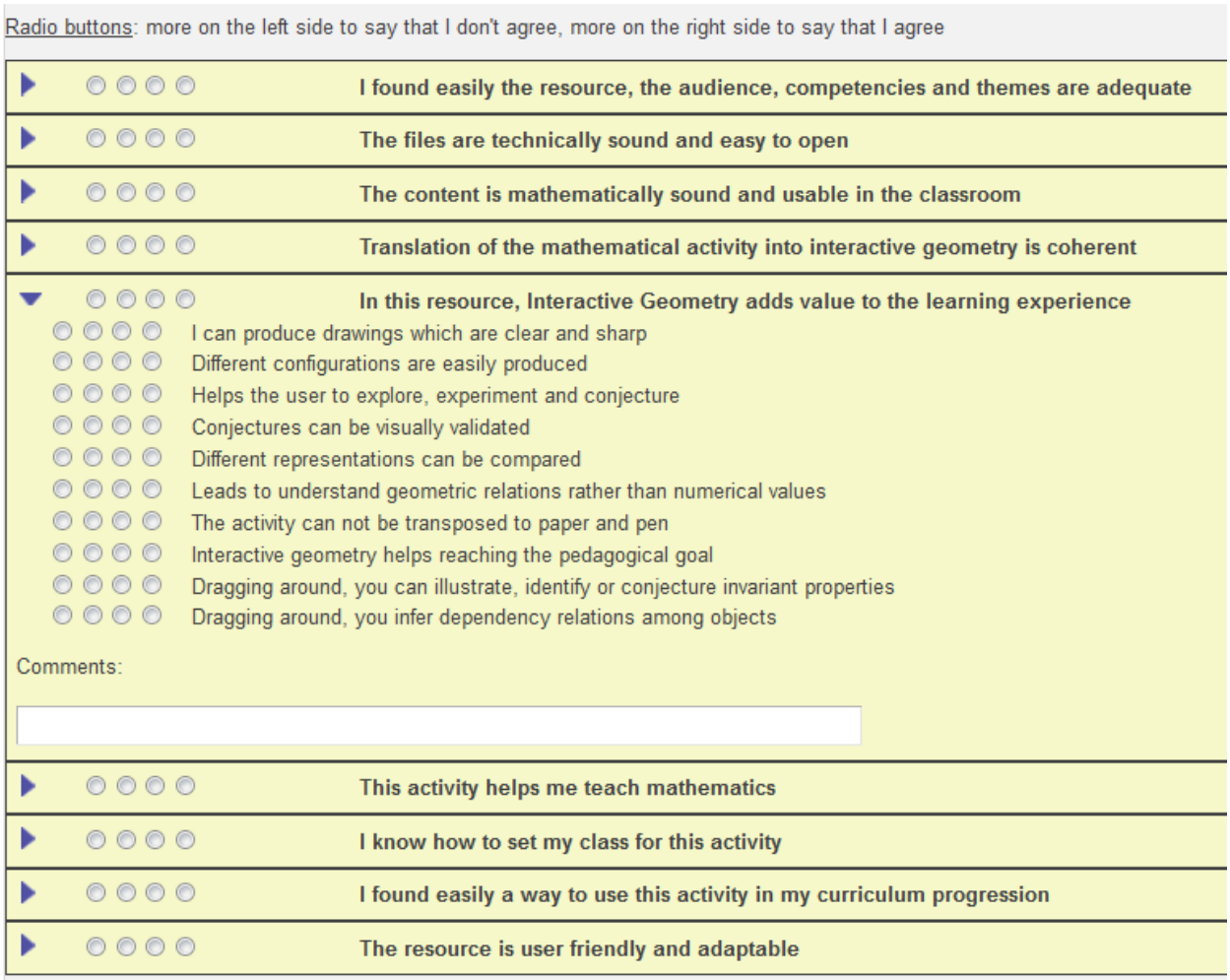


Figure 1: Online questionnaire for reviewing a DG resource in the i2geo repository

This methodology can be considered as a *design-based research*, in which "development and research take place through continuous cycles of design, enactment, analysis, and redesign" (DBRC 2003, p. 5), blending theory-driven design with empirical research. The first version of the questionnaire was designed by the two researchers drawing on research results related to the use of DG in math teaching and learning. It proposed eight general questions related to eight dimensions of a DG resource considered as critical with respect to the resource quality, such as technical aspect, mathematical content validity, instrumental aspect, and didactical and pedagogical implementation. Later, a ninth dimension related to the resource ergonomics has been added. Each of these questions can be developed into a set of more detailed criteria related to the corresponding dimension (Fig. 1). The theoretical considerations underpinning the choice of the dimensions and the definition of the criteria are exposed in some details in (Trgalová *et al*. 2011).

In order to make the questionnaire accessible to and usable by teachers, its elaboration has been done in a close collaboration with seven secondary mathematics teachers (DG-group), according to the schema in Fig. 2.

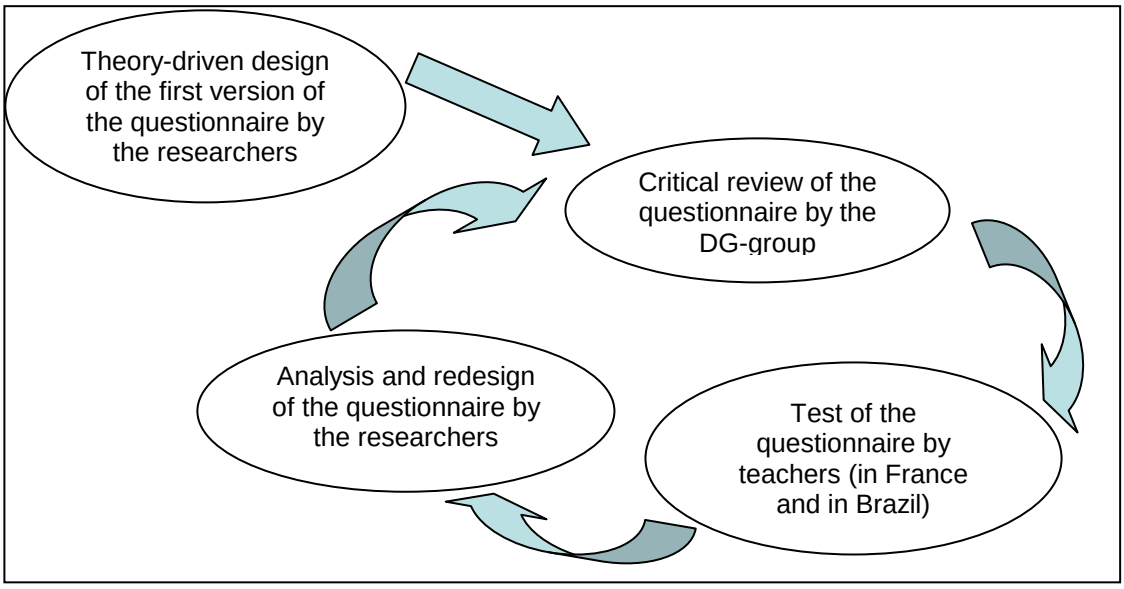


Figure 2: Schema of the resource quality design methodology

It is important to mention that besides the aim of the Intergeo community benefit consisting in improvement of shared resources, the definition of the quality criteria has also been driven by our ambition to enable each individual user engaged into a resource analysis to take a personal benefit by making her/him reflect on the purpose and the way DG is used in the resource, which may be different from her/his own practice. We supposed that the analysis of a resource would thus have an impact on the user in at least two aspects: (1) it would help her/him get a deeper insight into what the resource is about and thus facilitate its appropriation for a potential implementation in her/his classroom, and (2) it would contribute to the evolution of the user's practices with using DG by getting awareness of its possible contributions to the teaching and learning math and of various ways it can be used.

The empirical studies conducted within the questionnaire design cycle provided some evidence tending to confirm that teachers perceive the questionnaire as a very useful tool in the process of resource appropriation (Jahn *et al*. 2009, Trgalová *et al*. 2011, Trgalová & Richard 2012). However, these rather short experiments did not allow studying the impact of the questionnaire and the quality criteria either on the design and the improvement of the resources in the repository, or on the teachers' practices. Currently, we pursue our research in two directions. On the one hand, we attempt to study whether and how the questionnaire is used by the Intergeo community and to what extent it leads to the improvement of the resources in the repository. On the other hand, we try to explore out whether and how the use of the questionnaire to analyse dynamic geometry resources impacts teachers' practices. In what follows, we present a part of the research related to the latter issue.

THEORETICAL FRAMEWORK AND METHODOLOGY

DG-group

The teachers from the DG-group being involved in the design and use of the quality questionnaire, they became naturally our first research objects. These teachers started collaborating in 1996 within the Institute for Research in Mathematics Teaching in Lyon [3] as a group of "users and designers of DG resources" (Bourgeat *et al*., to appear). In 2008, the group volunteered to join the I2G project and it collaborated until recently with math education researchers on the DG resource quality issues. Every year since 2003, the group offers training courses aiming at helping other math teachers master and integrate DG systems into their practices. All group members prepare together their courses during their regular meetings and produce various resources (documents, tools…), although only two or three of them, in turn, are in charge each time of the implementation of the courses.

Methodology

In order to identify possible evolutions of the DG-group members' practices as teacher educators, we gathered teacher training resources the group has produced since 2003. Moreover, recently the group was asked to reflect on changes in their own practices that the group members could observe since their involvement in the I2G project. This introspective activity yielded many interesting observations (Bourgeat *et al*., to appear) related mostly to the teaching practices, some of which are reported in this paper. Finally, we have analyzed as well some of the teachers' reviews and comments of resources in the Intergeo platform, which can also shed light on their practices with using DG in math teaching. The next section presents the theoretical framework we used to analyse these data.

Theoretical lens used to analyse the data

Integration of technologies and double instrumental genesis

Numerous research studies on the information and communication technologies (ICT) integration adopt the *instrumental approach* (Rabardel 2002) as a theoretical framework specifically designed for studying teaching and learning phenomena involving technology. The instrumental approach relies on a distinction between an *artefact*, a tool available to an individual, and an *instrument*, which is the result of a process of appropriation of the tool by the individual when s/he uses it in order to achieve a given task. The process of transforming an artefact into an instrument is called *instrumental genesis*. Some of these studies stress the complexity of technology integration, which requires a *double instrumental genesis* in teachers: a first genesis of an instrument for achieving mathematics tasks, and a second one of an instrument for achieving educational tasks (Acosta 2008). Haspekian (2011) evokes a *personal genesis* transforming a given tool into a *mathematical instrument*, and a *professional genesis* transforming it into a *didactical instrument*. According to Trouche (2004), the ICT integration requires from the teacher to be aware of the

potentialities and constraints of artefacts, which is necessary to design suitable mathematical tasks. Moreover, the teacher has to be able to implement these tasks into the classroom and to foresee the spatial and temporal classroom management. The author introduces the term *instrumental orchestration* to refer to the didactical management of the artefact in a classroom. Drijvers *et al*. (2011) define the instrumental orchestration as "the intentional and systematic organisation and use of the various artefacts available in [a] computerised learning environment by the teacher in a given mathematical task situation, in order to guide students' instrumental genesis" (p. 1350).

These considerations will frame our analysis of the teacher training resources produced by the DG-group. We will look for elements in these resources showing whether the group is aware or not of the necessity of the double instrumental genesis in teachers wishing to integrate DG. We will also try to highlight the way the group orchestrates DG activities both in math classroom and in teacher training.

Potentialities of dynamic geometry

A dynamic geometry environment is computer-based software that allows the user to create geometrical figures and manipulate them into different shapes and positions by dragging their elements, mostly points. One of the distinctive features of DG is that when dragging, the geometrical properties of the figure defined in its construction are preserved. Three main modalities of dragging have been identified in the literature (Healy 2000, Laborde 2001, Arzarello, Olivero, Paola & Robutti 2002): (1) *dragging for verifying* consists in dragging to check the presence of the supposed (known) geometrical properties in the figure. According to Hölzl (2001), uses of DG are often limited to this modality, in the sense that students are expected to drag figures to confirm empirically the properties which are more or less given; (2) *dragging for conjecturing* consists in dragging to look for new properties of the figure through the perception of what remains invariant when dragging; (3) *dragging for validating/invalidating* consists in dragging to check whether the constructed figure preserves its geometrical properties when dragging.

In the analysis of the DG-group resources we will focus on the modalities of dragging in the activities it proposes and especially whether there are changes in the teachers' perceptions of the role of dragging.

DATA ANALYSIS

The analysis of the resources produced by the DG-group shows a significant shift in the practices of the DG-group teachers in three main aspects. The first two are related mostly to their practices as mathematics teachers, the third one to their practices as teacher educators.

Modalities of dragging

Relying on the teachers' auto-analysis of their own practices before and after their involvement into the I2G project (Bourgeat *et al*., to appear), it appears that with their students, the teachers used DG mostly to obtain robust constructions aiming at highlighting invariants in geometric figures:

"Yesterday, obtaining robust geometric constructions and highlighting invariants were the main goals assigned to students: they should construct a figure by using known properties […] which they could validate by the invariance of the figure when dragging."

Nowadays, they propose new types of activities in which they ask students "to explore figures in order to highlight invariants and/or conjecture new properties" (ibid.). The teachers seem to have acknowledged the importance of dragging for conjecturing in students learning and they thus propose various and richer tasks using different modalities of dragging.

The following comment [4], written by one of the DG-group members about a resource in the repository illustrating, with a robust construction, the equality of three ratios in a triangle with a parallel line to one of its sides, shows her awareness of the interest of soft constructions in geometry learning:

"Several improvements are possible: 1. [The point] N can be set free, which will allow visualizing the difference between the case proportional-parallel and the cases where the ratios are not equal".

Although it is difficult to establish a direct link between the questionnaire and the evolution of the teachers' awareness of the DG contributions to the teaching and learning geometry, we can suppose that the numerous discussions about this issue, that eventually led to the definition of criteria related to the added value brought by DG to the math activity, are at the origin of this evolution.

Instrumental orchestration

Regarding the classroom management, the teachers confess to have struggled to combine phases of students' work on computers in a computer lab with collective phases of debate, which often needed to be postponed until the next session in an ordinary classroom, as they say (ibid.):

"Before, the activities with ICT took place in a computer lab in the conditions that postponed the debate and the students-teacher interactions regarding their observations and manipulations in a digital resource."

Nowadays, the teachers orchestrate their ICT-based lessons in a more effective way: the use of a video projector allows articulating individual and collective phases. Indeed, the teachers say: "[Now] we observe the interactions in a genuine triangle "students – teacher – digital resource" (ibid.).

This shift can certainly be related to the criteria related to the pedagogical implementation of the resource, one of the nine aspects that we consider critical for determining the resource quality.

Awareness of the double instrumental genesis

The analysis of the teacher training resources produced by the DG-group reveals that the training programmes the group proposed before 2007, i.e. before its involvement into the I2G project, aimed mostly at helping trainees to master DG environment tools. The training activities consisted in series of exercises to solve with DG chosen to illustrate the use of a particular DG tool. Figure 3 shows a typical training activity: the trainees were asked to solve the exercise and indicate what DG tools they have used.

Exercise n°4:
a) Given a segment [AB], construct a square with [AB] as a side.
b) Given a segment [AB], construct a square with [AB] as a diagonal.
c) Construct a square with a side [AB] without using « parallel line » and « perpendicular line » tools.

N.B. For each question, verify that the construction remains stable.

Figure 3: Example of a teacher training activity proposed by DG-group in 2005

The focus of the teacher training programmes in this period was clearly on technical aspects of mastering a DG environment. In terms of instrumental genesis, the DG-group accompanied trainees' personal geneses of mathematical instruments.

Since 2009, the teacher training proposals of the DG-group show a significant shift towards considering didactical and pedagogical aspects of DG integration. Indeed, in a training programme proposed in 2009, the group announces the following objective: "The aim of this training programme is to accompany the teacher wishing to take her/his students to a computer lab". After a short phase during which the trainees solve exercises aiming at getting them acquainted with the main DG tools, they are invited to reflect on the types of activities suitable for the use of DG, the goal being to bring forward the following aims: introduce a new mathematical concept, construct figures, and put students into a research activity with DG. After having solved a given exercise with DG, they are asked to explore it in light of a possible implementation in a classroom: envisage possible adaptations, anticipate classroom management. Figure 4 shows a training resource, in which the two phases, solving an exercise and exploring it from didactical and pedagogical points of view, are present. The trainees are also led to create their own activities related to a math domain of their choice and adapted to the level of their class. They have to specify the teaching goals and envisage the classroom implementation of the activity. The DG-group has developed specific resources to help the trainees with this task, such as a description sheet of a session using DG (Fig. 5) or a checklist with questions to ask before using ICT in a classroom, e.g., when to use ICT, do the ICT contributions

favour students' learning, or how to integrate an ICT session into an ordinary teaching sequence.

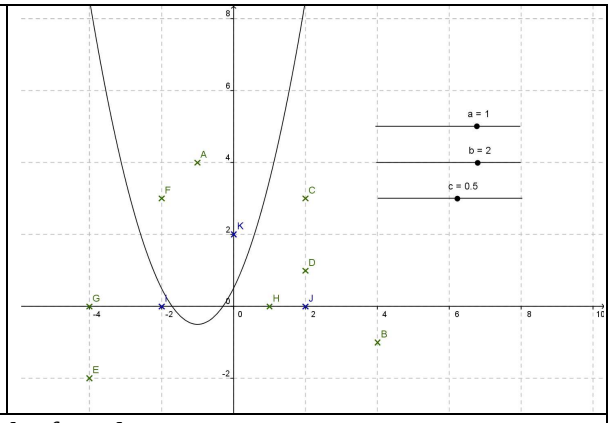
3. Studying polynomials 3.1. Relationship between graphical representation and expression of a first and second degree function The trainees open the file « stage1-exo-fct - degre 1 et 2.ggb » They are given the document « stage1-exo-fct degre 1 et 2.odt » Let the trainees solve the exercise. Ask them to reflect on possible adaptations for a Grade 11 classroom for the next training day.	
2nd day: exploitation of the exercise 3.1 solved during the first day: Ask for possible adaptations. A specific attention should be paid to the method used by the students (successive trials without the properties). The need to review this point with them… how? How to manage this activity?	

Figure 4: Excerpt of a training plan elaborated by the DG-group in 2011

DG software	
Class	
Mathematical topic	

TYPE OF OBJECTIVE OF THE SESSION

Represent a math object ❒	Research problem ❒	Discover a property ❒

OBJECTIVES OF THE SESSION

CONTRIBUTIONS OF THE DG

MODALITIES OF USE (COMPUTER LAB, VIDEO PROJECTOR, INTERNET…)

DEVELOPMENT OF THE SESSION

STUDENTS' PRODUCTIONS (IN A COMPUTER LAB)

Figure 5: DG session description sheet

These elements show that the DG-group has gotten awareness of the necessity to assist the trainees' professional instrumental geneses so that they can transform DG software not only into a mathematical instrument, but also into a didactical one. Moreover, the resources produced by the group entail signs of the influence of the quality questionnaire, namely considerations of several dimensions such as

contributions of DG, didactical exploitation of DG potentialities or instrumental orchestration. This seems to confirm a highly positive impact of the DG-group involvement into a design of DG resource quality questionnaire.

CONCLUSION

In this contribution, we reported about a research on the issue of DG resource quality, conducted by a mixed group of math education researchers and in-service teachers. We attempted to show a positive impact of this collaboration on the teachers' practices both as math teachers and teacher educators. Regarding the use of DG in their classes, we observe significant changes in the nature of tasks the teachers propose to their students: these are richer and more challenging, asking the students to explore figures and conjecture properties, rather than just verify supposed or known properties or validate robust constructions. The teachers are also able to envisage more productive instrumental orchestrations allowing a genuine integration of DG in their math classes. As teacher educators, the group seems to be now much more sensitive to didactical and pedagogical questions related to the DG integration than before. Initially, it focused mostly on technical aspects of mastering a DG software, thus accompanying trainees' instrumental geneses yielding a mathematical instrument, whereas nowadays, its programmes include activities allowing the trainees to develop a didactical instrument as well.

NOTES

1. Interoperable Interactive Geometry for Europe, 2007-2010, http://i2geo.net/xwiki/bin/view/Main/About.

2. i2geo.net

3. Institut de Recherche sur l'Enseignement des Mathématiques (IREM).The IREMs gather together primary, secondary and university teachers to conduct research on problems in math teaching and learning at all school levels, to offer teacher training programmes based on research results, and produce and disseminate pedagogical resources.

4. http://i2geo.net/xwiki/bin/view/Coll_cdording/Egalitedes3rapportsavecunedroiteparalleleauncoteduntriangle?bc=&viewer=comments

REFERENCES

Acosta, M. (2008). *Démarche expérimentale, validation, et ostensifs informatisés. Implications dans la formation d'enseignants à l'utilisation de Cabri en classe de géométrie*. Thèse de doctorat, Université de Grenoble 1 et Université de Genève.

Arzarello, F., Olivero, F., Paola, D., Robutti, O. (2002). A cognitive analysis of dragging practices in Cabri environments, *ZDM – The International Journal on Mathematics Education*, 34(3), 66-72.

Baccaglini-Frank, A. & Mariotti, M.A., (2010). Generating Conjectures through Dragging in Dynamic Geometry: the Maintaining Dragging Model. *International Journal of Computers for Mathematics Learning* 15(3), 225-253.

Bourgeat, F., Calpe, A., Digeon, M. *et al*. (to appear). De l'utilisation à la formation : géométrie dynamique et logiciels mathématiques. In G. Aldon *et al*. (Eds.), *Représentations dynamiques des mathématiques : quels outils pour faire, pour apprendre et pour enseigner les mathématiques ? Actes des Journées mathématiques de l'IFE 2012*, 12-13 juin 2012, Lyon (France).

DBRC (Design-Based Research Collective) (2003). Design-Based Research: An Emerging Paradigm for Educational Inquiry. *Educational Researcher* 32(1), 5–8.

Drijvers, P., Doorman, M., Boon, P., van Gisbergen, S. (2011). Instrumental orchestration: theory and practice. In V. Durrand-Guerrier *et al*. (Eds.), *Proceedings of CERME 6* (pp. 1349-58), Jan. 28-Feb. 1 2009, Lyon (France).

Haspekian, M. (2011). The co-construction of a mathematical and a didactical instrument. In M. Pytlak et al. (Eds.), *Proceedings of CERME 7* (pp. 2298-2307), February 9th-13th 2011, Rzeszów (Poland).

Healy, L. (2000). Identifying and explaining geometric relationship: interactions with robust and soft Cabri constructions. In *Proceedings of the 24th conference of the IGPME* (Vol. 1, pp. 103–117) Hiroshima, Japan.

Hölzl, R. (2001) Using dynamic geometry software to add contrast to geometric situations – A case study. *Int. J. of Computers for Math Learning* 6(1), 63-86.

Jahn, A. P., Trgalová, J. & Soury-Lavergne, S. (2008). Analyse de ressources pédagogiques et amélioration de leur qualité : le cas de la géométrie dynamique. In *Actes du 2° SIPEMAT*, 28 juillet-1 août, Recife (Brésil).

Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-Geometry. *Int. J. of Computers for Math Learning* 6(3), 283–317.

Rabardel, P. (2002). *People and technology - a cognitive approach to contemporary instruments*. http://ergoserv.psy.univ-paris8.fr

Restrepo, A. M. (2008). *Genèse instrumentale de déplacement en géométrie dynamiqe chez des élèves de 6ème*. Thèse de doctorat, Université de Grenoble 1.

Trgalová J., Richard P. (2012). Analyse de ressources comme moyen de développement professionnel des enseignants. In J.-L. Dorier, S. Coutat (Eds.) *Enseignement des mathématiques et contrat social : enjeux et défis pour le 21e siècle – Actes du colloque EMF2012* (GT6, pp. 908–918).

Trgalová, J., Soury-Lavergne, S., Jahn, A. P. (2011). Quality assessment process for dynamic geometry resources in Intergeo project: rationale and experiments. *ZDM – The International Journal on Mathematics Education* 43(3), 337-351

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *Int. J. of Computers for Math Learning* 9, 281-307.