THE NATURE OF ARGUMENTATION IN SCHOOL TEXTS IN DIFFERENT CONTEXTS

Chrissavgi Triantafillou[1,*], Vasiliki Spiliotopoulou[1,*], Despina Potari[2]

[1]The School of Pedagogical and Technological Education (ASPETE),

[2]University of Athens

In this study we explore the substantial argumentation developed in Greek mathematical and science texts in specific topics related to the notion of periodicity. After analyzing a number of texts from both subjects the nature of argumentation was realized in a form of a systemic network. This network presents the complexity of the argumentative activity (the process developed and the modes of reasoning identified) in the different subjects and the tools mediate this process. Finally, by comparing and contrasting the argumentation in two texts that share a closely related thematic content we get some evidence of how the contextual activity via reasoning is shaped.

Key words: argumentation, mathematics, science, modes of reasoning, nomo-logical, logical-mathematical, logical-empirical and empirical inferences

INTRODUCTION

Biehler (2005) argues that the systematic reconstruction of the meanings of mathematical concepts used in different activities remains an important didactical task to be faced: "*All the various spheres of practice in which mathematics is used are in principle relevant sources of meaning in general education"* (Biehler, 2005, p.61). The study of mathematical and pedagogical practices is important as these influence students' conceptions. Two factors are considered critical for the formation of students' pedagogical practices: the textbooks used and the teachers' cognitive and didactical knowledge. In most countries (Greece included) textbooks are used by teachers as the main source for their classroom activities. In textbook writing, meaning is not constructed on verbal language alone, but on the basis of graphical information and the produced argumentation and reasoning, as well. Love and Pimm (1996) denote, that although the implied relation between the reader and the text is inherently passive, "*the most active invitation to any reader seems to be to work through the text to see why the particular 'this' is so*" (p. 371). Chi and her colleagues' (Chi, deLeeuw, Chiu & LaVancher, 1994) research in the science context highlighted the importance of the argumentation developed in textbooks in the meaning-making process. Specifically, they argue that students in order to understand the text material

[*]The current research is part of a research project that is implemented within the framework of the Action «Supporting Postdoctoral Researchers» of the Operational Program "Education and Lifelong Learning" (Action's Beneficiary: General Secretariat for Research and Technology), and is co-financed by the European Social Fund (ESF) and the Greek State.

generate self-explanations, since even quality expositions require the reader to fill in substantial details.

In spite of the crucial role that textbooks play in schooling and educative practices, few research studies have focused on textbook analysis and particularly on the argumentation adopted by the authors in these texts. Stacey and Vincent (2009) by analyzing the nature of reasoning presented to students in Australian mathematical textbooks in specific topics identified the following categories: (a) deductive reasoning (by using a model, or a specific or a general case); (b) empirical reasoning (concordance of a rule with a model and experimental demonstration), (c) external conviction (appeal to authority) and (d) qualitative analogy or metaphorical reasoning. Cabassut (2005) by comparing the reasoning presented in proofs in French and German school mathematics textbooks argues that deductive arguments often occur in conjunction with empirical arguments, presumably to obtain the additive effect.

The work presented in this paper is part of a research project that aims to identify epistemological and didactical aspects among different educational practices concerning the concept of "periodicity". Periodicity is an essential scientific concept because it plays a central role in the school curriculum and is expressed in different educational fields where it acquires practical importance. By adopting the position that argumentation and concepts are interwoven inside a text, we analyze the nature of reasoning developed in topics related to periodicity in math and science textbooks. The argumentation developed in school texts in different subjects for a common topic is rather limited. Analysing the argumentation produced in this case is didactically important since these texts are addressed to the same student who is 'responsible' for making the appropriate conceptual connections.

Our specific research questions are:

- What is the nature of argumentation that is employed in the textbooks to support the meaning of periodicity?
- How is argumentation differentiated in the mathematical and scientific context?

THEORETICAL FRAMEWORK

We adopt Vergnaud's (2009) theory of conceptual fields that addresses the process of conceptualization of reality. It is a pragmatic theory as it presupposes that knowledge acquisition is shaped by situations, problems and actions for the subject. It is, therefore, through the situations that a concept acquires meaning to a student. Vergnaud considers that a concept is a triplet of a set: **C=(S, I, L)** where **S** stands for the set of situations which give sense to a concept (*the referent*); **I** stands for the set of operational invariants associated to the concept (*the meaning*); **L** stands for the set of linguistic and non-linguistic representations which allow for the symbolic representation of a concept, its attributes, the situation to which it applies and the procedures it nourishes. In this paper, the thematic units where the concept of periodicity appears in school texts are considered as situations (S); the argumentation developed by the author of the textbooks in these units as operational invariants (I) as well as the rules that generate the reasoning activity for the establishment of new knowledge; the tools employed by the author in the argumentation process as linguistic and non-linguistic representations (L).

Since our interest is on argumentation techniques or methods used in textbooks to reason about the presented new knowledge we are interested in what Toulmin (1969) calls 'substantial argumentation' (p. 234). Substantial argumentation does not have the logical stringency of formal deductions but is used for gradual support of different statements. Toulmin establishes the importance of practical arguments and their logical canons, which may not be entirely safe as formal mathematical arguments, but are necessary tools of thinking in general. Argumentation here is taken to mean the use of reasoning for the construction of knowledge presented in a text for the purpose of convincing the students of the truth of a conclusion. This is considered to affect students' ways of understanding and conceptualizing the field.

Argumentation and reasoning in mathematics and in science context

The argumentative process that mathematicians develop to justify the truth of mathematical propositions, which is essentially a logical process, is usually called formal proof. Balacheff (1991) makes a distinction between argumentation and mathematical proof since the argumentation could include intuition, experimental methods or everyday practices taken from outside a mathematical theory. In our paper, since formal proofs are very rare in the topic of periodicity even in the context of mathematics (Dreyfus & Eisenberg, 1980), we consider them as part of the argumentation developed in a text.

Szu και Osborne (2012) claim that arguments in the science context may be either deductions about the world based on a set of a priori premises; inductive generalizations when reasoning is typified by laws; or inferences to the best explanation as in Darwin's development of evolutionary theory. Daily life reasoning is characterized as informal since people draw inferences from uncertain premises and with varying degrees of confidence (e.g. Over & Evans, 2003). In contrast, scientific reasoning is based on experimental verification and it has a validating intention which leads to generating scientific knowledge. This type of reasoning is characterized as empirical or pragmatic (e.g. Recio & Godino, 2001; Cabassut, 2005).

Stinner (1992) classifies the knowledge provided in science textbooks in three planes: (a) The logical where he encounters the finished products of science, such as laws, principles, models, theories, and the mathematical and algorithmic procedures establishing them; (b) The empirical where he encounters the experimental, intuitive, and experiential connections that support the logical plane and (c) the psychological where he encounters relations to students' prescientific knowledge. Kuhn (1962) in his influential work, '*The Structure of Scientific Revolutions*', claims that textbooks are pedagogic vehicles for the perpetuation of science and argues that frequent contact between the logical-mathematical and the evidential-experiential levels of activity is promoted through problem-solving activities.

Harel and Sowder (1998) defined empirical proof schemes (inductive and perceptual), analytical proof schemes (transformational and axiomatic) and external convictions in order to characterize students' argumentation in mathematics. Especially, the perceptual proof schemes are made by means of 'rudimentary mental images' and there is no evidence that 'operational thought' is taking place (p. 255).

METHODOLOGY

A grounded theory research approach (Strauss & Corbin, 1998) is adopted in this study. Our methodological framework is based on the qualitative inductive content analysis. Moreover, the technique of systemic networks (Bliss, Monk & Ogborn, 1983) has been adopted not only as a form of representing our scheme of categories, but also as an analytic tool. In particular, we aim to produce a quantitative elaboration of the arguments which underlie the text in 11 Greek textbooks on topics related to the notion of periodicity.

The sample: The texts analysed are taken from the subjects of Mathematics, Physics and applied technologies (Electrology, Electronics and Informatics) used in Greek lower secondary and upper secondary General and Vocational school. In each textbook we restrict our analysis to topics that are related to periodicity. Specifically in Mathematics the topics are trigonometry and periodic functions, in Physics the topics are related to Periodic phenomena (e.g. oscillations, simple harmonic and circular motion) while in applied technologies the topics are related to Alternate Currents. In order to implement our analytic plan, we divided the text into units of analysis by restricting analysis to all the parts which aim at delivering mathematical and scientific knowledge (we did not include worked examples, exercises and historical notes). In this paper the word 'text' is used to denote a section of textbook material and the accompanying visual representations.

Unit of analysis: Our unit of analysis is every conceptual thematic unit that has an independence from the rest of the text and produces an argumentation. It is conceived as a part of the topic that we analyze; it has a beginning and an end; and has a relative independence in its content: we can identify it and distinguish it from the other units. Each unit of analysis is characterized by its thematic content (e.g. "Define periodic function" or "Define periodic motion" or "Describe the generation of alternate current") which is organized in a particular way. One unit of analysis several times coincides with a textbook unit as it is defined by the author. But in some cases we have to split the textbook unit in more units of analysis when a change in its thematic content and the argumentation produced is identified. After defining each unit of analysis we separately analyze the structure of the argumentation developed it terms of its process and its ingredients (parts). The process of argumentation is realized as a sequence of interdependent and logically connected statements. So, a secondary unit of analysis is chosen, expressed by a sequence of sentences. This usually corresponds to one or more paragraphs and the accompanying visual representations, and supports the generation of argumentation developed in the unit. Semantically, in each unit of analysis we can identify different explanations, justifications and/or proof of new knowledge. We call these types of reasoning as ‘modes of reasoning’ as this term is used in Stacey and Vincent’s (2005) study.

Analysis of data: Subsequently, we analyze the kind of modes of reasoning applied and the tools mediated this reasoning. These tools are in the linguistic (i.e. verbal language) or non-linguistic form (i.e. physical models or mathematical representations). Within a feedback loop the codes developed and negotiated among the researchers. Those codes were revised and eventually reduced to main categories and checked in respect of their reliability. As a result, categories and subcategories were formed and their interrelations were recognised by matching our emerging classification to our data. Finally, the nature of argumentation was realised in the form of a systemic network (Bliss, et al. 1983), presented in Figure 3.

RESULTS

In the first part of this section we exemplify our analysis in two texts. At the end we compare the nature of the argumentation developed in each case. The texts are from the subject of science and mathematics and share a closely related thematic content. In the second part we present in the form of a systemic network the structure of the argumentation as it developed after analyzing a number of texts from the above subjects.

Examples from texts

We present below a text from the subject of Physics. The text is from the topic ‘Oscillations’ and its specific thematic content is: “Define periodic motions”


ΠΕΡΙΟΔΙΚΕΣ ΚΙΝΗΣΕΙΣ

Figure 1: The text

“When you were younger you would have got into a swing many times or you would have even noticed the other kids playing with it. The swing has a high starting point, goes up and down and back to its starting point and keeps on moving in the exact same way. The yo-yo is a popular game, widely used in many countries in the world (maybe you have played with it several times). You hold the string from the one edge and you let the circle move. The string winds and unwinds around the spinning axle several times in exactly the same way.

The movements of the swing or the yo-yo are examples of ***periodic motions. This means that they are motions that are repeated at equal intervals.***

The normal circular movement of the Sun is periodical as well as the movement of the Earth round the sun, which is repeated annually. The muscle of the heart performs a periodic motion as presented at the electrocardiogram"

(Physics, 3rd Grade of Lower Secondary School. (2008). Athens, Greece: OEDB, p. 89)

In the first paragraph two examples of periodic motions are presented. The examples are taken from everyday life (the swing and the yo-yo game). These properties that seem to characterize every periodic motion are presented in the key phrases: "moving in exactly the same way" or "repeated at equal intervals". Finally, in the third paragraph additional examples of periodic motions are provided. The process of argumentation was realized as follows: introducing two special cases - providing scientific generalisation with the aim of solidifying the scientific truth - giving more examples in order to reinforce students' understanding. Therefore, the process of argumentation is forming the following pattern: Special$\rightarrow$General$\rightarrow$Special. The parts of this process are the modes of reasoning applied i.e. *empirical inferences* based on every day experiences (when moving from the special to the general case), *nomo-logical* inferences when defining periodic motions and finally *logical-empirical inferences* when starting from a general idea of logical type (the definition) and ending up by implementing it in certain empirical situations.

We present below a text from the topic of Trigonometry. Its thematic content is: "Define a periodic function and its period".

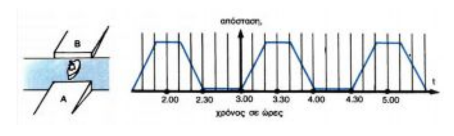

Figure 2a

"Suppose that a ferry travels between two ports, A and B, and the graphic representation of its distance from port A as a function of time is presented in the following graph [Figure 2a]. *We notice that every 1 and 1/2 hour the ferry repeats the exact same movement. This means that in whatever distance it is from port A in some time (t) it will be at the same distance at the time (t+1½) hours and it was at the same distance on the (t-1½) hours. Consequently, the function that presents the distance of the ferry from port A, in respect to the variable t takes the same values at t, t + 1½ and t-1½ .We suggest that this function is periodic with a period of 1 ½ hours.*

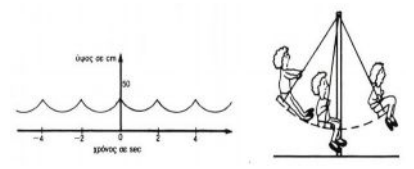
Figure 2b

The following graph [Figure 2b] *is a graphic representation of the height of the swing as a function of time (t). We notice that despite the height of the swing in a certain moment (t), it will have the same height at the time (t+2)s as well as at (t-2)s. We say that the function (that models the height of the swing with respect to t) is periodic with a 2 sec period.*

In general: A function f with domain the set A is called periodic, when there is a real number $T > 0$ *so as for every* $x \in A$*: i)* $x + T \in A$, $x - T \in A$ *and ii)* $f(x + T) = f(x - T) = f(x)$. *The real number T is called the period of f."*

(Algebra, 2rd Grade of Upper Secondary School. (2012). Athens, Greece: OEDB, p. 73)

Examples of periodic functions are presented in the first two paragraphs while in the third paragraph the definition of periodic function comes as a generalization of the two examples. Hence, the structure of the argumentation in terms of its process is inductive since it moves from special cases of the situation to a general case (S_1, S_2 $\rightarrow$G). The modes of reasoning identified are based on empirical observations on mathematical models of periodic motions. These observations were 'evidence' identified in the graph representations where the reader must 'spot' the specific points on the graphs, identify patterns and end up in a general conclusion. We classify this reasoning as *logical-empirical inference* since reasoning starts from empirical situations and ends up in a general conclusion. Finally, we acknowledge this conclusion as a *nomo-logical inference* that emerges as a result of previous generalizations.

Comparing the structure of argumentation in the two texts

Both texts refer to periodic motions and the functions that model their behaviour. The structure of the argumentation developed in mathematics and science differs in its process (S_1, S_2 $\rightarrow$G$\rightarrow$S) and (S_1, S_2 $\rightarrow$G) and its parts (the modes of reasoning employed). The modes of reasoning identified when moving from the special to the general cases were empirical in the science text and logical-empirical in the mathematical text. Although a common example was used in both texts (the periodic motion of a swing) different modes of reasoning were employed. Particularly, the empirical inference in the science text engages the reader in a holistic perspective of the periodic motion while the logical-empirical inference in the mathematical text engages the reader in a point-wise perspective (Van Dormolen & Zaslavsky, 2003). Moreover, the definition of periodic motions in the science text comes as a generalization of verbalized properties while the definition of periodic functions in the mathematical text comes as a generalization of mathematical and symbolic properties. Some of these differences could easily be explained due to the difference in readers' school level (different school grades), while some others characterize the context in which each argumentation is developed.

The systemic network

After analyzing a number of textual units (this research is still in progress) in our attempt to synthesize our results, we present the nature of argumentation developed in a systemic network (Fig. 3). The BAR ([) notation signifies that all the categories are mutually exclusive, whereas the BRA ({) notation signifies that any number or even all of the categories can be selected simultaneously.

The structure of the argumentation was characterized in terms of its process and its parts. The parts of the argumentation are the modes of reasoning acknowledged in this process. These two dimensions are viewed in interrelation. Four processes were identified: (a) Moving from special to general case and then exemplifying. (b) Moving from examples to general case (c) moving in the inverse way i.e. from general to special cases and (d) deductive reasoning i.e. remaining in the general case through out the entire textual unit.

The parts of the argumentation were acknowledged in terms of the kind of modes of reasoning applied by the author and the tools that mediated this reasoning. The modes of reasoning identified are in the following categories:

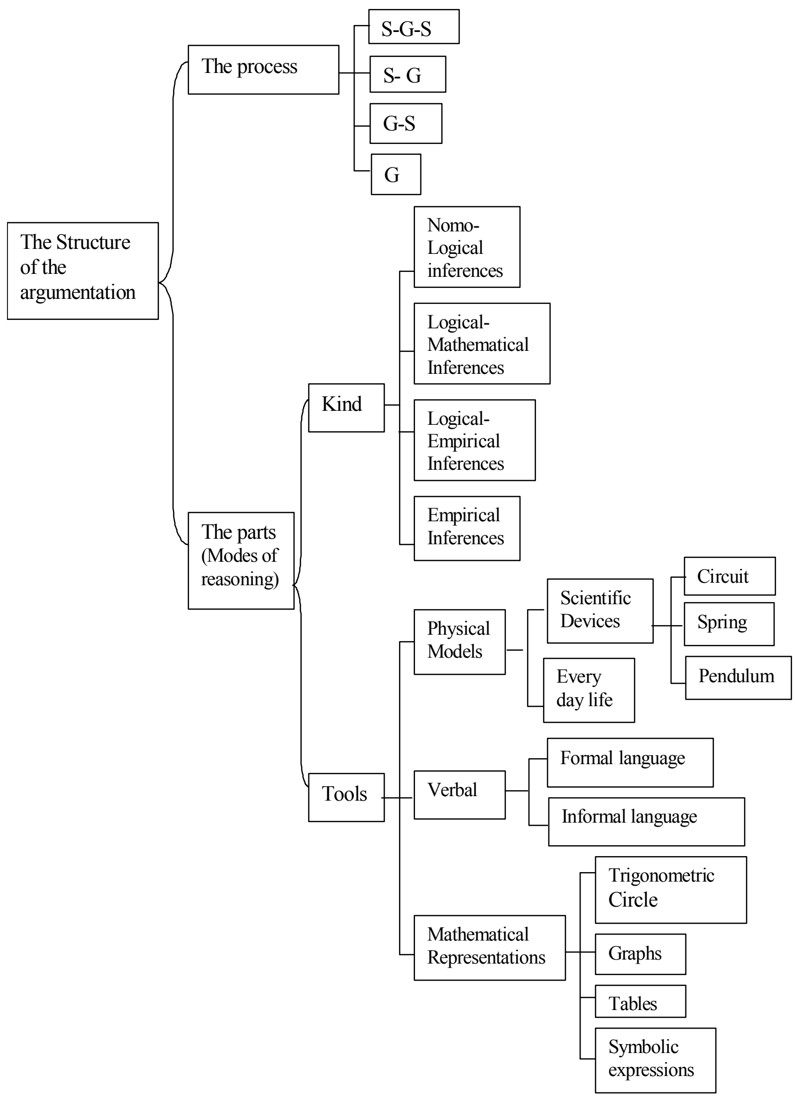

Figure 3: The systemic network

- *Nomo-logical inferences* when a definition or a law emerges as a result of previous generalizations.
- *Logical-Mathematical inferences* when reasoning is based on mathematical relations and techniques.
- *Logical – Empirical inferences,* when reasoning either starts with general statements and ends to specific situations (as in the science text) or the other way around (as in the mathematical text).
- *Empirical inferences* when they are based on experiences either from everyday life or from experimental activity.

Different tools mediate the reasoning process such as physical tools (either scientific devices or every day life); mathematical representations (i.e. the trigonometric circle, graphs, tables and symbolic expressions) and verbal (i.e. informal or formal language). Finally, all the above categories and subcategories were associated and interrelated in the argumentation process.

CONCLUDING REMARKS

In this study we explore the nature of substantial argumentation developed in Greek mathematical and science texts in specific topics related to the notion of periodicity. In order to implement our plan we developed a methodology of analyzing the argumentation developed in texts from different subjects. Our methodology defines two units of analysis: the conceptual thematic unit and the mode of reasoning. In this way we can compare the argumentation, the generative activity and the tools mediated this argumentation in different texts in different grades and/or subjects.

After analyzing a number of textual units from the subjects of science and mathematics, the nature and the structure of argumentation were realized in a form of a systemic network. This network presents the complexity of the argumentative activity and its ingredients in the different contexts. The modes of reasoning as parts of the argumentation process are realized in the form of nomo-logical, logical-mathematical, logical-empirical and empirical inferences. These inferences are mediated through a number of linguistic and non-linguistic tools. All these tools incorporate aspects of the notion while at the same time nourish the argumentation process. Finally, by comparing and contrasting the argumentation in two texts that share a closely related thematic content we get some evidence of how the contextual activity via reasoning is shaped in different subjects and in different grade levels. Particularly, through our analysis, we spotted differences in the argumentation produced and the tools that mediate it that could illuminate aspects of the notion in different ways.

REFERENCES

Balacheff N. (1991). The benefits and limits of social interaction: the case of mathematical proof *Mathematical knowledge: its growth through teaching*, pp.175-192, Kluwer Academic Publishers, Alan J. Bishop et al. (eds).

Biehler, R. (2005). Reconstruction of meaning as a didactical task: the concept of function as an example. In J. Kilpatrick, C. Hoyles, O. Skovsmose, P. Valero (Eds.) *Meaning in mathematics education*. Mathematics Educational Library V. 37; Springer. New York.

Bliss, J. Monk, M. & Ogborn, J. (1983). *Qualitative data analysis for educational research*. London: Croom Helm.

Cabassut, R. (2005). Argumentation and proof in examples taken from French and German textbooks. *Proceedings of 4th Cerme* (Congress of European society for research in mathematics education). Sant Feliu de Guixols. Online http://cerme4.crm.es/Papers%20definitius/4/wg4listofpapers.htm.

Chi, M. T. H., de Leeuw, N., Chiu, M., & LaVancher, C. (1994). Eliciting self-explanations improves understanding. *Cognitive Science, 18,* 439-477.

Dreyfus T. & Eisenberg T. (1980). On teaching periodicity *International Journal in Mathematics. Education. Science & Technology*, 1980, Vol. 11(4), 507-509.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from an exploratory study. In A. H. Schoenfeld, J. Kaput & E. Dubinsky (Eds.), *Research in College Mathematics Education III* (pp. 234-283). Providence, RI: AMS.

Kuhn, T. (1962). The Structure of scientific revolutions**.** Chicago, IL: University Press

Love, E., & Pimm, D. (1996). 'This is so': A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer Academic Publishers.

Over, D. E., & Evans, J. St. B. T. (2003). The probability of conditionals: The psychological evidence. *Mind and Language, 18*, 340-358.

Recio, A. M., & Godino, J. D. (2001). Institutional and personal meaning of proof, *Educational Studies in Mathematics*, 48, 83–99.

Stacey, K. & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics*, 72(3), p. 271-288.

Stinner, A. (1992). Science Textbooks and Science Teaching: From Logic to Evidence. Science Education, 76(1), 1-16.

Strauss, A. & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Szu, E & Osborne, J. (2012). Scientific Reasoning and Argumentation from a Bayesian Perspective , *Perspectives on Scientific Argumentation*, Part 1, 55-71.

Toulmin, S. (1969). *The uses or argument.* Cambridge, UK: Cambridge University Press.

Van Dormolen, J. & Zaslavsky, O. (2003). The many facets of a definition: the case of periodicity. *Journal of Mathematical Behavior*, *22*(1), 1-106.

Vergnaud, G. (2009). The theory of conceptual fields. Human Development, 52, 83-94.