USING "IRDO" MODEL TO IDENTIFY ERRORS MADE BY STUDENTS IN DIFFERENTIAL EQUATIONS EXAMS

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In this action research report, I [1] investigated the errors made by students on their Differential Equations (DE) exams to classify them and develop a model to identify the root causes of their low scores as a result of those errors. Knowing the causes would help the instructor to be more cautious about the roots of the students' errors and it would also allow the researchers to help students overcome their errors and enhance their understanding about DE concepts and type problems. This could improve students' abilities to perform better on their DE exams and develop their self-confidence too. In this study, four types of errors were identified as Identifying, Recalling, Doing and Overviewing (IRDO). Throughout the DE course that the first author taught, he witnessed the benefits of introducing the IRDO model to the class and using it to help the students' performances of their exams in one hand and improve the mathematical communication in class and allow the instructor to offer the IRDO model to his students to prevent them to have a better control on their exams, better organize their work and thus improve their exam scores.

KEYWORDS: *Differential Equations, The exam, errors, IRDO model.*

INTRODUCTION

As a mathematics instructor, at each term, I at least give two exams as midterm and final for the same purpose as determining the depth of students' understanding of what they learned. While grading the exam papers, it was disappointing to see that many students had poor performances on the exams. However, there were convincing evidences to show that the students had reasonable understanding of the DE concepts and those evidences were collected via different sorts of communications in class.

This encouraged the instructor to carry out an action research project to find out the reasons that caused this problem. In fact, the first author conceived the problem more as a mathematics education problem rather than the lack of students' understanding of the mathematics subject. As well, we agree with Barton (2011) that students are responsible for their own learning as well as for the "grade that they receive". Believing on this, we tried to find a way to enable students to take this responsibility and this was another motivation to design the study as an action research one was that the instructor realised this problem through his teaching and he as a teacher/ researcher wanted to solve it to improve his teaching. During the course of study, he realised that even those students who have good understanding of the subject matter, do not necessary have satisfactory performances do to the different types of errors that make on the exams. This is almost the same problem that Dimitric (2012) has observed, and suggests that although analysing the exam papers is “time consuming”, but its benefits are “worth the effort.”

PROBLEM AND PURPOSE

The justification for this study was the gap between the students’ understanding of DE and their performances on their exams. To do this, the study focused on the students’ exam papers and the instructor tried to first understand the roots of the students’ errors based on the evidences on their exam papers, then categorise them accordingly and last, put them together and generalise them into a model for identifying errors that students might make while dealing with DE exams.

LITERATURE REVIEW

Differential Equations (DE) is among the most general and useful courses for all students majoring basic sciences and engineering at the tertiary level. DE courses have different purposes for these students ranging from mere skills to ability for modelling the real life problems. Because of its importance and its necessity in the curriculum of sciences and engineering, some researchers have shown interest to study this topic from different angles. For instance, the ways in which university students understand DE concepts and become skilful in solving its problems have been studied by Arslan (2010), Kwon (2005 & 2002) and Camacho (2012). Their findings is useful for other instructors and interested researches in this field and help them to learn more about new approaches to teaching and learning DEs. However, students’ performances on DE exams have not been the subject of much scrutiny. In specific, students’ errors while writing the DE exams and root causes of them have not been investigated so far. Students’ grades on their exams are extremely important for them. And I think students’ grades and their mathematical beliefs are related. Students believe about understanding DEs related to the grades and the grades are heavily dependent on errors. As well, the real teaching experiences indicate that students’ grades are heavily affected by the errors that they make; errors that are mostly cognitive rather than being the result of carelessness or lack of understanding the DE concepts.

As is paraphrased by Wiens (2007), studying the type of errors in different mathematics fields was one of the research areas that attracted many researchers and is relevant at the present as well and he explains why: First, errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequence of definite processes whose nature must be discovered. Second, it seems to be possible to analyse the nature and the underlying causes of errors in terms of the individual’s information- processing mechanisms. Third, the analysis of errors offers a variety of points of departure for research into the processes by which students learn mathematics (P. 5.)

Thus, identifying and categorising the errors could help the instructor to implement the appropriate teaching strategies that might be useful for students to improve their performances on the DE exams.

The Study was as action research since the first author studied his own DE class since he faced a problem as students' poor performances on their exams due to the different errors that they made in their exams. This action research project was collaborative since the two authors/ researchers reflected on each other's observations and reflections. In this project, everything was documented and the first phase of it completed during the second semester of the 2011-2012. In total, 42 university students who took their first Differential Equation course at the university voluntarily participated in the study. All of them filled the content forms to allow the researchers to use their exam papers as the main sources of data and the content of two exams as data collection tools.

For the classification of errors, all the exam papers were reviewed to identify the errors and after the coding them, four categories of errors were made as the followings:

Identifying error

Mathematicians have three basic classifications system for DE as ordinary, partial, or differential-algebraic and they further could described by attributes such as order, linearity and degree. For each kind, the solution methods and the nature of the solutions, depends heavily on the class of DEs being solved.

While students study DEs, they need to learn to look at a DE and classify it into one of the above three groups. The reason is that the techniques for solving differential equations in each group are common within that group.

Our study showed that the "Identifying errors" usually happen, when students fail to distinguish between different classifications of DEs; namely "First Order", "Second Order", "Linear vs. Non-linear", "Homogeneous vs. Non-homogeneous", and such. Further, "Identifying errors" happen when students are not able to identify specific kind of DEs such as "Separable", "Exact", "Bernoulli", "Cauchy–Euler", "Abel's formula", "Riccati" and etc.

For example, some students were not able to realize that $x(1-x)\,y'' + \tfrac{1}{2}(x+1)\,y' - \tfrac{1}{2}\,y = 0$ is not "Second Order Exact DE" and it is not "Second Order Cauchy–Euler" either, but a "Second Order Homogeneous Linear Equations with non-Constant Coefficients" instead. Therefore, it will be solved by $y_1 = x+1$ and Abel's formula. Here is another example. In fig 1, a student cannot be able to identify that it is second order NOT first order.

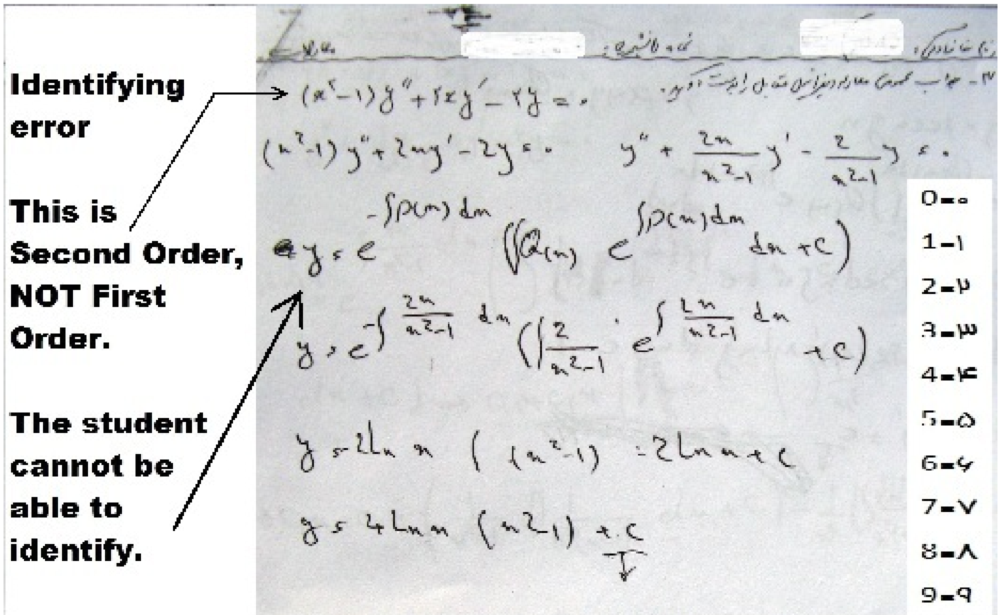

Fig 1: An example of identifying error

Recalling error

It is common to transform a DE of one type into an equivalent DE of another type whenever is appropriate, to be able to use easier solution techniques. Sometimes students have difficulty to make such transformation because of the lack of "Recalling" right things on the right places. We named this situation as "Recalling errors" in which, students are failed to preference easier solution techniques. Incorrect choice of solution techniques, might consume a large amount of students' time and energy and have negative effect on their motivations.

For example, students were asked to solve the following problem:

"In differential equation $(x+y)dx+(x-1)dy=0$ if $y(2)=-\frac{3}{2}$, the amount of $y(0)$ gain."

It has some different solution techniques.

First: Considering $(x+y)dx+(x-1)dy=0$ to the form of $(a_1x+b_1y+c_1)dx+(a_2x+b_2y+c_2)dy=0$ and gain the confluence (x_0, y_0) of two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ regardless of the constant numbers it is solved. Then in the answer, instead of x and y, students should substitute $x-x_0$ and $y-y_0$.

Second: By checking $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$, one might consider $(x+y)dx+(x-1)dy=0$ to the form of Exact DE. Fig 2 is an example of unsuccessful second solution technique.

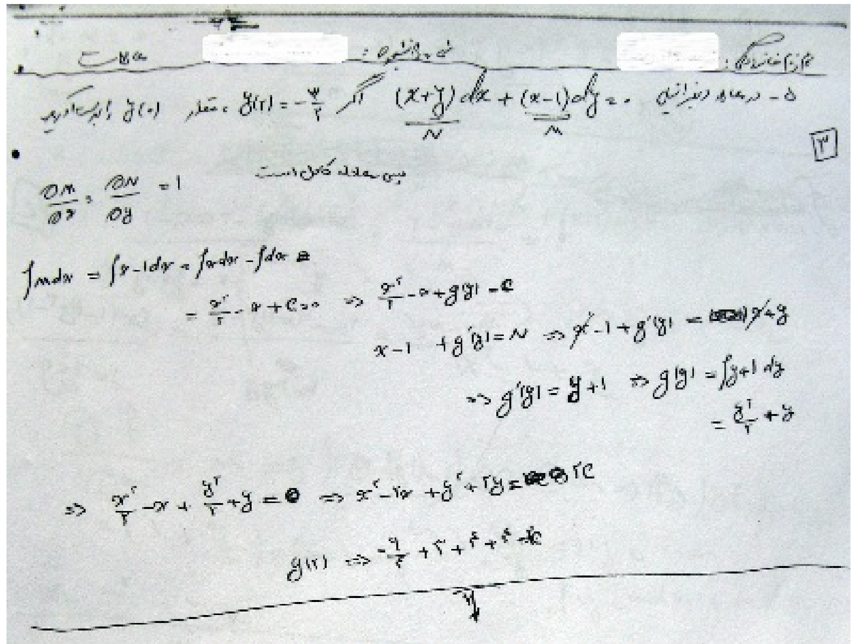

Fig 2: An example of recalling error, unsuccessful second solution technique

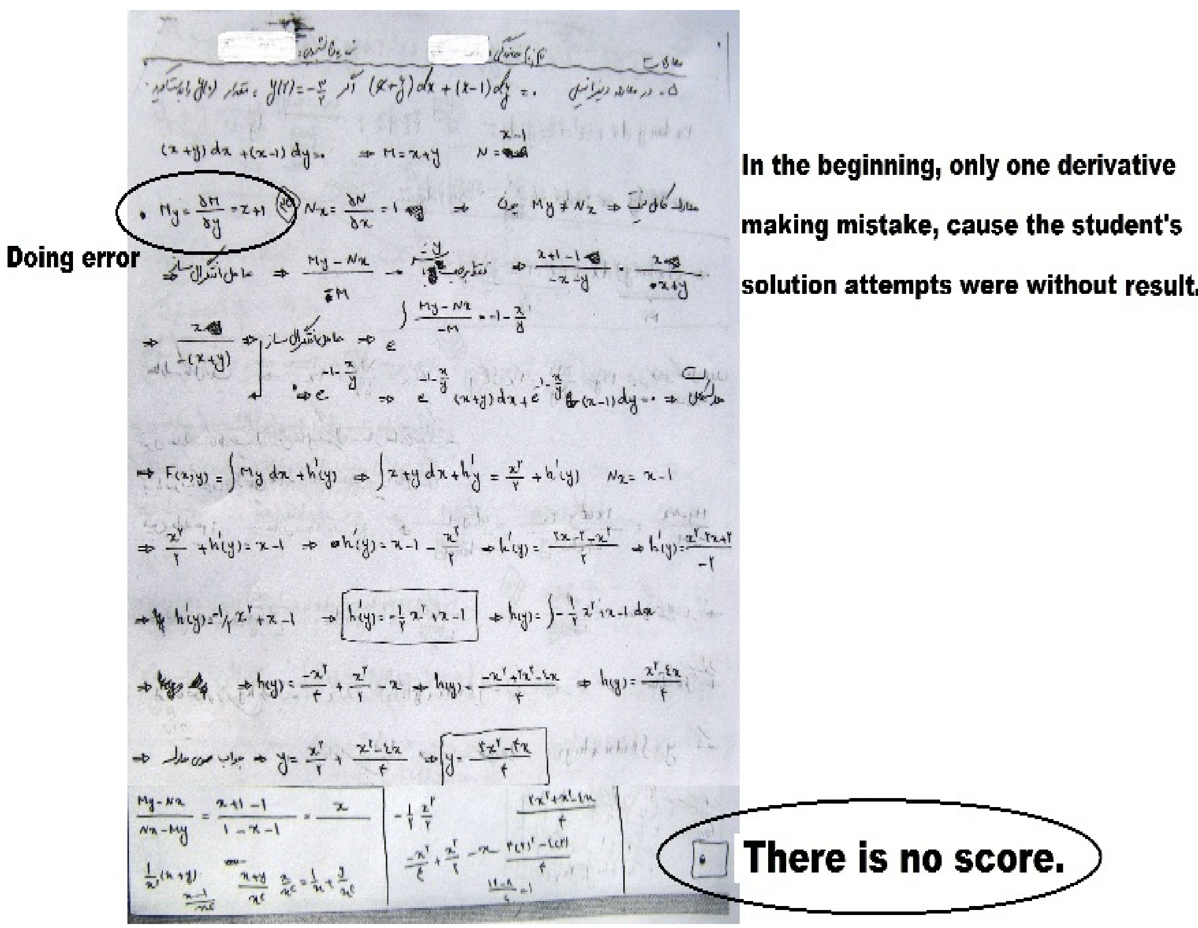

Fig 3: An example of doing error

Third: By transforming $(x+y)dx+(x-1)dy=0$ into $\frac{dy}{dx}+\frac{1}{x-1}y=-\frac{x}{x-1}$. The third is the easiest and the most efficient one considering the time. However, no student solved the problem by this method. Recalling errors are students' inability to recall other solutions apart from the first unsuccessful attempt. Ability to call various solutions can be very useful, especially when the previous solution did not lead to a successful solution.

Doing error

Doing errors involve all kinds of errors that are listed by Eric Schechter as "common errors [2]" and by Paul Dawkins as "Common Math Errors [3]" I address my students with the above lists. Fig 3, in previous page, is an example of doing error. Fig 4 is an example of overviewing error.

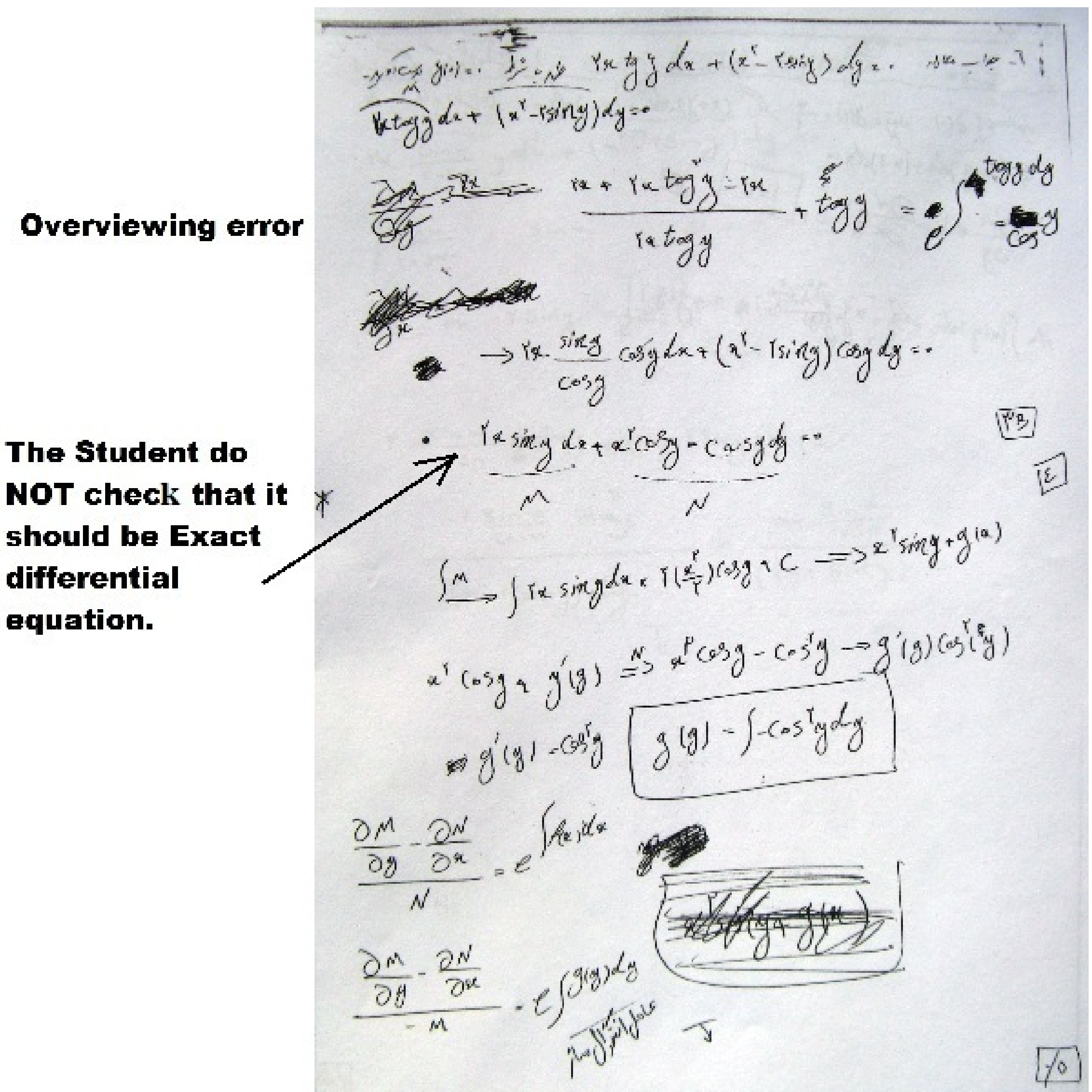

Fig 4: An example of overviewing error

Overviewing error

Long time ago, Schoenfeld (1985) offered a framework for analyzing how and why people are successful (or not) when they engage in problem solving which is still applicable. Schoenfeld believes that the following four factors are necessary and sufficient for understanding the quality and successful problem solving performances as the knowledge base or resources, problem solving strategies (heuristics), control; including monitoring, and self-regulation, or metacognition, and beliefs.

Overviewing errors happen when a student is not able to monitor his/her written exams. Sometimes students do not pay attention to "overviewing their work" as an important phase of their writing exam.

SUMMARY

In this study, we have determined the four types of errors in solving DEs, as Identifying, Recalling, Doing and Overviewing errors or IRDO errors. IRDO is responsible for a large portion of the points lost on the DE exams by many students.

PRILIMINARY FINDINGS

In this paper, the IRDO model was used to analyse the data that were collected through final exam; namely a Differential Equation (DE) course in the fall semester of 2011- 2012. At the beginning, the number and type of errors made by students according to the IRDO model were identified, the student's percentage and scores on the final exam were calculated, and the frequency of each type of error was determined.

The major finding at this stage was that "doing error" had highest frequency and occurred 108 times in the final exam, while "overviewing error" had lowest frequency of 19. Table 1 shows the IRDO errors for each question of final exam.

	Q1	Q2	Q3	Q4	Q5	Q6	Total
Identifying error	5	7	12	13	3	11	51
Recalling error	5	3	17	4	13	10	52
Doing error	21	17	10	22	18	20	108
Overviewing error	3	11	2	0	3	0	19
IRDO error	34	38	41	39	37	41	230

Table 1: IRDO errors frequencies in Q1-Q6, Final DEs exam

There was a correlation between the average score that students received for each question and the number of errors that they made. As it has illustrated in Table 2, the lowest average is 0.48 for Q3 that the frequency of its errors is 41.

1.81	Q1	$y'' + 4y = 4x \to y(0) = 1 \& y'(0) = 5 \to (Laplace\ Transform)$
o.65	Q2	$y'' + y = 0\ (Series)$
0.48	Q3	$(x^2 - 1)y'' + 2xy' - 2y = 0$
1.16	Q4	$y'' + y = \sec x + \tan x$
1.18	Q5	$(x+y)dx + (x-1)dy = 0 \to y(2) = -\frac{3}{2} \to y(0) = ?$
2.11	Q6	$2x \tan y dx + (x^2 - 2\sin y)dy = 0 \to y(0) = 0$

Table 2: Average of Q1-Q6, Final DEs exam

There are several factors contributing to these different types of errors that students make on their exams including attitude, self-management, motivation, self-confidence, and time, and each of them are potentially responsible for the students' incorrect answers.

IRDO errors in Q1 and Q6, the first and the last questions, had lowest frequency. IRDO errors increased from 34 in Q1 to 41 in Q3, decreased from 41 in Q3 to 37 in Q5, and again, increased from 37 in Q5 to 41 in Q6. We would like to speculate that the reason behind these increases and decreases is that students were unfamiliar with Q3 and Q6.

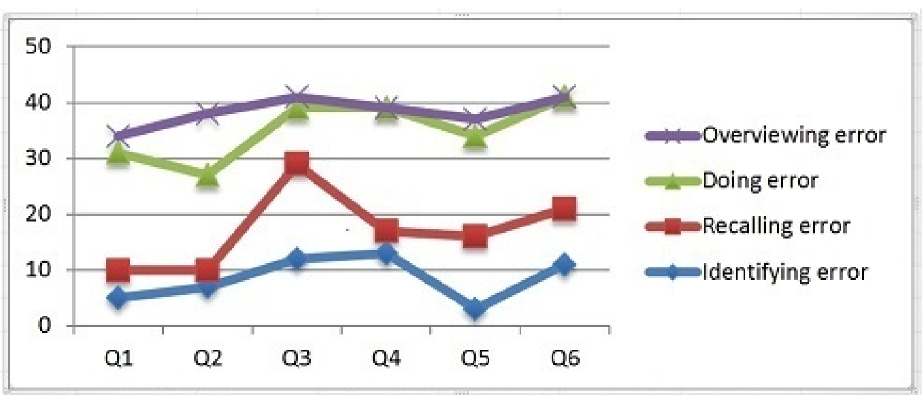

Table 3: IRDO errors, display the trend of contribution

The findings helped to realize that a large portion of the students made different kinds of errors on their exams. So, it is a serious issue to help students who need to pay more attention to what they are doing while writing their exams.

IMPLICATIONS

The PhD proposal of the first author has different phases and in this paper, we reported on the preliminary findings of the first phase or pilot study. Literature review on teaching and learning of the DEs helped us to realize that teaching and learning of the DE goes much more beyond memorizing a set of rules, algorithms, and procedures to solve a set of routine problems. For instance, the DEs could be studied through the use of graphical, numerical and contextual systems of representation as well.

As Artigue (1999) warned us before, “changes are not so easy to achieve; they require time and institutional support”. And she continued to add that “it is not enough to write or adopt new textbooks, the problems are related to… the forms and content of students’ assessment”. As a result of this study, we would like to use IRDO as an Instructional Template, without making any major changes in terms of textbook or assessment. We would like to measure the effects of this tool in another cycle of research project in a DEs course.

“Mathematicians reported extensively the students’ lack of both verbal ability and appreciation for verbal expression in mathematics” (Nardi, 2011). One way of improving the verbal ability might be through opportunities that could be provided for them to practice and identify their errors using mathematical communication (Fardinpour, 2011). One thing that we plan to do for the second phase of the study is to give students time to see how a student solves a problem and ask them to identify what he/she did wrong. We then documenting what will be the effect of using IRDO. We expect that IRDO model would help the researchers to more rationally identify the errors and then, highlight them in the mathematical communication in classroom. Then students’ experiences and skills will improve by mathematical communication in classroom.

NOTES

[1] In this paper, we designed a study research project that was carried out by the first author, and the data were analysed by two researchers. However, we used “I” pronoun instead of “We” whenever the class instances are narrated by the instructor- the first author.

[2] Retrieved at http://www.math.vanderbilt.edu/~schectex/commerrs/

[3] Retrieved at http://www.tutorial.math.lamar.edu/getfile.aspx?file=B,14,N./

REFERENCES

Arslan, S. (2010). Traditional instruction of differential equations and conceptual learning. *Teaching Mathematics and its Applications*. 29. pp: 94-107. Retrieved from: www.teamat.oxfordjournals.org

Artigue, M. (1999). The teaching and learning of mathematics at the university level: Crucial questions for contemporary research in education. *Notices of the AMS*. Volume: 46. Number: 11. p.: 1385.

Barton, B. (2011). Designing alternative undergraduate delivery. *Pre-Proceedings of CERME 7*. Poland. Retrieved from: www.cerme7.univ.rzeszow.pl

Camacho, M. ; and et al. (2012). An exploration of students' conceptual knowledge built in a first ordinary differential equations course (part i). *The Teaching of Mathematics*. Vol. XV- 1. pp. 1–20.

Dimitric, R. (2012). Using students' tests to improve teaching and learning. *Pre-Proceeding of 12th International Congress on Mathematical Education.* COEX. Seoul. Korea. P.: 1270.

Fardinpour, Y.K. (2011). Mathematics teachings' experiences in bilingual classrooms in north-west Iran. *Pre-Proceeding of ICMI Study 21 Conference.* Sao Paulo. Brazil.

Kwon, O. N. (2002). Conceptualizing the realistic mathematics education approach in the teaching and learning of ordinary differential equations. *Proceedings of the 2nd International Conference on the Teaching of Mathematics at the Undergraduate Level*. Hersonissos. Greece. University of Crete. Retrieved from: www.math.uoc.gr/~ictm2/Proceedings/invKwo.pdf

Kwon, O. N. (2005). Towards inquiry-oriented mathematics instruction in the university. *Proceedings of KAIST International Symposium on Enhancing University Mathematics Teaching.* Daejeon. Korea.

Nardi, E. (2011). 'Driving noticing' yet 'risking precision':university mathematicians' pedagogical perspectives on verbalisation in mathematics. *Proceedings of CERME 7*. Poland. Retrieved from: www.cerme7.univ.rzeszow.pl

Schoenfeld, A. H. (1985). Mathematical problem solving. Orlando. FL: Academic Press.

Wiens, A. (2007). An investigation into careless errors made by 7th grade mathematics students. Unpublished MA Theses. Department of Mathematics. University of Nebrasca- Lincoln. Paper 32. Retrieved from: http://digitalcommons.unl.edu/mathmidsummative/32