RESULTS ON THE FUNCTION CONCEPT OF LOWER ACHIEVING STUDENTS USING HANDHELD CAS-CALCULATORS IN A LONG-TERM STUDY

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Long-term studies of students using a handheld CAS-calculator encourage the hope that the gain of lower achieving students using digital tools is above average. The German CASI-project takes advantage of the three-tier school system to examine and evaluate the long-term use of handheld CAS-Calculators by lower achieving students in the 9th and 10th grade. The findings presented in this paper result from the pre-, post- and follow-up-test for the teaching unit on quadratic functions and equations which took place at the beginning, after approximately one third and near the end of 10th grade. The students had one year of experience using digital tools and already passed the tests on linear functions and equation systems. The theoretical framework for the test-design was based on the translation skills for functions.

INTRODUCTION

Functions have always been in the focus of research when the impact of digital tools on students' school achievements is evaluated. Especially, because

> the graphing calculator affords the user both the ability to create equations, tables, and graphs quickly and the facility to move among the representations rapidly (Hollar & Norwood, 1999)

there is special expectation of a handheld CAS-calculator to particularly foster the translation skills (Swan, 1982) between different representations of functions[1] in students. Indeed, Weigand and Bichler (2010) were able to observe an improvement regarding the transition between graph and equation of a function. Therefore this subject matter is an obvious starting point when focusing on the results of lower achieving students using digital tools.

O'Callaghan found

> that the CIA[2] students had a better knowledge of the individual components of modelling, interpreting, and translating as well as a better overall understanding of the function concept

supporting the idea of digital tools helping students to work on realistic problems involving functions. Schwarz and Hershkowitz (1999) claim

[1] See table 1 for a complete list of these translation skills as presented in (Swan, 1982).

[2] CIA=Computer-Intensive Algebra

> that more students learning functions in the interactive environment than in a traditional environment had rich function concept images

and the result of a meta-study by Barzel (2012) states that conceptual knowledge in general can be fostered by using CAS. This is why there seems to be reason to hope, the students would be able do better when working on examination tasks involving situational descriptions of functions and the corresponding translation skills.

The German School System

The findings presented in this paper emerged from a project which took place in Germany and in order to judge the meaning of the results some knowledge about the German school system is necessary. There are four types of secondary schools in Germany which all start at 5th grade.

The students who show the highest academic abilities in primary school attend the Gymnasium. They graduate to the German Abitur in 12th grade[3] which is comparable to British A-levels and qualifies to study at a university.

The students showing moderate academic abilities in primary school attend the Realschule until 10th grade and then choose between switching to the Gymnasium or other higher schools to get the Abitur or a similar degree and starting an apprenticeship. If they do switch to the Gymnasium they graduate after 13 years of schooling since they have to start over in 10th grade.

The students who show low academic abilities in primary school attend the Hauptschule and most likely start an apprenticeship after 10th grade even though there are possibilities to switch schools and reach Abitur or similar degrees like the students who attended the Realschule. In some German states Hauptschule and Realschule are replaced by a new integrated type of school.

The fourth type of school is the Gesamtschule which incorporates all of the three other types. This is done by differentiation through special courses on various academic levels in each grade; some students learning at Gymnasium-level others at Hauptschule-level.

Studies concerning Students using handheld CAS-Calculators

Some empirical studies regarding the usage of handheld CAS-calculators have been undertaken in Germany. In particular the CALIMERO-Project in Lower Saxony and the M³-Project in Bavaria have to be named. In Lower Saxony handheld CAS-calculators are employed at the Gymnasium from Grades 7 to 10 and up to the Abitur. The Bavarian study examined students at the end of the Gymnasium (Bruder & Ingelmann, 2009, Weigand & Bichler, 2010).

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[3] This has been subject to change quite often during the past ten years. Abitur after 12th grade is true for North Rhine-Westphalia at the moment, but this might also be about to change.

Since all major empirical studies on the usage of digital tools in mathematics education in Germany took place at the Gymnasium it is particularly interesting to note that while the overall test results of students using handheld CAS-calculators did at least not fall off in quality (Barzel, 2005) especially lower achieving students benefited from the use of handheld CAS-Calculators. The increase in school achievement found in this group of students was significantly greater than the average (Ingelmann & Bruder, 2007). Weigand and Bichler (2010) stated, that in the 10^{th} grade particularly the weaker students my benefit from the use of digital tools. International meta-studies even observe substantially better test results in classes using digital tools (Hembree & Dessart, 1986, Ellington, 2003). The results presented in the introduction give reason that the research on even lower-achieving students should be started with a closer look on their performance when solving tasks involving funktions and translation skills.

THE CASI-PROJECT

The Project Computer Algebra Systems used in lower secondary schools (CASI-Project) examines the long-term use of handheld CAS-calculators at the German Realschule respectively courses of the same academic level at the Gesamtschule. It aims to support, test, and examine the use of digital tools by students taught in 9th and 10th grade. The focus lies on the design and evaluation of educational concepts for the use of handheld CAS-calculators regarding these by definition lower achieving students. Another part of the project concept is the aim to encourage and support diverse use of digital tools. The intent is to use the handheld CAS-calculator not only to calculate, but also to experiment, visualise, create algebraic models and control (Greefrath 2010).

The examined sample of students consists of 10 project classes at 5 different schools in North Rhine-Westphalia with 10 different project teachers who voluntarily signed up for the project without knowing which students they would be teaching except for their grade for obvious reasons. These 10 project classes were then equipped with ClassPad handheld CAS-Calculators. Aside from the loose planning of certain special content areas described below at project teacher meetings and the possibility to call for technical support at any time, the researchers did not influence the teachers' work. Additionally, the six parallel classes at the same schools serve as a control group.

Test-Design

To achieve the goals specified in the paragraph above, several different research methods have been employed. During the 2 year duration of the project five content areas (linear functions and equation systems, Pythagoras theorem, circles and the number p, quadratic functions and equations, mathematic growth) were chosen to be of special interest and the educational concept of these is jointly planned and carried out by all project teachers and includes the definition of competencies the students are meant to achieve with digital tools, but also without any technological support at all. These study projects have been monitored using a pre-/post-test design as well as qualitative studies on mathematical test tasks and solution structure or strategies. Aside from mathematical tests, questionnaires on the students' opinions about digital tools and mathematics as well as journals of the teachers about the actual usage of the ClassPad during the lessons are evaluated.

The results presented in this paper follow from tests on the teaching unit on quadratic functions and focus on the quantitative side. These tests consist of two parts: While working on the first part of the test, students are not allowed to use any technology at all. The first task in this part is to solve two equations or equation systems to test for the algebraic abilities of the student. The second task involves drawing graphs of linear or quadratic functions and reading off zeroes or intersection points.

Question 1. A candle burns down. At this moment, it's height is 9 cm high. It shortens by one half cm per 10 minutes. Draw a graph into the given coordinate system that represents this situation.

Question 2. A stone drops from the top of a tower which is 12 m high. After one half second, it has dropped 1.25 m, after one second 5 m and after one and a half second 11.25 m. Draw a graph which assigns the elapsed time to the height of the falling stone over the ground.

Figure 1: Example questions for translation from situation to graph

After finishing this part, the students have to pass the first part to the teacher in order to be allowed to use their calculators to solve the following examination questions. In fact most of these tasks are solvable with little to no technological help and would by classified "tasks with optional or neutral benefit from digital tools" according to the scale introduced by Brown (2003). The questions have been designed to be parallel in matters of the involved translation skills after Swan (1982) – some examples for the translation between situation and graph from different tests being shown in fig. 1. As already pointed out in the introduction, emphasis was placed on the translation skills including situations. The translation skills taken into account for the tests' design are underlined in table 1.

From \ To	Situations	Tables of data	Graphs	Algebraic Expressions
Situations	---	measuring	<u>sketching</u>	descriptive modelling
Tables of data	reading	---	plotting	fitting
Graphs	<u>interpreting</u>	<u>reading off</u>	---	curve fitting
Algebraic Expressions	<u>formula recognition</u>	<u>tabulating or computing</u>	<u>curve sketching</u>	---

Table 1: Translation Skills (Swan 1982)

The lack of translations to algebraic equations takes the low age and therefore low experience in designing algebraic expressions into account. Also there might be some "curve fitting" and "fitting" involved due to the tabular design of the corresponding task (see fig. 3 for details), but the other direction is most likely the more important one.

The tests take place in the second year of the CASI-project. At that time the students attend the 10th grade and the chosen teaching unit takes place several weeks after the summer break. The pre-test occurs right before the subject changes to quadratic functions and with no repetition happening before. The post-test concludes the teaching unit and can optionally also be taken as written exam. The follow-up-test takes place about 3 month after the post-test without any kind of repetition.

RESULTS

The results presented in this chapter have to be sorted by the number of students involved in the analysis. This unfortunate premise is due to organizational problems with not-returning tests and decreasing numbers when working only with those students participating in all tests.

Therefore the results will be given in two sub-paragraphs. The first one will work with the data of all students who participated in all three tests to give the main impression and provide results in the most rigid form available. This first sub-paragraph will show an effect visible in the follow-up-test. To include the results of a substantially higher percentage of project and control group students and focus on the findings regarding the follow-up-test the second one will include the students who participated in the follow-up-test and at least one test of the other two. The combination of the results in this form proved to be the most accurate way to look at a larger number of students without flawing the results and also provide the more rigid view on the data.

Results of the students participating in all tests

The overall results of this group of students are presented in fig. 2. The analysis includes the data of 131 project students and 67 students of the control group.

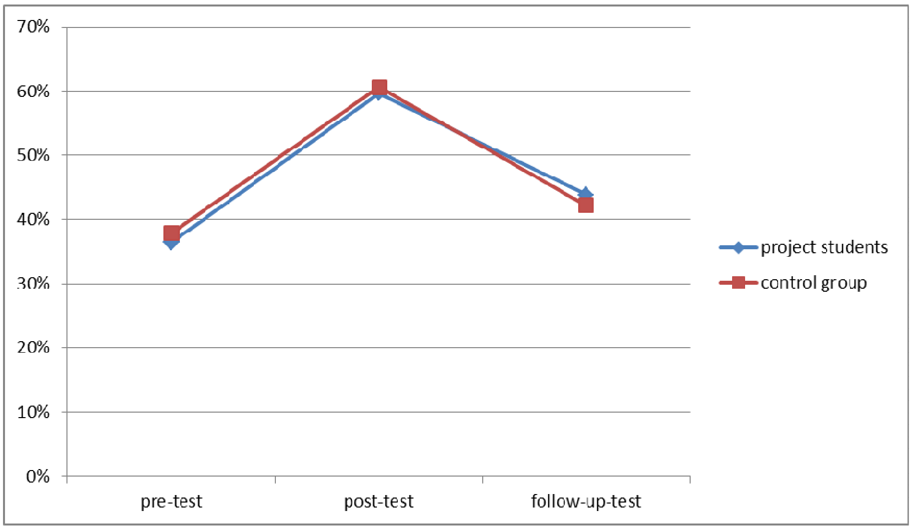

Figure 2: Test-results of the students participating in all three tests

The results show the usual peak at the post-test and then fall off until they reach a slightly higher level in the follow-up-test than in the pre-test. The small differences between the two groups are not significant, but the fact that the project students achieved a higher test-result only in the follow-up-test qualifies for a more detailed analysis of this special part. When analysing the examination questions on their own, mostly there are no significant differences between the project students and the control group. The few significant distinctions between these groups are summarised in the next paragraph.

There is a significant better result of the control group in the pre-test examination question on the translation from graph to situational description, t(89.3)=-2.779, p<.02, r=-0.226. In the post-test the control group has a significantly better outcome in an examination question where they had to read off values of graphs and insert modified versions of these graphs into the same coordinate system, t(169)=-2.297, p<.03, r=-0.161. The follow-up-test shows a significant advantage of the project students over the control group when solving tasks similar to the one shown in fig 3, t(196)=2.453, p<.02, r=0.171.

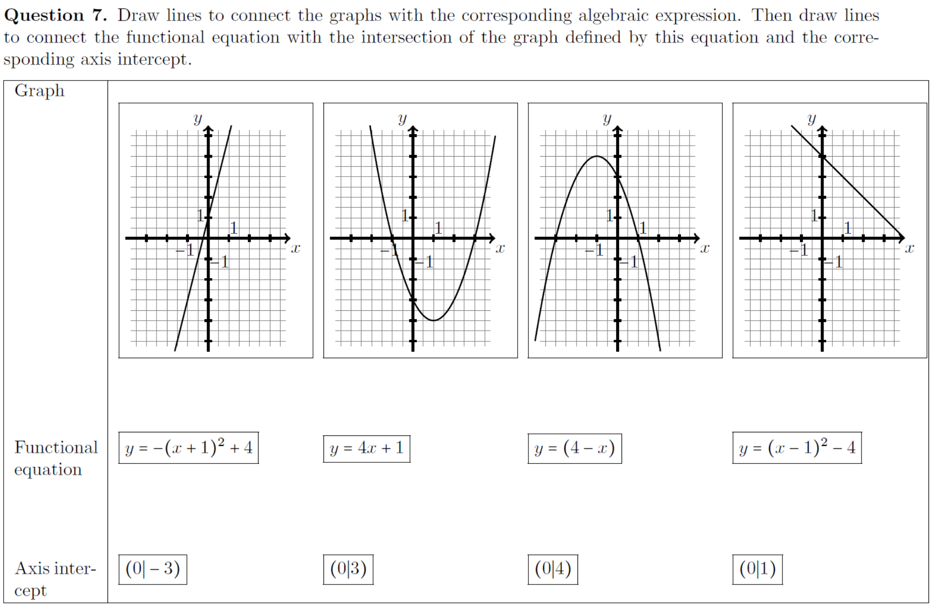

Figure 3: Example examination question from the follow-up-test

There are multiple translations involved in this examination question. Mainly they involve the translations between graphs and algebraic equations and algebraic equations and points on the graph which can be considered as a certain type of tables of data.

Results from the students participating in the follow-up-test

The results of the students who participated in both tests give ground to the assumption that a closer look on the results of the follow-up-test can lead to a better insight. So in this paragraph the students' total is defined by the participation in the follow-up-test and at least one of the other two tests in order to eliminate students for whom there is no data on their long-term progress. Admittedly, this way of looking at the data may be problematic when it comes to generalizations, but the strong tendencies found in the results of these larger set of students give some hint of explanation for the larger picture. Participation in either pre- or post-test and the follow-up-test still guarantees the students' participation over more than 3 month such that the students are familiar with project material and the way the parallel tasks are designed. Fig. 4 summarises the results in the three tests.

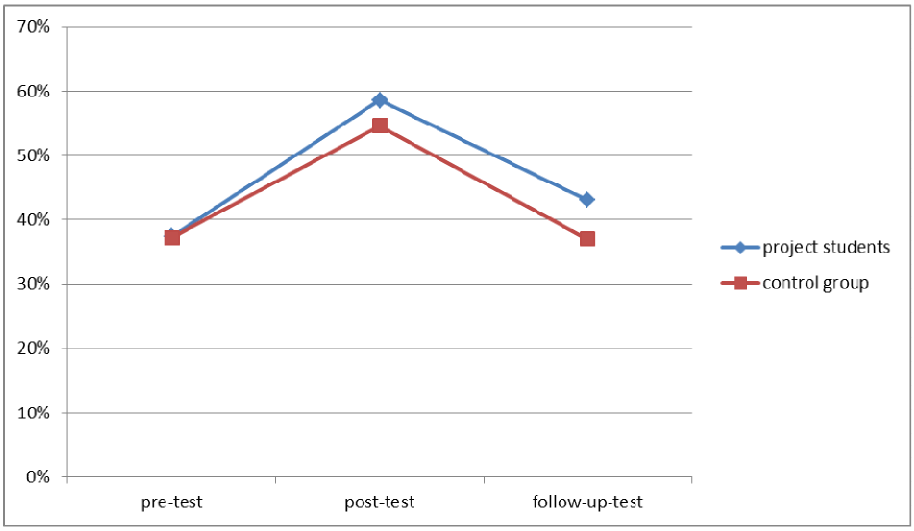

Figure 4: Results of the students participating in the follow-up-test

Both curves show the usual peak at the post-test's result, but there are more differences between the groups. The project students and the control group already differ significantly in the post-test and that gap gets bigger in the follow-up-test. The exact data is summarised in table 2.

	pre-test	post-test	follow-up-test
N (project stud.)	161	152	182
N (control group)	74	90	97
M (project stud.)	37%	59%	43%
M (control group)	37%	55%	37%
SD (project stud.)	15%	13%	13%
SD (control group)	16%	16%	15%
t	0.104	2.029	3.48
df	233	240	277
p	.917	.05	.001
r	0.007	0.137	0.204

Table 2: T-test results and effect size for the three tests

So there is no statistical difference at all in the pre-test, a significant difference with a small effect size in the post-test and a highly significant advantage of the project students over the control group with a small to medium effect size in the follow-up-test. A closer look at the examination questions yields the following findings[4].

The significant advantage of the control group when translating from the graph of a function to a situational description in the pre-test is still visible, t(233)=2.294, p<.03, r=-0.155, but equals out in the post-test and turns to a significantly better result of the project students in the follow-up-test, t(277)=2.11, p<.4, r=0.125. Additionally, there are still better results of the project students visible when solving examination questions of the type shown in fig. 3 with small effect sizes in the pre- and post-test and medium effect size in the follow-up-test. Table 3 contains the t-tests' results on these tasks.

	M (PS)	M (CG)	SD (PS)	SD (CG)	t	df	p	r
pre-test	59%	48%	34%	36%	2.294	233	.03	0.148
post-test	91%	84%	19%	23%	2.342	157	.02	0.157
follow-up-test	83%	64%	26%	33%	4.917	161	.001	0.301

Table 3: Results of the examination question of the type shown in fig. 2[5]

The project students' results (M=45%, SD=32%) also significantly exceed the control group's results (M=35%, SD=30%) in the post-test examination questions on drawing functions and reading off values without any technological help, t(240)=2.699, p<.01, r=0.171. Furthermore there is a difference (project students: M=54%, SD=30%; control group: M=46%, SD=29%) in solving equations by hand, t(277)=2.073, p<.04, r=0.123.

CONCLUSION

Conducting a project on the use of handheld CAS-calculators at the German Realschule takes the research on digital tools to a class of students which has not been taken into account yet. It has been unclear whether students of this age and academic ability would be able to profit from technology at all or perhaps even be hindered by the necessity to learn the language of the computer in addition to everything else. The lessons during the CASI-project were conducted by interested but otherwise normal teachers and in spite of advice and planning of certain aspects for the special subject matters mentioned above in no way influenced by the researchers.

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[4] Only results with significance p<.05 are reported.

[5] PS=project students, CG=control group

The results presented in this article regarding the students who participated in all three tests show that the test results under these circumstances do at least not fall off in comparison to the control group. This is valid for the tasks where all technical help was allowed as well as for the examination questions which have to be solved by hand only. The few significant differences regarding isolated examination questions cannot be observed in the corresponding parallel tasks and – with the exception of the task displayed in fig. 3 – don't show a tendency to be anything more than statistical artefacts.

Widening the pool of students by allowing students with only two written tests – as long as the follow-up-test is included – to enter the analysis leads to some additional findings. The project students achieve significantly higher results in the post- and follow-up-test with the effect even increasing towards the later test showing that mathematical knowledge is conserved better. This might partly be caused by the fact that the project students were able to use the handheld CAS-calculator to help solving the task shown in fig. 3 which is an examination question where there can really be a benefit from the use of digital tools . Even if this is the case it still shows the students' ability to autonomously use the digital tool for their benefit showing their progress in terms of instrumental genesis. (Guin & Trouche, 1999)

The results of the examination questions addressing translation skills which include translations with situational descriptions do not show any special advantage or disadvantage of either group of students. So there is no evidence to the assumption the richer conceptual images of the project students have an effect on these isolated tasks.

Additionally the abilities of the students to work on certain basic tasks without any technological help are at least conserved. This result was reached only by agreeing on certain competencies which are to be achieved and without the teachers being mandated by the project supervisors to train these skills with any special program. Students from the project group even had the tendency to be a bit better than the control group in these tasks.

A direct comparison of the effects observed in this project with the effects found by Ingelmann and Bruder (2007) as well as Weigand and Bichler (2010) examining students at the Gymnasium is difficult due to the different ages of the students and other ways the subject matters are addressed. The lack of improvement in examination questions addressing translation skills might be caused by the less abstract way of teaching functions at the Realschule. The main tendency of the results however is in line with the findings of these previous studies.

REFERENCES

Barzel, B. (2012). Computeralgebra im Unterricht : Ein Mehrwert – aber wann? Münster; New York, NY; München; Berlin: Waxmann.

Barzel, B. (2005). Open Learning? Computeralgebra … No time left for that. *Zentralblatt für Didaktik der Mathematik, 37(5),* 336–342.

Brown, R. (2003). Computer algebra systems and mathematics examinations: A comparative study. *The international Journal of Computer Algebra in Mathematics Education, 10,* 155-182.

Bruder, R., & Ingelmann, M. (2009). CAliMERO aus Sicht der Forschenden. *Mathematikunterricht (Seelze), 55(4)*, 13–19.

Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes, *Journal for Research in Mathematics Education, 34(5),* 433-463.

Greefrath, G. (2010). Mit dem Computer qualitativ arbeiten? *Praxis der Mathematik i. d. Schule, 52(31)*, 20-24.

Guin, G., & Trouche, L. (1999) The complex Process of converting Tools into Mathematical Instruments: The Case of Calculators, *International Journal of Computers for Mathematical Learning, 3,* 195-227.

Hembree, R., & Dessart, D. J. (1986). Effects of hand-held calculators in precollege mathematics education: a meta-analysis. *Journal for Research in Mathematics Education, 17(2),* 83-99.

Hollar, J. C., & Norwood, K. (1999). The Effects of a Graphing-Approach Intermediate Algebra Curriculum on Students’ Understanding of Function. *Journal for Research in Mathematics Education, 30(2)*, 220-226.

Ingelmann, M., & Bruder, R. (2007). Sinnvoller Einsatz von CAS in den Klassen 7 und 8. In Gesellschaft für Didaktik der Mathematik (Eds.): *Beiträge zum Mathematikunterricht 2007. Vorträge auf der 41. GDM Tagung für Didaktik der Mathematik* (pp. 94-97) Hildesheim: Franzbecker.

O’Callaghan, B. R. (1998). Computer-Intensive Algebra and Students‘ Conceptual Knowledge of Functions. *Journal for Research in Mathematics Education, 29(1),* 21-40.

Schwarz, B. B., & Hershkowitz, R. (1999). Brakes or Levers in Learning the Function Concept? The Role of Computer Tools. *Journal for Research in Mathematics Education, 30(4),* 362-389.

Swan, M. (1982). The Teaching of functions and graphs. In G. van Barnevold & H. Krabbendam (Eds.), *Conference on functions. Conference report. Pt. 1* (pp. 151-165). National Institute for Curriculum Development, Enschede.

Weigand, H.-G. & Bichler, E. (2010). Symbolic calculators in mathematics lessons: The case of calculus. *International Journal for Technology in Mathematics Education, 17(1),* 3-15.