THE USE OF ORIGINAL SOURCES AND ITS POSSIBLE RELATION TO THE RECRUITMENT PROBLEM

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Based on a study about using original texts with Danish upper secondary students, the paper addresses the possible outcome of such an approach in regard to the so-called recruitment problem to the mathematical sciences. 24 students were exposed to questionnaire questions and 16 of these to follow-up interviews, which form the basis for both a small quantitative analysis and a qualitative elaboration of this.

Keywords: Original sources; recruitment problem; view of mathematics; HAPh.

INTRODUCTION

Usually when discussion falls on the use of original sources in mathematics education focus is on the teaching and learning of mathematical in-issues such as abstract mathematical concepts, mathematical ideas and notions, theorems, proofs, etc. (e.g. Jahnke et al., 2000). And it is true that original sources indeed have a lot to offer in this respect – for recent empirical examples, see Clark (2012); Glaubitz (2011); Kjeldsen & Blomhøj (2012). But as occasionally suggested, a use of original sources may have *more* and quite *different* things to offer our educational systems as well. In fact, a use of original sources may be a way of dealing with more general problems such as: *recruitment* of students to the mathematical sciences; *transition* of students between educational levels, in particular between upper secondary level and university; *retention* of students once they have entered the mathematical sciences at university level; and the dimension of *interdisciplinarity* often appearing somewhat artificial to students when implemented in a classroom situation. In this paper, I shall address the first of these, i.e. the recruitment problem – for an elaboration of the three other possible roles of original sources, see Jankvist (preprint) – and I shall do so by relating to empirically collected data from upper secondary students, in order to both strengthen and unfold the claimed role of original sources in relation to this educational problem.

HAPH-MODULES AND ORIGINAL SOURCES

From February 2010 to May 2012, I followed a Danish upper secondary class of 26 students who through two teaching modules were exposed to extensive readings of original sources. An overall purpose of the modules were to introduce the students to aspects of history, application, and philosophy – abbreviated *HAPh* – of mathematics and to do this simultaneously in one module relying on original sources (for more

detailed description, see Jankvist, In press). In the first HAPh-module, implemented in April-May 2010, the students read Danish translations of the following there texts [2]:

- LEONHARD EULER, 1736: *Solutio problematis ad geometriam situs pertinentis*
- EDSGER W. DIJKSTRA, 1959: *A Note on Two Problems in Connexion with Graphs*
- DAVID HILBERT, 1900: *Mathematische Probleme – Vortrag, gehalten auf dem internationalen Mathematiker-Kongreß zu Paris 1900* (the introduction)

The overall theme of this module was mathematical problems, as addressed by Hilbert in his introduction to the 1900 lecture. To make Hilbert's quite general observations a bit more concrete, the students first were to read the two other texts, each of which addresses a mathematical problem. Euler's paper from 1736 addresses the Königsberg bridge problem: how to take a stroll through Königsberg crossing each of its 7 bridges once and only once – today this paper is considered the beginning of mathematical graph theory. With the dawn of the computer era two centuries later, graph theory (and discrete mathematics in general) found new applications. Dijkstra's algorithm from 1959 solves the problem of finding shortest path in a connected and weighted graph – today it finds its use in almost every Internet application that has to do with shortest distance, fastest distance or lowest cost. Furthermore, Dijkstra also discusses a method for finding minimum spanning trees, a problem relevant for the building of computers at the time, and since then used in telephone wiring, etc. (see also Jankvist, 2011).

The second HAPh-module was implemented in September-October 2012, and here the students read Danish translations of the following three texts:

- GEORGE BOOLE, 1854: *An Investigation of the Laws of Thought on which are founded the Mathematical Theories of Logic and Probabilities* (chapters II and III)
- CLAUDE E. SHANNON, 1938: *A Symbolic Analysis of Relay and Switching Circuits* (first parts)
- RICHARD W. HAMMING, 1980: *The Unreasonable Effectiveness of Mathematics*

The title of Hamming's paper made up the theme for this module. Hamming discusses 'the unreasonable effectiveness of mathematics' from the viewpoint of engineering (and computer science) asking why it may be that so comparatively simple mathematics suffices to predict so much, this making up the 'unreasonable' aspect. To provide the students with a possible concrete example of this, they were first introduced to Boole's idea of a two-value algebra – and the context in which this was conceived by Boole in 1854, namely that of trying to describe language (and thought) from a logical point of view. Next, the students were to study a later – to some degree contemporary – application of Boolean algebra by Shannon from 1938. By relying on a set of postulates from the now further developed Boolean algebra ($0\cdot0 = 0$; $1+1 = 1$; $1+0 = 0+1 = 1$; $0\cdot1 = 1\cdot0 = 0$; $0+0 = 0$; and $1\cdot1 = 1$) and by interpreting these in terms of circuits, Shannon is able to deduce a number of theorems which can be used to

simplify electric circuits and thus the building of such considerably (see also Jankvist, 2012a).

EMPIRICAL SETUP

As part of the empirical setup the students were exposed to a series of questionnaires and follow-up interviews during the 2-year period in which I followed them (for specific details, see Jankvist, 2012b). In order to get an indication of the possible effect of the modules and the students' readings of original sources in relation to the recruitment problem, their final questionnaire of March 2012 included the following set of questions:

1. Have the two modules provided you with a different view of what mathematics is; how it comes into being; and what it is used for? If yes, explain how and in what sense. If no, then why not?
2. Did the two modules encourage you to study or in any way concern yourself with mathematics (and/or natural science) after upper secondary school? If yes, how and why? If no, why not?
3. Whether you answered 'yes' or 'no' to the above question (2), do you then consider the two modules to have provided you with a more *enlightened basis* on which to either select or deselect mathematics (and/or natural science) to be part of your future education? If yes, how? If no, why not?

Question 1 above of course concerns the students' beliefs about mathematics as a scientific discipline, which was one of the main objectives of the overall study (Jankvist, 2012b), but which is also relevant for the following reason: Often when students either select or deselect mathematical sciences as part of their higher education, they may in fact be basing their choices on 'incorrect' assumptions. In the panel on empirical research at HPM2012, David Pengelley referred to this problématique as "reality vs. fantasy" [3]. This phrase has to do with students – including upper secondary ones – not having an (accurate) idea of what mathematics is about when practiced as a scientific discipline at the tertiary levels, e.g. by pure and applied mathematicians at universities. As found in Jankvist (2009), upper secondary students' answers to the question of what professional mathematicians do typically range from having no clue at all to believing that they perform some kind of 'clean-up job' consisting in finding 'errors' in already existing formulas and proofs, more efficient ways of calculating already known quantities, etc. Often such views has to do with their belief of mathematics as something a priori given; static and rigid – a belief of course not unrelated to textbooks' usual presentation of mathematical topics. Only very few students seem to believe it possible that mathematicians can come up with actual *new* mathematics. Therefore the students know neither what they accept to study if they choose to engage with the mathematical sciences, nor what they reject to study if they do not. The claim, which I am of course trying to make, is that a study of original sources may provide students with a truer image of mathematics as a scientific

discipline, both pure and applied, because history in general and original sources in particular show mathematics-in-the-making as opposed to mathematics-as-an-end-product (Siu & Siu, 1979), i.e. the usual textbook presentation. Question 3 addresses this aspect by asking the students if they think that the modules enabled them to make their choice on a more enlightened basis. Question 2 is a more straight forward question asking if the modules in any way encouraged the students to pursue a study of the mathematical sciences – or to put it on the edge; if the study of original sources 'attracts' or 'rejects' in terms of recruitment.

I shall split my analysis of the students' answers into two parts: one in which I perform a small scale quantitative analysis of the students' questionnaire answers; and another in which I try to elaborate and deepen this by drawing on the follow-up interviews with the students.

QUANTITATIVE DATA AND RESULTS

Out of the 27 students, 24 answered the final questionnaire. Of these 24 students, 16 were exposed to follow-up interviews. The possible answer combinations of the students are given in table 1. The reason for distinguishing between students who were exposed to interviews and students who were not is that sometimes students would change their answer during interviews. In particular to question 3 (Q3) some students would alter their original answer, since apparently the phrasing in the questionnaire was not entirely clear to all of them. Any such changes are taken into account in the column 'Quest.+Int.'.

Combination			**Quest. + Int.**	**Quest. only**	**Total**
Q1	*Q2*	*Q3*	*16 students:*	*8 students:*	*24 students:*
Yes	Yes	Yes	6	0	6
Yes	No	Yes	7	1	8
No	No	Yes	2	0	2
Yes	Yes	No	0	1	1
Yes	No	No	1	5	6
No	No	No	0	1	1

Table 1: Number of student answers according to occurring combinations [4].

A first observation based on table 1 is that a total of 21 students (88%) agreed to the HAPh-modules having provided them with a different view of what mathematics is, how it has come into being, and what it is used for (Q1). 7 students (29%) agree to the

HAPh-modules having encouraged them to study mathematics or natural science as part of their future studies. Out of these 7 students, 4 already had decided to pursue a higher education related to the mathematical sciences, but the remaining 3 may be characterized as possible 'win over' students. Regarding question 3 (Q3), a total of 16 students (67%) agree to the HAPh-modules having provided them a better foundation to either select or deselect mathematics and/or natural science to be part of their future education. Taking into account that the 15 of these were the ones exposed to follow-up interviews and that 4 of these actually altered their answer during the interview after having been explained the meaning of question 3 more clearly, it is reasonable to assume that the total number could have been considerably higher had all 24 students been exposed to follow-up interviews [5]. The observation that the majority of these students actually answers 'Yes' to question 1 (5+1 students) only supports this further, since the majority of the interviewed students who answered 'Yes' to question 3 also answered 'Yes' to question 1. I shall return to this observation in the final discussion.

SELECTED QUALITATIVE DATA

In order to deepen some of the reported findings, I shall display a selection of students' answers from questionnaire and follow-up interviews in order to illustrate students' rationale behind their answers as well as possible changes in these [6].

Christopher, who represents the 'Yes-Yes-Yes' combination in questionnaire as well as interview, replied to question 1 that the modules provided him with "a different way of seeing things, that it [mathematics] isn't only calculations with numbers". When asked to elaborate in the interview, he replied:

> Well, you can say that what gave me some [insight] was all this philosophy, which lies behind, but also the way in which it has evolved... that it has evolved in order to describe a certain thing; for example that Boole used it to describe one thing, and then Shannon saw, okay, I apply it for this other thing and then develop it according to that. This connection; that it is two completely different things they are working with and they then can use the same [mathematics]... that this mathematics can be applied in so many different contexts. (Christopher, March 29th, 2012)

Regarding question 2, Christopher was already set on studying something related to the mathematical sciences, but he stated in his questionnaire answer that "the modules definitely did not reduce this desire". Also Christopher stated that he simply had been "thinking more" while working with the original texts during the modules, than he usually did when working with the textbook in his regular classes.

Another example of a 'Yes-Yes-Yes' student is Katharine, who provides the following answers. Question 1: "Yes, since here connections are made between problem, solution, and practice, so that we in the end can exploit it. Then you get the 'whole ride' which makes it easier to understand." Question 2: "Yes, because it wakes you up! And you then want to find out how other things were created also." Question 3:

"Yes, because they [the modules] have provided me with an insight in what mathematics at a higher level can do." When asked, immediately after the second HAPh-module, about reading original sources, Katharine said:

> I like the original [texts] better. You kind of get inside the head of those people and think, well that's how they... Because, if there is another one [a secondary source] trying to interpret it, then I feel that they can't really figure out the original, so they take it to a lower level. Whereas I feel that you are challenged more when reading the original [text]. You get to sense how he [the author] has structured it, how he has thought, and so. Of course, it is okay to have the small explanations afterwards on what is meant with this and that... So you see, okay, that was a quick summery, and I understood that. Then you feel that you have won something, because you understand his [the author's] intention, how he carried it through, and so on. That was incredibly exciting. You felt that you got to know them a little more personally and how they expressed themselves using mathematics, explained [things], and so. Also, it gave you ideas on how to express yourself mathematically, in your hand-in tasks etc. I found that very exciting. (Katharine, November 3rd, 2011)

To illustrate the 'Yes-Yes-No' combination and the change of this into 'Yes-Yes-Yes', we shall look at an extract from the interview the student Tobey:

> Tobey: Yes, they [the original texts] gave me an understanding of how you need to think completely different. [...] It has been quite an instructive experience in that regard; kind of an *aha*-experience once you began thinking about it in relation to all the [questionnaire] questions afterwards.
>
> Interviewer: Besides you being surprised due the two modules, did they have any other impact on you?
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> Tobey: What they impacted is that I now may consider, well not to study mathematics directly, but to study something where you use mathematics to more than what you use it for in physics... Because it is a deeper discipline than what you usually think it to be, with just formulas, plus, and minus. [...] There's more to it. It can be applied to several things, at least in relation to these... It would be cool to look at those problems which have been posed, but which have no solution yet... It would be cool to be involved in finding the solution to just a single one of them. It would be completely awesome.
>
> Interviewer: But then you answer ['No' in question 3] here... What I might have hoped is that regardless of you wanting to study math or not, then the modules might have provided you guys with a more... well, done that you could either select or deselect on a more enlightened basis?
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> Tobey: But it has! I mean, after these [questionnaire] questions my answer has definitely changed, because yes, they can do that. (Tobey, March 29th, 2012)

As Tobey, the student Nikita also expressed being more open to possibly pursue a study related to mathematical sciences after the modules than she had been prior to the HAPh-modules:

> I do think that I have got more of a reason to select it, than I had in the beginning [of upper secondary school], because we've seen several different aspects of it [mathematics] due to these modules. If I had only been working with the textbook and so, my answer would definitely have been 'No', I believe... because it's very monotonous and much of the same, whereas with the two modules we've had the opportunity to think differently and view things through different lenses and... Yeah, see the interrelations in a more comprehensive way than we usually get things presented. So, personally I've discovered that there is much more to mathematics than what it says in the textbook. [...] I think it surprised me that someone actually has been sitting and working with these things, and then arrived at this. Because before I've only thought about mathematics as something just being there, and us as just having these and these things which we could make use of. I've never given it a thought that someone had sat down and worked on it and arrived at something to be used in certain contexts... I've never thought about it like that, only in the way that it's in the textbooks and that's just the way it's given. (Nikita, March 29th, 2012)

In both questionnaire and interview, Nikita is quite clear on answering 'Yes' to question 3, almost as if this is implicit to her due to her answer to question 1. Due to Nikita's positive change from questionnaire to interview regarding question 2, she is counted as a 'Yes-Yes-Yes' student in table 1. Regarding her encounter with original sources, Nikita had on a previous occasion expressed herself positive regarding this:

> ... not only did you have to understand what it was about, you also had like the language of it, and it has been a different way of thinking compared to the mathematics we are usually taught, where we have this formula and it works like this, this, and this. Here you got all the background knowledge, and how he arrived at it, etc. For me, I personally think that I get much more interested, when I see it all, than if I'm only told that now we are studying vectors and we must learn how to dot these vectors and then we must be able to calculate a length, right. That's all very good, but what am I to use it for? Whereas, when you know about the background, the development up till today, that I think was exciting. (Nikita, November 3rd, 2011)

As a representative of the 'Yes-No-Yes' combination we shall take a look at an extract from the interview with the student Liza, who is also quite settled on her positive answer to question 3:

> Interviewer: Let us take *you*: You are not interested in studying mathematics?
>
> Liza: No.
>
> Interviewer: No. And after you've followed these modules, do you then feel that you know what you say 'No' to, to a higher degree than if you had not been through the modules?
>
> Liza: For sure, I do.

Interviewer: You do?

Liza: Definitely, yes. I would never have thought mathematics at the university to be about things such as Euler...

Interviewer: As [Euler's] graph theory?

Liza: Graph theory! Or I might have thought, okay, there is something related to graphs. But graph theory, no. So yes, it certainly did provide me with a more enlightened basis. (Liza, May 22nd, 2012)

Another student representing the 'Yes-No-Yes' combination is Salma, who give the following three questionnaire answers. Question 1: "Yes. It has shown me how mathematics develops, and at the same time how mathematicians work with mathematics. And that mathematics is its own language." Question 2: "No, I must admit that it hasn't. I do find it [mathematics] quite interesting, but there are things which excite me more." Question 3: "I have never considered studying mathematics. But if I had, then it would have been nice with these modules, since I feel that you will know much better what you agree to study."

A third representative of the 'Yes-No-Yes' combination is Sophia, who explains her encounter with original sources in mathematics as follows:

> Regarding the modules, even though it has been a little dry from time to time, I do think that it has been nice to get the historical [dimension], to read the original texts, and do it the way they did, the people who developed things. [...] ... in order to get it at a slower pace... to try and figure out 'what the fuck is going on here?' That is, instead of just sitting and doing exercises, which you do in school. To try something completely different, something which might be more similar to what they [the originators] actually did. (Sophia, March 27th, 2012)

DISCUSSION AND RESULTS

Both Tobey and Nikita are representatives of the previously mentioned potential 'win over' students to the mathematical sciences as a consequence of their studies with the original texts in the HAPh-modules. For both of them it seems quite clear that this 'encouragement' to possibly pursue mathematics further is due to the effect of the original texts on their view of mathematics. Nikita gives as reasons the interrelations between different parts of mathematics which the texts reveal, the fact that mathematics has come into being by the hands of human beings, and not least the different way of working when studying an original source as opposed to the regular textbook. Tobey stresses the dimension of creativity in research mathematics and refers enthusiastically to the posed but yet unsolved problems in mathematics (with an implicit reference to the text by Hilbert). Also, Katharine was an example of a 'win over' student, since she prior to the second HAPh-module had no intentions of possibly pursuing the mathematical sciences. Regarding the four other students who gave positive answers to question 2 (table 1), the thing to notice is that the modules –

and the reading of original sources – did not diminish their desire to pursue the mathematical sciences. As illustrated by the quotes of Christopher and Katharine, it may have even enhanced it.

Although the above suggests that a use of original sources may have as an outcome that some students can be encouraged to study the mathematical sciences, the more important finding of the study is that in relation to question 3. Namely, that 2/3 of the students agree to the modules having enabled them to either select or deselect future studies involving mathematics and/or natural science on a more enlightened basis. For the majority of these students this appears to be directly related to the original texts having provided them with a different view of mathematics as a discipline (Jankvist, 2012a). Of course, that a use of original sources can change students' view of mathematics is not a new finding. Still, the present study confirms it once again. What is new in this respect, however, is that this study, although small in scale, suggests a direct connection between a students' positive answer to question 1 (Q1) and a positive answer to question 3 (Q3) [7]. Hence, the more important finding of this study is not necessarily that a use of original sources may 'win' some students over to the mathematical sciences, but that the students who are 'won over' are done so on a more enlightened basis. (Equally important is of course that the students who deselect the mathematical sciences also do so on a more enlightened basis.)

This again has a direct relation to the problem of retention, as mentioned in the introduction. Because if the students who enter the mathematical sciences at tertiary level have a more realistic image of the discipline which they are about to study, then surely one would expect a higher degree of retention among such students.

NOTES

1. The present work has – as part of the STAR-project – been supported by the *European Social Fund* through grant no. ESFK-09-0024. The development of the two HAPh-modules was supported by the *Danish Agency for Science, Technology and Innovation*.

2. The precise references to the original sources may be found in the teaching modules, which are available as texts no. 486 and no. 487 at: http://milne.ruc.dk/ImfufaTekster/

3. The panel on "Empirical research on history in mathematics education: current and future challenges for our field" at HPM2012 in Deajeon organized by Uffe Thomas Jankvist along with panellists David Pengelley, Yi-Wen Su, and Masami Isoda.

4. No answers of the combinations 'No-Yes-Yes' and 'No-Yes-No' occurred.

5. As in any other interview situation, there is always the possibility that the interviewee is trying to please the interviewer by answering what (s)he thinks the interviewer wants. The way of trying to avoid this here was to invite the students to elaborate on their answer to question 3, and when doing so some students would realize more clearly the meaning of the question and change their answer from 'No' to 'Yes'.

6. All student quotes have been translated from Danish.

7. If we follow through with this, we may assume that the 6 'Quest. only' students who answered 'Yes' to Q1 but 'No' to Q3, might have altered their answers had they been explained the question more thoroughly in an interview session. Taking the 2 'Quest. + Int.' students who answered 'No' to Q1 but 'Yes' to Q3 as a source of error, we would get 20 out of 24 (83%) instead of 16 out of 24 (67%). But this is of course to some degree speculation.

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