STUDENTS' PERCEPTIONS OF HOW THEY LEARN BEST IN HIGHER EDUCATION MATHEMATICS COURSES

Svein Arne Sikko and Birgit Pepin

Sør-Trøndelag University College, Trondheim, Norway

This paper reports on results from an on-going project investigating the transition from upper secondary to higher education mathematics. From a survey of students in their second year in higher education, we report on the teaching and study methods from the first year which, according to the student survey, gave them the greatest dividends as learners of mathematics. Results show that work assignments and collaboration with fellow students in tutorial groups or informal groups were the methods students felt they learnt most from. Lectures were not perceived as equally important in the learning process. The implications of the results are discussed in the paper.

INTRODUCTION

The transition from secondary to higher education mathematics has been widely studied and it appears to be a difficult phase for many students (e.g. Gueudet, 2008). In addition to causing disappointment and distress for students personally, student failure and/or drop-out represent a significant loss of money for the university/college, a concern for teachers and a loss of potential for society as a whole (Gamache, 2002). It is said that students coming into higher education are more numerous and have more diverse backgrounds than previously, and they have different and often vague views of mathematics, its learning and its role in their future careers and lives (Kajander & Lovric, 2005). Students struggle with university studies because they have a distorted perception of what learning is and what the acquisition of knowledge entails and many students see knowledge as a collection of facts that can be absorbed passively (Gamache, 2002). Moreover, undergraduate mathematics does not yet seem to accommodate the diversity of its student body in its offerings and learning mode opportunities (Barton, Ell, Kensington-Miller, & Thomas, 2012).

In this study we investigate how students perceive their 'learning milieus', why students continue with mathematics and what could be reasons for dropping out. We examine how students develop their identity as mathematics learners, at transition and through their first year university mathematics, and how they perceive the 'use' of mathematics for their further studies and lives. In this paper we report on the results from the second of two data points performed in the second year of the project. The question we ask is "Which forms of study did the students perceive as having been most useful in their first year of study?"

CONCEPTUAL FRAMEWORK

The larger study was conceptualised in collaboration with, and it is very similar to, the TransMaths project at the University of Manchester. The project's aim is to develop a deeper understanding of how student experiences of mathematics education practices may interact with various (identified) factors to shape students' development as learners of mathematics, their dispositions and their decision-making at this crucial time. It appears that students experience difficulties at different stages, and they develop different strategies to make these transitions successful (e.g. Brown & Rodd, 2003). Wenger (1998) contends that learning involves both *practice* and *identity*, that is learning develops as students engage and participate in a particular 'world' and in a *practice*. At the same time institutional practices afford, or hinder, students developing a mathematical disposition and an identity (e.g. Boaler, 2002) that supports their engagement with mathematically oriented subjects. The on-going project studied students' identities in relation to their experiences of different mathematics learning-and-teaching practices.

The literature (e.g. Wingate, 2007) argues that at transition to university students are often expected to become 'independent learners', hence the importance of "learning to learn". In a previous article (Pepin, Lysø, & Sikko, 2012) we argued that the strategies for learning to learn mathematics were not adequately addressed in the higher educational institutions we studied, except in elementary teacher education, although according to our survey, this was just what students said they needed most. There are, however, selected 'innovative practices' (e.g. Croft, Harrison, & Robinson, 2009) which appear to have led to improved student learning experiences.

In terms of learning-and-teaching practices, face-to-face lectures remain a standard component of most higher education mathematics courses, despite being widely criticised, and even with advances in information technology and access to the internet, and it is claimed that lectures often are, in practice, where students' learning starts (Pritchard, 2010). These criticisms relate to the lecture as a mode of teaching (mathematics) which promotes superficial learning and 'transmissive' teaching, an environment where 'right-and-wrong' answers are encouraged, amongst others. However, there is evidence that lectures, appropriately 'conducted', are likely to provide opportunities for student learning, for students to take responsibility for their own learning and to engage in activities that are conducive to collaborative learning (e.g. Barton et al., 2012).

Research (e.g. Crawford , Gordon, Nicholas, & Prosser, 1998) shows that students' views of mathematical learning and knowledge relate to their experiences of learning as a whole, which indicates the need for shifting lecturers' attention from just focussing on the mathematical content (and presentation of their course) to a more systemic view of the learning environment. For example, Schoenfeld (1998) proposes an environment that fosters "a community of sense-making in which exploring ideas is highly valued" (p.61) and in his mathematical problem-solving courses teachers encourage students to conjecture and propose solutions where the validity and accuracy of the solutions are decided by the group. Barton et al.'s study (2012) shows that lecturers are crucial in establishing new social norms (Yackel & Cobb, 1996) where the lecture goals are changed from 'covering the content' to 'developing mathematical understanding' which includes active engagement of students and where "students spend some time working in informal groups engaged in mathematical activity" (p.6). Hence the literature advocates student activity, collaborative learning and informal group work as having overall positive effects for the learning of mathematics, in the cognitive domain as well as the social and affective domain in higher education mathematics (e.g. Barton et al., 2012).

This change of focus is also supported by new technology, albeit more research is needed in this relatively new domain, and one could ask whether 'technology would be helpful in fostering novice students' autonomy by using appropriate online resources' (Gueudet, 2008, p. 252). E-learning is now proposed in the secondary-tertiary transition in mathematics (e.g. Bardelle & Di Martino, 2012), with the purpose of transforming practice and hence learning- advocates believe that it can transform thinking and attend to individual students' needs for personalisation (of paths) and collaboration (in a single activity) (Bardelle & Di Martino, 2012).

RESEARCH DESIGN

The research design (of the whole project) was based on a theoretical framework of mixed methodology involving student longitudinal survey; student biographical interviews; and case studies of practice, all at two data points (DP1 and DP2). The questionnaires were developed based on the Manchester examples, subsequently tested and calibrated for the Norwegian context (which included the validation of each question), and appropriately translated.

The data chosen for analysis reported in this article were the questionnaire/survey data from students at the following institutions: at City University (CU) the study was conducted with students in in three different courses (Calculus 1 for civil engineers specializing in maths and physics; Basic Analysis; and Mathematics for applications). The Basic analysis course was followed by students enrolled in the teacher program in the sciences or mathematics, but also included students in various undergraduate programs and a one-year mathematics program. The Mathematics for applications course is usually followed by students who need a somewhat less theoretical dominated mathematics course and focuses on applied mathematics. At River University College (RU), we conducted the survey in both the three-year engineering program (RUE) and the teacher education (RUT) for years 1-10. In engineering at RUC students from three different programs were following the same math course. At DP 1 (autumn 2010) we collected questionnaires from 720 students (and interviewed a total of 49 students spread across the various programs and courses - see Pepin, Lysø, & Sikko, 2012). At DP 2 (autumn 2011/spring 2012) we collected 562 questionnaires.

The DP 2 Questionnaire had a total of 26 questions. In this paper we will look at three of the questions, Question 16, Question 17 and Question 18. In Question 16 respondents were asked to consider 10 different statements regarding teaching and learning methods in the mathematics they experienced during their first year. For each of the statements they were asked to respond on a scale from 1 (Strongly disagree) to 5 (Strongly Agree). It was also an opportunity to check for Do not know / Not applicable. In Question 17, they were asked to give more detailed comments on which of the learning methods and study forms mentioned in Question 16 they believed they had benefited most from; while in Question 18 they were asked to give reasons for possible non-attendance in lectures.

In order to develop deeper insights into students' experiences at transition from school to university mathematics education, the study implied robust elected methodological principles. These included:

- the principle of 'extended time' survey: in the case of the students, this involved following their development from their first weeks at university (DP1: case observations; individual student interviews; informal talks/interviews with lecturers) throughout their first year and into the second (DP2: as above);
- the principle of 'continuity': selected students were 'followed' inside sessions (e.g. lectures, tutorials) and outside these sessions (informal group discussions/sessions);
- the principle of 'seeing it through the student's eyes': when following the students and observing their 'work environments', data collection was conducted as far as possible through the 'lense' of the students' eyes and their work practices (e.g. observing them in lectures writing, listening, discussing with their peers, etc.);
- the principle of 'reflective investigation': this included discussing what they had written down, or submitted, or said before, in a reflective discussion.

In addition, and in order to counter threats to the validity of the data, and to further strengthen our rigorous data collection across different sites, the teams of researchers/investigators changed in the following ways: always one person was responsible for a particular case, but the second person changed from data point to data point. This allowed each investigator to see different cases 'in situo', and in turn to reflect on his/her own case and its students. In terms of ethics, we adhered to the code of conduct for surveys (observation and interviews) in Norway (NSD), which included student anonymity in questionnaires and voluntary 'opting-in' for interviews.

The methodology clearly has limitations, and in particular our survey. Although we had two investigators at each data point who both had experience with studies of this kind, and we had good rapport with the lecturers at the institutions, we were limited in terms of time and manpower: it was clearly not possible to understand and research the full range of students' experiences. Our survey data also has limitations regarding the longitudinal aspect. At DP 2 all students present at the lecture where the survey was conducted was allowed to take part, regardless of their participation at DP 1. Combined with the option they had to remain unidentifiable, this accounts for the fact that only around 40 % of the surveyed students at DP 1 are identified at DP 2. This also has implications for the significance of comparison of results at the two DPs. Hence, the results cannot be generalised across other sites in Norway, or other subject areas. The extensive nature of our investigations could however deepen our understandings of what students may experience when studying mathematics courses in selected institutions. Our plans are to work with more institutions (e.g. in Norway, and with Finnish and Swedish universities) in order to compare and deepen our results, as well as to develop more detailed insights into selected methodological phenomena. This work has started with colleagues at Manchester university and three Finish institutions.

THE FINDINGS

The 10 different statements in Question 16 can be divided into four different categories related to 1) Lectures, 2) Organized tutorials, 3) Informal self-selected groups, and 4) Use of computer programs.

Questions about the benefits of lectures was measured through three statements which are summarized in Table 2

	Mean	Standard deviation
I benefitted a lot from lectures in mathematics	3.75	1.06
I generally feel that lecturers responded to my needs in the mathematics courses	3.30	1.09
I was able to understand most of what was being taught in the lectures	3.41	1.01

Table 2: Benefit of lectures. 1) Strongly disagree, 2) Disagree, 3) Neutral, 4) Agree, 5) Strongly agree

There were some differences between the institutions. Engineering students at RUE (3 year engineering programme) seemed to report most positively on their benefit from lectures, as 42 % strongly agreed with the statement "I benefitted a lot from lectures in mathematics", and they also strongly agreed with the statement "I generally feel that lecturers responded to my needs in the mathematics courses" (20 %). However, only 14% of students at CU strongly agreed with the former (and 22 % at River UC Teacher education), and 7-8% to the latter. When asked about understanding what is going on in lectures, the percentages were more equal: 14 % of the River UC teacher education students; 11 % of the engineering students; and 6 % of the City U students strongly agree.

How the students felt they benefitted from organised tutorials and working with obligatory exercise hand-ins was also measured through three statements. Results can be seen in Table 3.

	Mean	Standard deviation
I learned a lot of mathematics by working with the obligatory hand-in exercises	4.12	0.97
I learned a lot of mathematics from working with my fellow students	4.04	0.94
I benefitted a lot from the tutorials with teacher assistants	3.41	1.19

Table 3: Benefit of organised tutorials. 1) Strongly disagree, 2) Disagree, 3) Neutral, 4) Agree, 5) Strongly agree

It is noticeable that the mean score on the statements about obligatory hand-ins and working collaboratively with fellow students were considerably higher than the score for lectures. It is also noticeable that the score for benefits of the tutorials with teaching assistants was much lower than the two others. The standard deviation was also much higher. A closer look at the data makes it clear that the RUT (a teacher education programme) pulled the score down; in fact, as many as 41 % of the RUT students checked “Don’t know” on this statement. In comparison, the “Don’t know” percentage was only 5 % at CU and 9 % at the RUE program.

Even if working with obligatory exercise hand-ins was given a high score at all education programs, CU students were those who saw this as most beneficial, as 90 % agreed or strongly agreed to the statement “I learned a lot of mathematics by working with the obligatory hand-in exercises”. For students at RUC the percentage was 78 % and the percentage at RUE was 59 %.

In addition to or parallel to the organised tutorials, students organised themselves in informal or self-governed groups (see Pepin, Lysø, & Sikko, 2012, p. 357). The benefits from working in such groups were measured with two statements (Table 4). The statement about working together with fellow students fits both in the category of organised tutorials and the category of informal group, and is therefore included twice.

	Mean	Standard deviation

I learned a lot of mathematics from working in informal groups with friends and colleagues	3.90	1.10
I learned a lot of mathematics from working with my fellow students	4.04	0.94

Table 4: Benefit from informal groups. 1) Strongly disagree, 2)Disagree, 3) Neutral, 4) Agree, 5) Strongly agree

The RUE and RUT students agreed more to these statements than their peers at CU.

The benefits from working with computers appeared to be quite small. The statement "I learned a lot of mathematics from working with different computer software" was given a mean score of 1.79; whereas the statement "I learned a lot of mathematics by communication on the LMS "It's learning etc." got a mean score of 1.70. The students at RUT and RUE seemed to agree somewhat more to these statements, but across programs and institutions it appears that computers did not add significantly to the students' learning experiences.

It is interesting to note that the statement "I preferred the teaching at upper secondary school to the teaching last year at university/college" got a relatively low score, the mean being 2.73. This indicates that the students did not want the higher education institutions to adapt more to the way teaching was conducted at their previous schools. This can also be seen from the findings at Data Point 1 (see Pepin, Lysø, & Sikko, 2012), where it was found that students appreciated that they had to take more responsibility for their own learning at university.

The three statements that got the highest mean scores were the following: (1) 'I learned a lot of mathematics by working with the obligatory hand-in exercises' - Mean 4.12; (2) 'I learned a lot of mathematics from working with my fellow students' - Mean 4.04; and (3) 'I learned a lot of mathematics from working in informal groups with friends and colleagues' - Mean 3.90. These results point unambiguously towards the fact that students perceive working in groups with hand-in exercises as meaningful activities for their learning. They gave statements concerning these much higher scores than statements about how much they learnt from attending lectures. In terms of open comments, exercises, hand-ins and group work were mentioned 409 times in the open comments.

DISCUSSION OF RESULTS

(1) The findings from the survey provide evidence for the claim that students perceive the most important learning to take place when they are working with the mathematics themselves, and in particular when they are working together with their colleagues in small groups. Concrete comments from students at all three institutions support that. A student at CU claimed that he rarely went to lectures and that the "obligatory hand-in

exercises …these were my main tool for learning mathematics”. In terms of seeking help, the tutorials and informal group work appeared to provide the main support. “I benefitted most from the tutorials, there one could ask and get help when you didn’t understand something.” (CU student 1). “I benefitted a lot from working in a small group with fellow students. To get help, and giving help back to others, this helped me a lot in my learning.” (RUE student 1)

Although the participants in our survey came from different backgrounds (strong mathematics, engineering, teacher education), there were commonalities in their ‘discourse’ and survey answers, in the sense that students largely welcomed collective work and team-based learning. It appeared that they felt safe in these groups where they could ask questions, seek help from and provide help to others. This supports previous research findings (e.g. D’Souza & Wood, 2003) in terms of the benefits of collaborative learning for the creation of an environment of active, involved and goal-directed learning. It also allows students to exercise a sense of control on the tasks they had to perform and is likely to enhance self-management skills. It appears that collaborative learning has an overall positive effect in the cognitive/mathematics learning domain, as well as the social (and possibly affective, domain in higher education mathematics, see (Bardelle & Di Martino, 2012)

(2) Many students claimed that their learning outcome of lectures was low, and lecturers were worried that the average lecture attendance was too low. Asking students about their reasons why they did not attend lectures, a common thread in answers from all students, regardless the institution, was that students felt that the pace of lectures was too fast. As the pace was too high, they could not follow the teachers’ explanations, and as a consequence they felt that they did not learn. Others claimed that they could not follow the lecture and take notes at the same time.

“It was very difficult to hang on during the lectures as the tempo was too high for me”. (CU student 3) “I felt the lectures went along at too high a pace, and I really could not follow the teacher. I have never understood so little in mathematics” (CU student 5). Even those who persisted in going to lectures found themselves at a loss with the mathematics - resilience was not the route to success in terms of learning.

“I attended all lectures, but I often had trouble understanding what was going on. Since then I have studied chemistry and I have had to use a lot of the maths from the maths course, but I soon found that I really had not understood the theories and the methods.” (CU student 4)

As lectures are typically held in large auditoria, students apparently had little opportunities for asking questions. Only when they prepared the lecture, did it seem to make sense and provided positive experiences.

“I felt I had a lot of benefit from working with my pals in stead of attending the lectures, as the explanations were better and it was easier to ask questions”. (RUE student 3)

“I attended the lectures, but I only benefitted from them when I had read about the subject prior to the lecture.” (RUT student 2)

For these students the social and socio-mathematical norms (Yackel & Cobb, 1996) of the lectures they experienced did not help them to understand and engage in the mathematics. Lecturers would need to develop their pedagogic practice, e.g. in terms of more skilled questioning and more active student involvement, in order to offer a diversity of learning approaches for their large and diverse audiences. This may also change student perception of learning in and through lectures.

In conclusion we claim that students are clear how they can learn best- and this is collaboratively and actively engaged. Not ‘remedial instruction’ (e.g. Gamache, 2002), but innovative practices, either face-to-face (e.g. Barton et al., 2012) or by using technology (e.g. Bardelle & Di Martino, 2012; Borba & Llinares, 2012), have proven to be beneficial for ‘re-invigorating’ large-group pedagogic practice, such as lectures, or individual/small group tuition.

ACKNOWLEDGEMENT

This study was supported by funds from Sør-Trøndelag University College, and we are also grateful to colleagues, especially Knut Ole Lysø, and students at both "River University College" and "City University".

REFERENCES

Bardelle, C., & Di Martino, P. (2012). E-learning in secondary-tertiary transition in mathematics: for what purpose? *ZDM Mathematics Education 44*, 787-800. doi: 10.1007/s77858-012-0417-y.

Barton, B., Ell, F., Kensington-Miller, B., & Thomas, M. (2012). Teaching undergraduate mathematics: Perspectives and interactions. Retrieved from http://www.tlri.org.nz/sites/default/files/projects/9285_Summary.pdf

Boaler, J. (2002). The development of disciplinary realtionships: Knowledge, practice and identity in mathematics classrooms. *For the Learning of Mathematics, 22,* 42-47.

Borba, M.C., & Llinares, S. (2012). Online mathematics teacher education: overview of an emergent field of research. *ZDM Mathematics Education, 44,* 697-704. doi: 10.1007/s11858-012-0457-3.

Brown, M., & Rodd, M. (2004). Successful undergraduate mathematicians: A study of students in two universities. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*. (pp. 97-104).

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998). Qualtiatively different experiences of learning mathematics at university. *Learning and Instruction, 8,* 455-465.

Croft, A.C., Harrison, M.C., & Robinson, C.L. (2009). Recruitment and retention of students – an integrated and holistic vision of mathematics support. *International Journal of Mathematics Education in Science and Technology, 40,* 109-125.

D'Souza, S.M., & Wood, L.N. (2003). Tertiary students' views about group work in mathematics. *Educational Research, Risks and Dilemmas – New Zealand Association for Research in Education (NZARE) and Australian Association for Research in Education (AARE) Joint Conference.* Retrieved from http://www.aare.edu.au/03pap/dso03154.pdf

Gamache, P. (2002). University Students as Creators of Personal Knowledge: an alternative epistemological view. *Teaching in Higher Education, 7(3),* 277-294. doi: 10.1080/13562510220144789.

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics*, *67*, 237-254. doi: 10.1007/s10649-007-9100-6

Kajander, A., & Lovric, M. (2005). Transition from secondary to tertiary mathematics: McMaster University experience. *International Journal of Mathematical Education in Science and Technology, 36 (2-3),* 149-160.

Pepin, B, Lysø, K.O., & Sikko, S.A. (2012). Student educational experiences at transition from upper secondary to higher education mathematics. In F.Rønning, R.Disen, H.Hoveid, & I. Pareliussen (Eds.), *FoU i praksis 2011. Rapport fra konferanse om praksisrettet FoU i lærerutdanning* (pp.275-285). Trondheim: Tapir Akademisk Forlag.

Pritchard, D. (2010). Where learning starts? A framework for thinking about lectures in university mathematics. *International Journal of Mathematics Education in Science and Technology,41,* 609-623.

Schoenfeld, A. (1998). Toward a Theory of Teaching-in-context. *Issues in Education, 4(1),*1-94.

Wenger, E. (1998). *Communities of Practice. Learning, Meaning, and Identity.* Cambridge: Cambridge University Press.

Wingate, U. (2007). A framework for transition: Supporting 'learning to learn' in higher education. *Higher Education Quarterly, 61*(3), 391-405.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458-477.