COMPARING MATHEMATICAL WORK AT LOWER AND UPPER SECONDARY SCHOOL FROM THE STUDENTS' PERSPECTIVE

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As part of a comparative study between how students experience and perceive their mathematics education at lower and upper secondary school, a classroom episode is analysed from a theoretical approach combining key concepts from the anthropological theory of didactics and Bernstein's theory of pedagogic discourse. The findings are discussed with reference to the aim of the overall study.

Key words: mathematical knowledge, transition, praxeology, recognition rules

INTRODUCTION

The overall purpose of the study reported from here is to investigate to what extent and how the studies of mathematics influence students when they apply for a certain programme in upper secondary school and how they experience and perceive the transition from lower to upper secondary school with respect to the teaching and learning of mathematics. In addition to the general relevance of increased knowledge about similarities and differences between mathematics education at the compulsory and non-compulsory school levels, low pass rates on the national tests on the first mathematics course during the first year at upper secondary school in Sweden have been reported, especially on vocationally oriented study programmes (Skolverket, 2012). It is therefore of interest to compare students' experiences of their mathematics studies during this transition process, and rationales behind their choices of study.

Differences between lower and upper secondary school mathematics that students can meet may be related to curriculum issues, differences in styles and focus of teaching or textbooks, kinds of examination tasks and evaluation criteria, pace and work load. Non-mathematical factors such as socio economic background are also related to differences in achievement between students or groups of students. However, this paper sets its main focus on how to identify and describe possible differences in the classroom teaching that may (partly) account for transition problems.

Knowledge in mathematics can refer to different things, such as technical skills in solving tasks by using a certain method, the ability to justify why a method works or to prove mathematical theorems. A common psychological approach for addressing these issues draws on the distinction between procedural and conceptual knowledge. However, to compare the character of the mathematical knowledge as it is being practised within the two different institutions (lower and upper secondary school in Sweden), including criteria for what counts as an accepted knowledge production by the institution, an approach that also takes institutional factors into account is needed. One such tool is provided by the anthropological theory of didactics, ATD (see e.g.

Bosch & Gascón, 2006), within which the notion 'mathematical organisation' (or *praxeology*) addresses different aspects of the mathematical content in for example a school context activity. More general structural issues regarding the distribution of knowledge in pedagogical contexts are described by Bernstein (2000), where some of the key concepts of his theory such as *classification* and *framing* of knowledge, and *recognition* and *realisation* rules, can be used as analytical tools when researching what possibilities the students have to succeed in the mathematics classroom. In this paper, the use of these theoretical tools for investigating the transition will be discussed and illustrated by preliminary empirical data from grade 9 and first year of upper secondary school. We thus address the question about how and to what extent a combination of these two theoretical approaches will support the comparison of how students experience their mathematics studies at the end of lower and the beginning of upper secondary school.

BACKGROUND

In Sweden, children begin at the age of seven in the nine years compulsory school. Almost all students proceed to upper secondary school, which is non-compulsory and consists of 18 different national programmes, theoretically as well as vocationally oriented. The students apply for upper secondary school during the last semester of grade 9 in compulsory school. One of the required qualifications for a national programme is to have passed in mathematics in grade 9. In the core school subjects, such as mathematics, national tests are compulsory in order to set a common national standard. Results from May 2011 show that 18 % of the students in grade 9 did not pass the national test in mathematics (Skolverket, 2012). However the national tests are consultative and 2/3 of those students finally passed in mathematics in grade 9.

There are three versions of the first mathematics course in upper secondary school. Course 1a is studied mainly in vocationally oriented study programmes, course 1b e.g. in the social science programme and course 1c e.g. in the science programme. To acquire a general eligibility for university studies a student has to follow the study programmes including courses 1b or 1c. Results from May 2012 show that around 35% of the students did not pass the national test for the first mathematics course. Of these, for course 1a 48 % of the (around 25000) students did not pass the national test, for course 1b 30 % (of around 30000 students) and for course 1c 9 % (of around 7000 students). (Skolverket, 2012) Since all those students passed in mathematics in grade 9 (although not necessarily on the national test), these results point to a concern about the transition from lower to upper secondary school mathematics.

The different levels of the Swedish school system are regulated by national curriculums, including syllabuses describing 'core content' and 'knowledge requirements' (Skolverket, 2011a, 2011b). While core content is naturally expanded in the first mathematics course in upper secondary school compared to grade 9 in compulsory school, the formulations of the main goals for the subject do not differ considerably between the two school sectors, both emphasising conceptual and procedural proficiency, problem solving and modelling skills, and mathematical reasoning and communication ability. Classroom practice, though, is not regulated in the steering documents and can differ between groups, teachers, schools and school sectors. To understand how the students experience their mathematics studies during the transition stage it is therefore necessary to investigate potential differences in classroom practices with analytical tools that are flexible enough to make a comparison of these practices possible.

STUDIES OF TRANSITIONS

Most of the existing research on transitions between school-sectors deals with other levels than the transition from lower to upper secondary school. At this level, most studies have focused on less domain specific issues such as identities and motivation (in relation to mathematics, see for example Midgley, Feldlaufer, & Eccles, 1989). With a focus on the character of the mathematical work in classrooms, Sdrolias and Triandafillidis (2008) investigate the transition from primary to secondary school geometry in Greece using a semiotic approach, concluding that students' continued construction of mathematically more developed signs may not be supported by the teaching they receive. They observe an increase in logical rigor in secondary school that does not build on children's primary school experiences. In addition, a "rushed move /…/ towards the production of a general law" is seen in teachers at both school levels, with a negative influence both on students' participation and on the "clearness of a mathematical idea" (ibid., p. 167). In Norway, Nilsen's (2012) study employs a semiotic perspective to compare the teaching and learning of linear functions at lower and upper secondary school, concluding that a lack of flexibility in the mathematical teaching, tasks and tests was impeding students' transition towards a more abstract notion of gradient at upper secondary level. The same author also investigated differences and similarities in teachers' beliefs about mathematics teaching at the two school levels (Nilsen, 2009). It was found that teachers at the lower secondary level put more emphasis on "reaching the individual student" (p. 2502), while upper secondary teachers focus more on good explanations and mathematical techniques for students' individual work on tasks.

Research on the transition from upper secondary to university level mathematics has focused on several of its aspects, including differences regarding the content and character of mathematics at the two levels of education. In the Swedish context, Brandell, Hemmi and Thunberg (2008) point to problems created by a gap in topics covered, as well as a change towards a more theoretical mathematical discourse, while Stadler (2009) characterises the transition in terms of how students refer to the different available resources. In an on-going project it has been shown that students' recognition of what counts as the promoted institutionalised mathematics is related to their achievement levels (Jablonka, Ashjari, & Bergsten, 2012). In their study Bernstein's concepts 'classification' and 'recognition rules' have been used to study changes in institutionalised mathematical discourse during students' transition between two levels of education.

THEORETICAL APPROACH

Previous research on transitions has pointed out several features within the receiving institution that seem to be "new" in relation to the students' experiences of mathematics education from the institution they are leaving. These relate to an increase in logical rigour and a more theoretical discourse combined with an increased pace, new mathematical content and work with more advanced mathematical signs, and a teacher focus more on subject matter explanations than "reaching the individual". In order to account for what students experience as different (and possibly problematic) in the passage from lower to upper secondary mathematics education, an analytical tool is needed that makes it possible to analyse the mathematical work at both institutions in a way that makes it open for comparison. Semiotic analyses of the treatment of specific mathematical concepts or ad-hoc comparisons of mathematical content covered will then need to be complemented with a more holistic analysis of the type of mathematical work and knowledge requirements that students experience in the classroom (cf. Nilsen, 2009). This will require a tool that can describe structures in the mathematical work that are *general* across institutions and relevant enough to capture those potential differences that transition research has pointed at, as well as others that have not yet been observed, and *flexible* enough to be applied to both institutions which are steered by different curriculums and different teaching traditions. This would suggest the use of a theory in the sense of Jablonka and Bergsten (2010), with the potential to describe relations between categories and account for aspects not yet observed.

A potential such tool is found within the anthropological theory of didactics, ATD, with its theoretical construct *mathematical organisation* or praxeology (e.g. Bosch & Gascón, 2006). By locating the praxeological analysis in relation to different *levels of co-determination* (ibid.), curriculum and teaching traditions can be adhered to. However, while the praxeological analysis accounts for the organisation of the mathematical knowledge in the classroom but not for the overall organisation and structure of the classroom work, we also need to employ some notions from Bernstein (2000) to be able to analyse differences and similarities along that dimension at the two school levels, which can be captured by 'classification' and 'framing'. These two theoretical approaches are compatible as they both consider institutional dimensions and share the same intellectual roots (cf. Bergsten, Jablonka, & Klisinska, 2010).

In this paper the theoretical construct 'praxeology', a key concept in ATD, will thus be employed to characterise the mathematical work in an activity such as a mathematics lesson: what types of tasks are given to the students and what techniques are used to solve these tasks (the praxis part of the praxeology, or the 'know-how'), what kind of arguments are used to justify the use of these techniques, and what theoretical background these justifications are based on (the logos part of the praxeology or the 'know-why'). For the study, it is of interest to investigate to which degree there is a difference in the character of the praxeologies, for example in terms of balance between the praxis and logos levels, developed in lower and upper secondary school mathematics. To account for constraints on classroom work coming from outside the classroom, such as curriculum, pedagogy and teaching traditions, the ATD employs the theoretical construct ‘levels of co-determination’. The ATD is a general theory in the sense of Jablonka and Bergsten (2010) and has been applied in different subjects and school levels (see e.g. Bosch & Gascón, 2006) pointing to inconsistencies and constraints in educational contexts.

Also Bernstein’s theory of pedagogic discourse (e.g. Bernstein, 2000) is such a general theory that has been applied in different cultural contexts, and it is here used for comparing the level of explicitness of the knowledge criteria for the students: are there differences in the strength of the framing in the two institutions (ibid.)? Furthermore, how do students differentiate between what the mathematically relevant aspects of the tasks are and what is less relevant, such as specific context issues. This dimension of comparison may be described in terms of classification (ibid.). There may also be differences with respect to the balance between *instructional* and *regulative* discourse (ibid.), where the balance between these two constituent parts of the pedagogic discourse is expected to change during the transition to a stronger dominance of instructional discourse.

Similarities and differences between the two school levels may concern what aspects of mathematical work are emphasised in the mathematics classroom teaching. For example, is solving the task enough or is the student also required to explain why and how a method is working? A comparison of this issue will be possible by way of a praxeological analysis combined with a focus on the framing of the classroom work. Furthermore, knowledge of mathematics includes the ability to communicate about the subject, according to the syllabus. One aspect of communication is writing down the solution of a task. To be able to do this in a way that is legitimate within the institution, the student must know what is expected from him/ her, that is, has to be in possession of a recognition rule. The character of the praxeologies, framing of the knowledge criteria and possession of recognition rules are factors that likely influence students’ experiences of mathematics at their respective school level.

AN EMPIRICAL STUDY

The study as a whole will contain analyses of curriculum documents, observations of mathematics lessons at the two school levels, interviews with students and a questionnaire survey. In this paper the main focus will be on the comparison of the classroom work at the two school levels. For this purpose, two mathematics classes were video and audio recorded during three consecutive lessons at two different occasions during the last semester of grade 9. Some of these students volunteered for follow up interviews and this group of students was revisited in their mathematics classes in the first semester of upper secondary school. Thus, three classes, following the mathematics courses 1a, 1b, and 1c, respectively, were also video and audio recorded in a similar way as the grade 9 recordings, along with follow up student interviews.

For the purpose of illustrating how the theoretical approach suggested may support the analysis of empirical data that can be used to study the transition process in focus, an episode from a mathematics lesson in one classroom in grade 9 will be discussed. The teacher-student communication selected here took place when the class was repeating the content of the course before the national test. The lesson pattern is typical for Swedish mathematics lessons, at lower as well as upper secondary level: a main part consists in individual work, where the students solve tasks from the text-book; the teacher walks around in the classroom helping the students, who raise their hands to call for help; teaching in front of the class mainly is done at the beginning of the lesson and for a relatively short time (Skolverket, 2003).

A teacher-student dialogue

A student has asked the teacher for help. The task is to calculate the cost of 0,9 kg shrimps, when the price is 95 SEK per kg. The student has written $95 \cdot 0{,}9$ in the notebook but does not know whether she should use the calculator or do the calculation by hand. The dialogue in focus here is about how to calculate $95 \cdot 0{,}9$ without a calculator. The teacher writes a standard algorithm and starts calculating:

1	Teacher:	Nine times five is forty-five and nine times nine is eighty-one, eighty-five [81 + 4], which gives eight five five, and then one decimal [writes a decimal sign between the fives]
2	Student:	How do you know that? [that it will be *one* decimal]
3	Teacher:	I was going to make another suggestion, but this might answer your question. If you take nine times ninety-five
4	Student:	Mm
5	Teacher:	it will be eight hundred fifty-five.
6	Student:	Mm
7	Teacher:	But you shouldn't take nine [with emphasis] times ninety-five, but zero point nine [with emphasis].
8	Student:	Yes
9	Teacher:	And because zero point nine is ten times less than nine
10	Student:	Mm
11	Teacher:	it means that if we answer like this [855], the answer will be ten times too large.
12	Student:	Mm
13	Teacher:	Yes

Here the teacher switches between describing techniques and providing justifications. In line 1 the teacher explicitly performs an algorithmic technique for multiplication that the student does not seem to question apart from the last expression, the claim "and one decimal", for which the students asks for a ground (line 2). As a response, the teacher then in line 3 starts to present an idea of how to explain the position of the decimal sign. The argumentation from line 5 through lines 7, 9 and 11 is used by the teacher to justify the claim made in line 1, where the warrant is implicitly included in a reasoning about powers of ten, again including calculations.

It is the student who raises the question (line 2) about *how* you know that it will be one decimal, though it seems (from line 3) that the teacher had prepared to warrant his claim some way. This question of justification can be either a question about the technique required to find the number of decimals or an explanation about *why* you can come to that conclusion. Due to the other very short contributions from the student, it is likely to conclude that the question *how* was about ‘how to do it’ rather than to understand. However, the teacher’s immediate answer to the question was to provide a rational explanation. In the continued dialogue the same pattern continues. The teacher provides another justification (warrant) of why the product contains one decimal. When the student raises a question about how to do when the numbers have many digits and decimals, the teacher employs a backing strategy describing a rule that can be used. As this rule is again a technique given without ground, he uses estimation to assure that the answer is correct, employing the qualifier ‘reasonable’. This pattern is typical for the dialogues during the three lessons. One of the students asks for help and the teacher gives an explanation including explanations both of how to do it and why it works. The student’s responses are mainly short, but sometimes s/he asks another question about *how* to solve the problem. In the excerpt the teacher’s explanations are intra mathematical. However, in other discussions he uses models or metaphors when switching between technique and justification. The teacher also uses metaphors, for example ‘common economy’ when explaining addition or subtraction of positive and negative numbers. However, he also mentions sign rules as techniques to help doing the calculations.

Recognition rules

Explicit discussions about knowledge criteria, in the sense of what is expected when you write down a solution to a task, are rare during the three lessons. There is one episode from the beginning of the first lesson when the teacher informs that they will get copies of national tests from earlier years, along with solutions, for practicing: “… and then we will go through them with solutions so you will think of that you not just write answers or what is important to include in a solution”. That is the single part where this is clearly mentioned. There is also one question from a student about if you just should write down the answer after you have completed a calculation by hand. It is not clear whether this question really is about how to present a solution or if it just aims to check if then the task is completed. In summary, explicit information about how appropriate solutions should look like is not frequently offered during the lessons to support students’ development of recognition rules for what counts as legitimate knowledge in this mathematics classroom.

Summary of the preliminary empirical findings

The excerpts show that both mathematical techniques and reasoning at the level of justification are frequent in the dialogues between students and the teacher. The teacher’s explanations often contain different ways of attacking the problem, as well as types of justifications suggested, thus providing a base for a more general praxeology to be developed (cf. Barbé, Bosch, Espinoza, & Gascon, 2005, pp. 237-

238). This strategy may also promote the students to get a broader view of the subject mathematics than ‘just finding an answer’. However, the students’ contributions to the mathematical discussions are mostly short, containing very few questions or conclusions about explanations and do not show signs of engagement in the justification of the mathematical techniques. Hence it is not possible, from these data, to know to what extent the students are aware of the teacher’s endeavours of explaining more than just the technique to find the answer to the task. From a more detailed look into what types of tasks and techniques are used, including those appearing in the examination tests and described in the textbook with justifications offered there, it will be possible to describe the character of the praxeology developed in the lessons (cf. Barbé et al., 2005).

The teaching observed does only to a very small extent explicitly discuss with the whole class the knowledge criteria in the three lessons, thus not providing many opportunities for the students to develop appropriate recognition rules. This is then possible only in the individual discussions with the teacher, or with other students. The data presented above point to a rather weak framing of the knowledge criteria also in such situations. Otherwise there is neither discussion of what is important in mathematics nor discussions about students’ solutions of tasks or standards for written solutions. However, that does not exclude that recognition rules are developed when students work with tasks in relation to solved examples in the textbook, or when they get their result, perhaps with their teachers’ comments, on written exams.

Preliminary analyses from the upper secondary school classroom observations show that there are significant differences between the three classrooms visited. In the class studying course 1a, the praxeology is constituted mainly by the ‘know-how’. The teacher’s explanations are short and mainly inform the students exactly how to solve a task. In the course 1b class there are more explicit arguments why a certain technique works and the framing of the knowledge criteria is visible. In the third class, studying course 1c, there is more teaching from the front, in some lessons up to 30 minutes. Questions from the students are rare, even when they work individually, but when someone asks a question, the teacher’s answer includes both an appropriate technique and its justification.

In the data from the grade 9 classroom presented, the mathematical knowledge in focus during the indivdual discussions between the teacher and the students was strongly classified, though in some explanations metaphors from outside mathematics were used. However, a weak framing of the knowledge criteria was found, indicating that students may develop only weak recognition rules. The preliminary data from the upper secondary school classrooms suggest a more visible pedagogy in the more theoretical courses.

DISCUSSION

Differences between the two school levels that may account for transition problems, or challenges, may be searched at different levels of co-determination. As the general goals and knowledge requirements for the mathematics studies as formulated in the national curriculums are found to be not very different, and the mathematics lessons generally structured by an overall similar pattern, a conceptual framework has been suggested for analysing differences in the organisation of the disseminated mathematical knowledge and the criteria (and their explicitness) for legitimate knowledge productions from the students at the two school levels. By the focus on general characteristics of the organisation of mathematical knowledge in terms of its technical and theoretical levels, and the framing through the teaching practice, it has the potential to detect critical differences, between the two school levels, in the character of the mathematics taught, including the knowledge criteria and their explicitness for the students, as well as differences in pacing and selection and sequencing of the content. The preliminary data indicate such differences as well as differences with respect to the balance between *instructional* and *regulative* discourse as predicted, but this also seems to differentiate *between* the programmes at upper secondary school.

It was also a preliminary assumption behind the study that school mathematics in compulsory school is more "mixed up" with for example everyday contexts than the mathematics in upper secondary school, thus pointing to the relevance of employing Bernstein's notion of classification. The relation between mathematics and other school subjects is also an issue of concern here. Some observations could be done on this dimension already in the preliminary data reported here.

From the rather small empirical basis reported above, it has been possible to characterise the mathematical work in the classroom in terms of the praxeology that seems to be developing, as well as issues related to the classification and framing of the pedagogic discourse, and thus establish links between the empirical data and the key descriptive terms from the theoretical framework suggested. When analysing empirical data from sequences of mathematics lessons at different occasions and in different classrooms at both school levels, the framework has the potential to support the kind of qualitative comparison aimed at here. For the purpose of the overall transition study, students' experiences of the mathematics taught at the two school levels will be investigated, complemented by theory guided analyses of transcripts from student interviews, curriculum documents, textbooks, and examination tests (such as the national tests). By this theory based approach for comparing two different educational contexts, we hope to avoid projecting categories developed within one context on the other which would run the risk of missing out on some critical differences. However, the approach can only work by remaining sensitive to the dialectic between the theoretical and the empirical (cf. Bernstein, 2000, p. 135).

REFERENCES

Barbé, J., Bosch, M., Espinoza, L., & Gascon, J. (2005). Didactical restrictions on the teacher's practice: the case of limits of functions in Spanish high schools. *Educational Studies in Mathematics*, *59*, 235–268.

Bergsten, C., Jablonka, E., & Klisinska, A. (2010). A remark on didactic transposition theory. In C. Bergsten, E. Jablonka & T. Wedege (Eds.), *Mathematics and mathematics education: Cultural and social dimensions. Proceedings of MADIF7* (pp. 58-68). Linköping: SMDF.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research and critique* (Rev. ed.). Lanham: Rowman and Littlefield.

Bosch, M., & Gascon, J. (2006). 25 years of didactic transposition. *ICMI Bulletin*, *58*, 51–65.

Brandell, G., Hemmi, K., & Thunberg, H. (2008). The widening gap – A Swedish perspective. *Mathematics Education Research Journal, 20*(2), 38-56.

Jablonka, E., & Bergsten, C. (2010). Theorising in mathematics education research: differences in modes and quality. *Nordic Studies in Mathematics Education, 15*(1), 25-52.

Jablonka, E., Ashjari, H., & Bergsten, C. (2012). Recognising knowledge criteria in undergraduate mathematics education. In C. Bergsten, E. Jablonka & M. Raman (Eds.), *Proceedings of MADIF8* (pp. 101-110). Linköping: SMDF.

Midgley, C., Feldlaufer, H., & Eccles, J.S. (1989). Student/teacher relation and attitudes toward mathematics before and after the transition to junior high school. *Child Development*, *60*, 981–992.

Nilsen, H.K. (2009). A comparison of teachers' beliefs and practices in mathematics teaching at lower secondary and upper secondary school. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the CERME6* (pp. 2494-2503). Lyon: Institut National de Recherche Pédagogique.

Nilsen, H.K. (2012). Transition from lower secondary to upper secondary mathematics in Norway: Gradients of linear functions. In G.H. Gunnarsdóttir, et al. (Eds.), *Proceedings of NORMA 11* (pp. 435-444). Reykjavik: University of Iceland Press.

Sdrolias, K.A., & Triandafillidis, T.A. (2008). The transition to secondary school geometry: can there be a "chain of school mathematics"? *Educational Studies in Mathematics*, *67*, 159-169.

Skolverket. (2003). *Lusten att lära – med fokus på matematik: Nationella kvalitetsgranskningar 2001-2002*. Stockholm: Skolverket.

Skolverket. (2011a). *Läroplan för grundskolan, förskoleklassen och fritidshemmet*. Stockholm: Skolverket.

Skolverket. (2011b). *Läroplan, examensmål och gymnasiegemensamma ämnen för gymnasieskola 2011.* Stockholm: Skolverket.

Skolverket. (2012). Statistik. Retrieved from www.skolverket.se/statistik-och-analys/

Stadler, E. (2009). *Stadieövergången mellan gymnasiet och universitetet. Matematik och lärande ur ett studerandeperspektiv.* (Dissertation) Växjö: Linnaeus University.