EVALUATION AND DESIGN OF MATHEMATICS CURRICULA: LESSONS FROM THREE HISTORICAL CASES

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ABSTRACT

Our central question is: Which factors and actors influence the content of mathematics curricula? In answering this historical question we also hope to provide inspiration and references for those involved in curriculum design and evaluation. We will base our answer on historical data from three mathematics curricula from the 17th, 18th and 19th century:

1. The curriculum of the Dutch Engineering School established in Leiden in 1600.

2. The 1756 curriculum for Utrecht orphans, funded from a huge legacy.

3. The curriculum of the HBS schools, established by the Dutch government in 1863.

We test whether the theoretical framework of current curriculum research also applies to these historical case, and sketch a comparative perspective.

HISTORICAL RESEARCH AS A BACKGROUND FOR CURRICULUM DESIGN

We became interested in historical decisions to define or renew the mathematical curriculum because of current and ongoing discussions about this issue in the Netherlands. In discussions and plans the arguments are always shortsighted. They refer to the actual situation and to the needs for the future. Never, or at least not often, crucial episodes in the past enter the scene. Yet, historical research has a valuable set of characteristics which makes it interesting for the curriculum evaluator and designer: the historical process is completed, and therefore it is possible to study the process and at the same time its outcome; also the current observer is not involved herself or himself in the discussions of the past, which makes an unbiased approach easier; and finally the current curriculum in any country in the world is a historical construct. Therefore we expect that historical research will produce results that are important not only for historians but also for curriculum specialists.

A first and superficial analysis reveals that personal preferences of influential individuals play an important role in curriculum development, next to arguments about societal needs. This first analysis brought us to our central research question

Which are the factors and actors that influence to a high degree the content of mathematics curricula?

and in order to structure the research we subdivide this question into the following three:

- Which motivations and whose ideals are influential on the content of the formal curriculum?
- Which factors and which people determine the interpretation of the formal curriculum and its implementation?
- Which factors and which people are important for successful implementation of a curriculum?

Method

First we select three cases of new mathematics curricula in the Netherlands, which were designed in three subsequent centuries (17th-19th). For each case the listed questions will be investigated. We will analyse the data and also make a comparison between the cases. The projected outcome is a historical description, and also a list of conditions, which are important for successful curriculum design and development. Comparison of these conditions with recent curriculum design theory and with developments in mathematics education should result in criteria which are decisive for successful mathematics curricula today. We refer to curriculum studies such as (Goodlad, Klein & Tye, 1979) and (Van den Akker, 2003), and 'borrow' their terminology, especially the following division in stages (or 'domains'):

- the intended curriculum, the ideal of persons who initiated the curriculum and the translation of the ideal into the formal curriculum;
- the implemented curriculum, interpretation and implementation of the formal curriculum, by teachers and through teaching materials;
- the attained curriculum, success (or lack of success) in relation to the students.

The selected historical cases are:

1. The Leiden Engineering School, established in 1600 and affiliated to Leiden University, also known as Duytsche Mathematique, shows the first known example in the Netherlands of a formal curriculum. The school flourished until ca. 1665.

2. The three Foundations of Renswoude, founded in 1756, offered professional education to talented orphans. Mathematics was the main subject, at least during the first two years of their education. The focus is on the Foundation in Utrecht, operational since 1761 and still existing today, although with different goals.

3. In 1863, the Dutch government established the **H**ogere **B**urger**s**chool (HBS), a secondary school for children of citizens who would not enter university, but who were to take up higher technical or administrative positions in society or enter the Polytechnic School in Delft, the present Technical University of Delft.

For each period the data will be presented and analysed separately. They will be combined and compared in the final section, which also presents our conclusions.

THREE HISTORICAL CASES OF CURRICULUM DEVELOPMENT

The curriculum of the Leiden Engineering School (Stevin and Van Schooten)

Leiden had a university since 1575, the first one in what in 1588 would be the Republic of the Seven United Netherlands, a federal state in which the House of Orange played a crucial role. Since 1568 the Dutch were at war with the Spanish king Philip II. The war, combined with strong economic development, led to the need for a type of professional that was not yet existing, the military and civil engineer. Therefore, in 1600, the Dutch leader Prince Maurits of Orange decided to connect to Leiden University an Engineering School called *Duytsche Mathematique*. Its name points at two central characteristics of the curriculum: it was taught in vernacular, i.e. Dutch, and mathematics was the main subject.

The formal description of the curriculum, written by Simon Stevin at the request of Prince Maurits, is still available. The elaboration and implementation by one of the first teachers, Frans van Schooten Sr, is amply documented, as well as the interpretation of Stevin himself in in part 1 and 2 of his *Mathematical Memoirs* (Wisconstighe Ghedachtnissen, 1605-1608).

Prince Maurits asked his former fellow student and later private mathematics tutor Simon Stevin to write an 'instruction' for the teaching at the Duytsche Mathematique. This 3 page manuscript, dated 9 January 1600 and signed by Maurits, is preserved in the archives of Leiden University library. It was published in (Molhuysen 1913, annex 338, 389*-391*) and in a five page pamphlet (published by Jan Paedts Jacobsz, Leiden 1600). Stevin's instruction is truly the intended curriculum, as the excerpt in Figure 1 shows:

Arithmetic: the four operation in whole numbers, rational numbers and decimal numbers, also the rule of three in those three types of numbers

Surveying on paper, that is calculating area with the use of decimal numbers

Measuring a circle, parts of a circle and area, ... learning to subdivide rectilinear figures and curvilinear figures into several parts, such as triangles or other figures, to check calculations

Measurements on paper of dykes and works to learn how many … feet the works contain.

Fieldwork, learning how to use tools properly.

Mapping on paper what is measured in the field and the reverse, from a map setting out stakes in the field

Fortification, learning the names of the parts from wooden or earth models. They will learn to make maps of towns. They will draw on paper the perimeter of fort or towns with four, five or more bastions and stake them out in the field.

Figure 1: excerpt from Stevin's 'Instruction'

The emphasis on practical geometry is striking. Practical geometry also involves the calculation of perimeter, area and volume, so arithmetic is part of the programme, but subjects as the area of circle sectors and measurements on conic sections are not "since engineers will only rarely hit upon these". Note that in the field of arithmetic Stevin prescribes "decimal numbers", which he had introduced himself in his *Thiende* (1585). The teaching of Van Schooten, who was one of the first to use decimal numbers, contributed to their introduction. Stevin also indicates that topics are to be taught both theoretically ("on paper") and in practice ("actual surveying in the field"), which is important since ruler, compass and right angle, useful for work on paper, are to be replaced by the typical tools for fieldwork. Students were expected to spend the summer months to service the army in helping with the fortification works. There is even advice how the theoretical course of arithmetic and geometry should be taught: every lesson to be split in half an hour group instruction and half an hour tutorial in which questions of individual students are answered.

An implemented curriculum soon followed. Stevin's own *Mathematical Memoirs* from the years 1605 to 1608 (Stevin 1608) can be considered as such. Yet, it is quite unlikely that this expensive folio-size book was used directly in teaching. Moreover Stevin's instruction of 1600 is well recognizable in the texts that Van Schooten used in his teaching at the Duytsche Mathematique, and therefore we take these documents, which are mainly in manuscript form, as an account of the implementations in actual teaching. Van Schooten succeeded the two first teachers of the school, Van Merwen and Van Ceulen. Both were appointed at the 10 January 1600 meeting of the Board of Leiden University in which the school was erected and Stevin's instruction was registered. They both served until 1610, the year of their death. By then Van Schooten had already assisted Van Ceulen in the fieldwork.

It took many deliberations and even some petitions by students until in 1615 Frans van Schooten (1581/2-1645) was appointed successor to Van Merwen and Van Ceulen. A series of manuscripts, kept in the university libraries of Leiden and Groningen, indicate how Van Schooten interpreted Stevin's instruction. A comprehensive description with detailed sources, is to be found in (Krüger 2010). The central source that informs us about Van Schooten's teaching is the Leiden manuscript *BPL* 1013, 256 leaves in folio, which bears the title "Mathematische Wercken door F. van Schooten" (… works by …). The manuscript can be dated c.1622; for a discussion of the evidence and further physical properties of the manuscript, see (Van Maanen 1987). *BPL* 1013 discusses arithmetic (extraction of roots, decimal numbers, calculation of area), geometry (definitions and axioms, propositions (Euclid), constructions), surveying (measuring distances in accessible lands and calculations), use of trigonometric tables (measuring in inaccessible lands, making maps, measuring heights (or depths), also measurements without use of tables, c.f. Figure 4), solids (calculations on all kind of shapes and materials, calculating content) and fortification (definitions, plans and calculations).

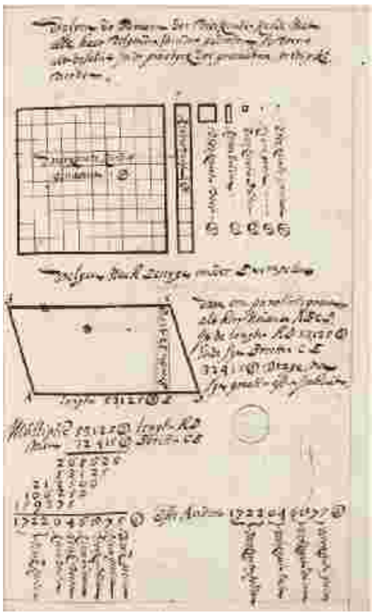

Figure 2: *BPL* 1013, f. 45 r, start of section on surveying

The structure of *BPL* 1013 is clear: each topic starts with an introduction. Van Schooten defines the new concepts (cf. Figure 2, the start of the practice of surveying; also interesting on this page is the use of decimal numbers; it shows the decimal comma as well as circled integer, Stevin's 1585 notation). The introduction is followed by a carefully chosen set of worked problems, in rising order of difficulty. Van Schooten designed these problems in an almost industrial manner. At least the section where he shows how to measure distances and heights contains drawings of landscapes prepared by an artist, before he himself drew the specific configuration of the problem. Some of the landscapes remained empty because Van Schooten had run out of problems (cf. Figures 3 and 4).

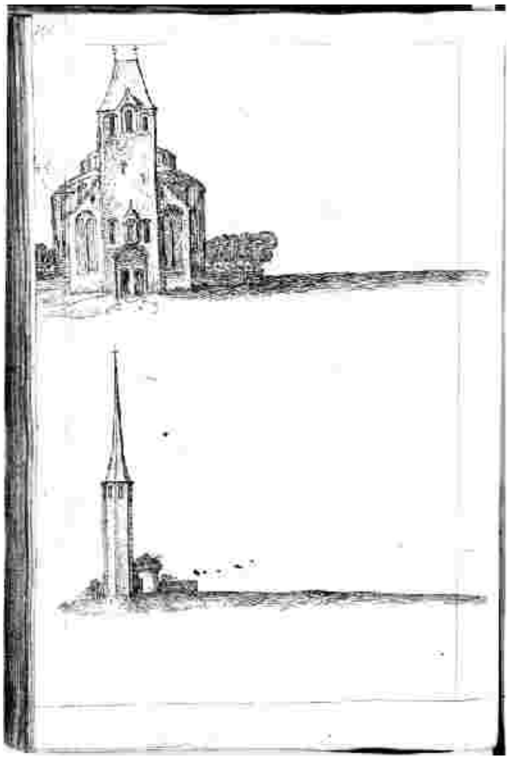

Figure 3: *BPL* 1013, f. 69v, unused diagrams

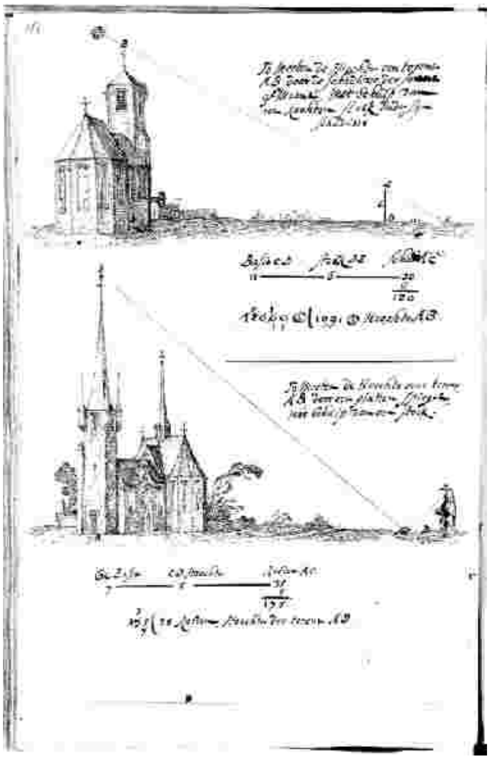

Figure 4: *BPL* 1013, f. 70v, heights

The curriculum of the Utrecht branch of the Foundation of Renswoude

The case that comes up now is a curriculum by private initiative, although eventually three institutes in three different Dutch cities were involved. The wish to establish a mathematical curriculum was a last wish: it was expressed in the last will of a very wealthy woman, Maria Duyst van Voorhout, baroness of Renswoude (1662–1754). The will, signed in 1749 and executed from 1756 onwards, allotted to orphanages in

Delft, The Hague and Utrecht each half a million guilders. These huge sums should afford the orphanages to

> … select some of the most talented and suitable boys, at least 15 years old, to set them apart from the orphanage in order to teach them Mathematics, Drawing or Painting Art, Sculpture or Stone Cutting, practices in building dykes to protect our Country against floods or similar Liberal Arts….” [Utrecht Municipal Archive: *HUA* 771, inv. 1)]

And so it happened, with some variation between the three independent but cooperating institutes, which all three were called after their founding mother “Fundatie van Renswoude” (foundation being the common name for an institution, funded by a legacy). The focus in this article will be on the branch in Utrecht. Its mathematics curriculum was described in (Krüger 2012). In a remarkable way the structure and progress through time of the Utrecht institution parallels the curriculum models of Goodlad et al. and Van den Akker, summarized above.

First, the last will of the Baroness van Renswoude made a concise and precise statement about her aims: promising orphan boys should receive an education in technical or artistic professions, which would respond to the needs of the nation. Mathematics was the first “practice” that the boys should learn. This concise visionary statement can be considered as the intended curriculum. It was elaborated into a formal curriculum by the executors of the testament in cooperation with the regents of the three orphanages. Together they composed an on 17 May 1756 signed “General Regulations”, a document that specified how the visionary statement would materialize. The Regulations describe a variety of aspects: the learning environment and structure of the course, the professions for which it should cater, the subjects to be taught (reading, writing, arithmetic, drawing, principles of mathematics and if necessary French and English language), the accountability of the professionals involved (governors, teachers and Foundation personnel), assessment of the students and also a consultation procedure between the three branches of the Foundation.

The implementation in the Utrecht branch required some strong measures. Up to 1756 few boys mastered reading, writing and arithmetic, so a schoolmaster was appointed to raise the entrance level of the boys. The Utrecht orphanage was so crowded that it had no room for further teaching activities, so a new spacious Foundation house was constructed. In 1761, an “Instruction” for the mathematics teacher was agreed upon. As to the mathematical topics the Instruction was brief: “General mathematics, Military and Civilian Architecture, Surveying, Etc.” The interpretation was left to the mathematics teacher (the “mathematician”). More detail was attached to the procedures of admission and examination (students were to be examined twice a year in the presence of the regents). The mathematician also had to advise on the future profession of a student. In this sense, the intended curriculum focused more on organizational aspects of learning than on its cognitive aspects.

The Utrecht regents made a lucky choice with their appointment of Laurens Praalder. Praalder (1711-1793) was a mathematical practitioner, versatile and widely respected.

He had no academic background, started his teaching career in a region where navigation and building dykes were central topics. In 1751 he was appointed at the Rotterdam Naval School, and in 1761 called to the Foundation. Praalder brought the implemented curriculum, which was still a draft, into practice. He taught at Utrecht until 1792, so for more than 30 years. He had a broad and practical view on the subjects and teaching methods, integrating mathematical theory with physical experiments and practical work with his students, especially in surveying. The duty in finding proper professions for his students was with Praalder in the good hands; he was linked in an extensive and varied network, where he established many useful apprenticeships for his students.

The Instruction had a rather general statement about the topics to be taught. So Praalder faced both the duty and the freedom of choice. He adopted the drafted structure of a course in three phases. Phase 1, which lasted two years and had two examinations per year, was devoted to theory and practical exercises in mathematics, drawing, French language and religion. In Phase 2 Praalder continued the theory but also shifted towards the preparation of the apprenticeship. Phase 3 was the actual internship, often out of town; some students even went abroad. The Foundation paid attention (and money) to providing the students with proper textbooks, which Praalder used next to his own extensive collection of notes; when he retired in 1792, 19 notebooks on a variety of subjects were copied for the sake of his successor.

From 1761 - 1810 the average number of students was 11.5, with a maximum of 15. Their average age was 15. They entered at an age between 12 and 18 and studied and lived within the Foundation for 8.2 years (an average again). A total of 71 boys were admitted, 52 of them completed their studies and started a professional career. Some succeeded well, e.g. as governmental officials, and in general the conclusion is that the Foundation provided education which made a seamless connection with a professional career. The programme was designed as such, and it was effective.

Secondary education for children of citizens, the HBS in the 19th century

In the Netherlands during the first half of the 19th century, secondary education continued to be a matter of private initiative. However booming industry and commerce required a nationally regulated educational system which would cater for subjects such as mathematics and science, languages and economics. Both the Royal Academy for Civil Engineers in Delft and the Royal Military Academy in Breda struggled with students who entered with insufficient knowledge of mathematics and science. The growing middle-class repeatedly proclaimed that it wanted better education possibilities for their children. After several unsuccessful attempts, the liberal statesman J.R. Thorbecke (1798 –1872), who also promoted and formulated the 1848 revision of the constitution, managed in 1863 to introduce the law on secondary education (WMO 1863). The WMO 1863 distinguished several types of school. A really new school type was the Hogere Burgerschool (higher secondary school for citizens) or HBS. It was modelled after the Prussian Realschule. As a consequence of WMO 1863, for the first time in Dutch history the government established and paid a limited number of exemplary national HBS and partly financed a number of HBS established by local councils.

The WMO 1863 defined two types of HBS, a three year school and a five year school. We focus here on the HBS with five year course. Its objectives were to provide general education with science and mathematics as important subjects for sons of the middle class and to provide admission to Delft Polytechnic School. Students with an HBS certificate were exempted of the first exam of the Polytechnic School. The school programme was ambitious, with 18 subjects: mathematics, mechanics and technology, physics, chemistry, natural history, cosmography, Dutch, English, French and German, geography, history, political science, two economic subjects, technical and artistic drawing, writing, and physical exercise. All subjects apart from writing and physical exercise were assessed in a regional final exam. As Latin was not a subject taught in the HBS, the HBS certificate did not give admission to university. Knowledge of Latin remained a condition for university admission.

Only for mathematics the syllabus was prescribed. In his "Explanatory Memorandum", Thorbecke mentioned the topics he deemed suitable for the HBS: (1) arithmetic (continuing where primary school had finished), (2) algebra, including quadratic equations, arithmetic and geometric series and Newton's binomial theorem, (3) plane and solid geometry, (4) trigonometry and goniometry and (5) descriptive geometry up to curved surfaces. This rather full programme was indeed assessed in the final exams, as is shown in the report on examinations of 1887. The mathematics programme of the Polytechnic School consisted of: higher algebra, spherical trigonometry, analytic geometry, descriptive geometry and applications, differential and integral calculus, surveying and geodesy. Thus the mathematics programmes of HBS and of the Polytechnic School fitted together, at least on paper.

At least two other people were influential in the preparation of the legislation. P.L. Rijke (1812–1899), physics professor at Leiden University, advised on the programme for the HBS and wrote a first draft of the law of 1863. He was in favour of experimental physics; Thorbecke, however, favoured a more mathematical physics programme. D.J. Steyn Parvé (1825–1883) taught mathematics at the Athenaeum in Maastricht and published his ideas on mathematics education in the Netherlands in 1850. Some of his ideas on the role of mathematics were very similar to those Thorbecke expressed in his Explanatory Memorandum. From 1858 until 1863 Steyn Parvé worked at the Ministry, where he was a close co-operator of Thorbecke. In June 1863 he was appointed Inspector of secondary education; the Inspectors were very active in supervising and guiding the schools for secondary education.

The HBS with five year course soon became very popular. Quite a number of students who passed their final exams took a course in Latin and gained entrance to university. Four of the five Dutch Nobel laureates received their education at a HBS. The fifth, Van der Waals, who was born in 1837, taught physics at a HBS. The WMO 1863 stated that teachers at the HBS needed a university degree or equivalent. Although there were worries about a possible lack of qualified teachers, the existence of these schools offered employment opportunities for graduates and so encouraged enrolment in a university programme for mathematics and science.

Our collection of data on the design and implementation of the HBS curriculum continues. The influence of supervision (national inspectors and local committees), of regional certificate exams, of the Polytechnic School and possibly the Royal Military Academy and of the financial aspects will receive further research.

A COMBINED AND COMPARATIVE PERSPECTIVE AND CONCLUSIONS

When we compare our findings for the three cases, we see them in this perspective:

- current curriculum theory, as described in the **Method** section, applies well to all three cases. In all cases there was a single person (Prince Maurits, the Baroness of Renswoude, Thorbecke), who translated a societal need into a visionary intended curriculum and who took care of funding. Co-workers took care of the further elaboration of the curriculum. Maurits' vision was materialized by Stevin and Van Schooten, the Renswoude testament turned into a regulation thanks to the regents of the Foundation, and Rijke and Steyn Parvé assisted Thorbecke in specifying and introducing his ideas about mathematics under the new WMO law.
- current emphasis in practical curriculum development differs considerably from the historic cases. In the 19th century the interest in detailed mathematical content is observed already, but in the two earlier curricula there is more emphasis on the societal context of the students. The Renswoude testament went very far in this.
- the attained curriculum is strongly depending on the quality and energy of the teacher. For the HBS case our data should first be analyzed, but the result in the two earlier cases is the merit of inspired teachers (Van Schooten, Praalder)
- in all cases assessment is a crucial part of the whole set-up. For the Duytsche Mathematique we have not paid much attention to the surveyors examinations, but these played an important role in the admission to official functions. In the case of the Foundation the examination system was strict. The regents attended the examinations, which implied that it was at the same time a quality check for the teacher. The HBS directly started with common certificate examinations for each region; here the inspection enters the scene, acting on behalf of the government.

There is clear evidence that vision towards society prevailed in the stage of the intended curriculum, that funding, a good team and trust was crucial in the stage of the implementation, and that the curricula attained results thanks to sound teaching, care for examination and coherence. We intend to be more specific and more operational at a later stage of our research, so that we can turn the above comparative observations into recommendations for the curriculum designer and evaluator.

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