BIOLOGICAL BASES FOR DEDUCTIVE REASONING

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This paper addresses the question "What is deductive reasoning, that humans can reason deductively, and what are humans, that they can reason deductively?" from a range of psychological, biological, evolutionary and neurological perspectives. The basic structure of deductive reasoning is seen as being common to all primates and based on evolutionary pressures to perceive causes and effects. However, deductive reasoning at higher levels of abstraction is unique to humans and linked to language use. The feeling of necessity that is associated with deductive reasoning is also accounted for by the presence of somatic markers for deductive conclusions, which also have an evolutionary basis.

Deductive reasoning, epistemology, necessity, abstraction, evolution

INTRODUCTION

The question I will consider in this paper, inspired by Warren McCulloch (1965), is "What is deductive reasoning, that humans can reason deductively, and what are humans, that they can reason deductively?" Starting from the observation that the biological systems called humans can reason deductively, I will ask what features of deductive reasoning and of humans make this possible. Two concepts will be central to my discussion of this question: the reliance of deduction on abstraction and the feeling of necessity associated with deductive reasoning.

It is my position that mathematics education must consider not only the nature of the content being taught, but also the nature of the people being taught. Philosophical and historical approaches to epistemology have told us much about the nature of deductive reasoning, but not in relation to the people who actually do it. For that we need to turn to what McCulloch called "experimental epistemology", inquiry into the physiological substrate of knowledge, or what Donald Campbell (1974) called "evolutionary epistemology", "an epistemology taking cognizance of and compatible with man's status as a product of biological and social evolution." (cited in Rav, 1989, p. 51 & 2006, p. 73).

> In contrast to the various philosophical epistemologies, evolutionary epistemology attempts to investigate the mechanism of cognition from the point of view of its phylogeny. It is mainly distinguished from the traditional position in that it adopts a point of view outside the subject and examines different cognitive mechanisms comparatively. It is thus able to present objectively a series of problems [including the problems of traditional epistemologies] not soluble on the level of reason alone [but, which are soluble from the phylogenetic point of view]. (Riedl, 1984, p. 220; 1988, p. 287, cited in Rav, 1989, pp. 51-52 & 2006, p. 73)

Here I will address the question "What is deductive reasoning, that humans can reason deductively, and what are humans, that they can reason deductively?" by breaking it into three parts: 1. "What is deductive reasoning?", 2. "What are humans, that they can reason deductively?" and 3. "What is deductive reasoning, that humans can reason deductively?".

WHAT IS DEDUCTIVE REASONING?

Deductive reasoning and mathematical proof in general have been the focus of study since the time of Aristotle (at least). It is not possible here to explore their many aspects (See Reid & Knipping, 2010, for a more thorough survey). Here I will focus on deductive reasoning, which is essential to mathematical proof, and on two aspects that are essential to deductive reasoning:

> Deductive reasoning is based fundamentally on a single rule of inference: *modus ponens*.
> Deductive reasoning is the only kind of reasoning that results in necessary conclusions.

Modus ponens links a datum to a conclusion via a general rule. For example, given the datum "546 is an even number" and the general rule "Even numbers can be written as the sum of two odd numbers." *modus ponens* allows us to conclude that "546 can be written as the sum of two odd numbers".

A conclusion deduced by *modus ponens* from a correct datum and a correct general rule in known with certainty. That is what is meant by calling the conclusions of deductive reasoning "necessary" conclusions. In contrast, conclusions drawn in non-deductive ways may be known with a great deal of confidence, but not with certainty. I would be surprised to encounter an even number that cannot be written as the sum of two prime numbers (based on the mass of data provided by Oliveira e Silva, 2012) but absolutely shattered if 546 could not be written as the sum of two odd numbers.

This is a preliminary answer to the question "What is deductive reasoning" from which I can consider the next question, "What are humans, that they can reason deductively?"

WHAT ARE HUMANS, THAT THEY CAN REASON DEDUCTIVELY?

I will begin by reviewing some evidence that suggests that humans cannot, in fact, reason deductively, at least not very well. I will then consider what humans must be able to do as a basis for deductive reasoning, including most importantly an ability to abstract. Finally I will turn to necessity and consider how humans might be able to detect the difference between necessary conclusions and probable conclusions.

Can humans reason deductively?

There is a large body of research that starts from the assumption that humans reason deductively (Cosmides, 1989, p. 191, cites e.g., Henle, 1962; Inhelder & Piaget, 1958; Johnson-Laird, 1982; Wason & Johnson-Laird, 1972), but it is clear that in some circumstances, humans do not reason deductively.

For example, on the Wason (1966) card selection task [1] “performance is relatively poor” (Goel, 2007, p. 436). “Relatively poor” is an understatement; in some studies only 10% of subjects turn over the correct cards. Given that the task involves only deciding what data and conclusions need to be checked to verify a general rule used in *modus ponens*, the difficulty many people have with this task makes it implausible that deductive reasoning is the basis for human thought.

There is also evidence that humans do not recognise that deductive reasoning leads to necessary conclusions. In mathematics education we can refer to the work of Fischbein (1982) who identifies necessity as a defining characteristic of mathematical proof.

> With reference to mathematics the way of proving is different: the statement we consider must be the logical, necessary conclusion of some other previously accepted statements. ... The universality of the truth expressed by the theorem is guaranteed by the universal validity of the logical rules used in the proof. (p. 15)

Fischbein and Kedem (1982) found that 80% of the 397 secondary school students they interviewed were not completely convinced by written proofs, suggesting that they did not recognise the necessity of the conclusions.

These results are discouraging in light of my intention to describe the aspects of being human that allow us to reason deductively. However, the fact remains that some people do succeed at the Wason task, and some recognise the necessity of the conclusions of deductive reasoning. Perhaps a closer look at what is involved in deductive reasoning will clarify things.

The basis for deductive reasoning

To reason deductively one must use a general rule. What does that mean? When a rat in a Skinner box that learns to press lever A before pressing lever B does it apply the general rule “If you press lever A before lever B, you get food”? Perhaps, but certainly without awareness of the rule. Animals can learn to behave according to a rule, when dealing with their immediate environment, but with no more awareness than a bicycle is aware of gear ratios.

Now, consider a 4 year old child who can give a logically correct answer to the following question: “All fish live in trees. Tot is a fish. Does Tot live in a tree?” (Richards & Sanderson, 1999, p. B2). In this case the general rule is clearly being used with awareness. What makes the difference here? The answer, of course, is language. Humans can express general rules in language, and rats cannot. It may be that the structure of language itself forms our thoughts into general rules (Bickerton, 1990) or that the social use of language does so (Vygotsky, 1986) or that general rules emerge from our making sense of the world and language reflects this (Devlin, 2000). In any case, the ability of people to reason using general rules is intimately tied to the ability of people to express general rules in language.

But how is it that a child can answer the question “Does Tot live in a tree?” while adults fail at the Wason task? Both have access to language and can express general rules. The key is in the presentation. “2-, 3-and 4-year-olds can solve deductive reasoning problems when they are given cues to use their imagination to create an alternative reality where different outcomes are possible” (Richards & Sanderson, 1999, p. B8). Language does not exist to refer to arbitrary general rules. It exists to allow people to function in reality, and humans reason quite well with general rules tied to a realistic context (even an alternative reality where fish live in trees).

Is it justified to say that people can reliably use deductive reasoning in realistic contexts even if they cannot in abstract contexts? It depends.

> A task is interpreted in accord with the normative model if and only if the rule is understood as unidirectional and deterministic, P & not-Q combination is considered to be the only relevant violating instance, and looking for violations is adopted as the testing strategy. However, in everyday hypothesis testing this is not warranted. (Liberman & Klar, 1996, p.146)

If the everyday context includes elements that suggest that the general rule might be probabilistic, or biconditional, or that trying to confirm the rule is a good way to test it, then people may seem not to reason deductively. However, if the task is presented in a realistic context, that clearly requires deductive reasoning, then most people can choose the correct cards.

If deductive reasoning is used in everyday contexts, like language, and is characteristic of humans, like language, does this mean deductive reasoning is learned, like language, at an early age? Or might humans be born already able to reason deductively, just as we can recognise faces and distinguish two from three? I do not know, and I know of no research on this question, but I suspect so. Research on animal cognition and infant cognition is producing fascinating new results every year, but deductive reasoning without language use has not yet been demonstrated. We do know however, that soon after children begin to use language, they begin to use deductive reasoning. The work of Richards and Sanderson (1999) quoted above provides one example, and Stylianides and Stylianides (2008) provide an excellent summary of the psychological research on deductive reasoning by children.

The ability to abstract

To express a general rule involves expressing an abstraction. The case must be seen as a specific element of an abstract class. Language includes both words for specific things (proper nouns like “Snoopy”) and for abstract classes (like “dog”), so humans can certainly express abstractions. Not only that, humans can invent new abstractions. It is worth looking more closely at that process, and especially at different levels of abstraction related to *what* is abstracted.

Devlin (2000) divides abstraction into four levels. Level 1 abstraction refers only to things that are present. My seeing many present birds as the same kind of bird is such an abstraction. This is the kind of abstraction animals engage in by perceiving in categories. Level 2 abstraction involves familiar things that are not present. Devlin (p. 121) claims that apes can engage in abstraction at this level. For example, when they find a nut that they cannot break with their teeth, apes go looking for a stone to drop on it. Level 3 involves "real objects that the individual has somehow learned of but never actually encountered, or imaginary versions of real objects, or imaginary variants of real objects, or imaginary combinations of real objects." (p. 121) like dodo birds, blue stop signs, unicorns and centaurs. For Devlin, level 3 abstraction amounts to having language, and so only humans can do it. Finally, level 4 abstraction involves objects that are themselves abstract.

Where do such abstract objects come from? From a process of abstracting that goes beyond forming categories. Rather than just perceiving a group of birds as "all the same" I can observe properties that are the same, for example, colour, or size, or the shape of the beak, or the pattern of the song. Such properties are also abstractions, but of a different kind. When I perceive that a cup is the same colour as a book, I create a new category of things that are that colour. The colour becomes an abstract object. "We reify our abstracting: the end of the process of abstraction—paying attention to only some of our experience—begins to be treated as a thing, an abstract thing." (Epstein, 2012, p. 252). The same thing happens with other properties, like number and shape. I form a category of all the (roughly) triangular objects I perceive, and then a triangle becomes a level 4 abstract object.

I suspect that Devlin's level 4 has further divisions. When thinking about triangles I can refer directly to a present triangular object to aid my thinking. But the same process that took me from level 2 to level 3 (recombining objects in my imagination) can be applied to abstract objects. From my abstract triangle and tetrahedron I can use my imagination to go to a four dimensional figure that is somehow like them, but at the same time even more abstract. I can model an abstract triangle with a concrete triangular object, but I cannot do that with an abstract object that is itself based on an abstract object. The higher the level of abstraction, the harder it is to think about it.

Deductive reasoning involves using general rules, and general rules involve using abstractions. This dependence means that deductive reasoning ought to be different if it involves different levels of abstraction. And it is. When dealing with level 1 and level 2 abstractions, deductive reasoning is essentially a way of describing causal relationships between things. If an ape selects a ball from a collection of objects in order to roll it, the ape could be said to conclude "The ball will roll." by deducing from "The ball is round." and "Round things roll.". As with the rat in the Skinner box, the ape is not aware of this rule, but behaves as if it has such a rule, and unlike the rat, can operate with level 2 abstractions, such as the ball it saw in the toy box yesterday.

Deductive reasoning with level 3 abstractions is more complicated, as it involves imagined things expressed in language. For example, the question "All fish live in trees. Tot is a fish. Does Tot live in a tree?" (Richards & Sanderson, 1999, p. B2) and the version of the Wason task using the rule "If a person is drinking beer, then he must be over 20 years old" (Cosmides & Tooby, 1992) involve level 3 abstractions. Children prompted to use their imaginations can answer the question about Tot correctly, and most adults succeed at the Wason task if it is clearly presented in a social context. Humans can reason deductively in these contexts because humans can use language and level 3 abstractions.

Deductive reasoning with level 4 abstractions, especially abstractions that cannot be modelled easily, on the other hand, is difficult for most humans. That is why most people cannot do the abstract Wason tasks, and why most people find mathematical proofs hard to follow. It is not that they cannot reason deductively; it is that reasoning deductively gets more difficult the more abstract it becomes.

Is deductive reasoning with level 4 abstractions really the same kind of thinking as with level 3 abstractions? I would agree with Devlin and Rav that it is, simply on the basis of the biological tendency to reuse existing attributes rather than evolving new ones.

> As it is a fundamental property of the nervous system to function through recursive loops, any hypothetical representation that we form is dealt with by the same 'logic' of coordination as in dealing with real life situations. (Rav, 1989, p. 61, 2006, p. 81)

Necessity

I now turn to a unique characteristic of deductive reasoning: necessary conclusions. What are humans, that they associate the conclusions of deductive reasoning with certainty, while conclusions reached in other ways are recognised as being only probable? When I come to a conclusion by deductive reasoning, I feel that not only that the conclusion is so, but also that is *must be so*. This feeling does not occur with other kinds of reasoning.

Damasio's (1996ab) concept of somatic marker offers a neurological basis that can be used to account for the feeling of necessity. A somatic marker is the juxtaposition of knowledge, emotion and bodily feeling related to a decision or a thinking process. Every decision a person makes activates not only knowledge relevant to making the decision but also emotional markers triggering bodily feelings. These "somatic markers" are not activated in patients with certain kinds of brain damage, with consequences for their everyday decision making.

If a deduction is seen as a kind of decision making, then the feeling of necessity is accounted for as a somatic marker associated with any use of deductive reasoning. Why do humans feel this way about deductive reasoning? There are two possibilities: the somatic marker may be acquired through individual experience, or it may now be innate, having been acquired at the species level in the course of human evolution.

If the somatic marker is acquired though experience this would account for the observation (reported Galotti, Komatsu, & Voelz, 1997, p. 77) that while young children (circa seven years old) have higher confidence in conclusions reached through deductive reasoning that conclusions reached through inductive inferences, they are less certain of the conclusions of deductive reasoning than they should be.

However, as we have seen above, the ability to reason deductively is tied to the degree of abstractness of the context, and it may be that children are less certain simply because they recognise that they are not completely fluent in abstract reasoning. Children may already have the somatic marker for necessary conclusions, but feel less certain because of unfamiliarity with abstractions. In that case, we must account for the presence of this somatic marker in humans by making reference to evolutionary pressures.

Some somatic markers (such as fear of snakes) are clearly innate, and Damasio accounts for the origins of somatic markers evolutionarily.

> Let us assume that the brain has long had available, in evolution, a means to select good responses rather than bad ones in terms of survival. I suspect that this mechanism has been co-opted for behavioural guidance outside the realm of basic survival. … It is plausible that a system geared to produce markers … to guide basic survival, would have been pre-adapted to assist with 'intellectual' decision making. (1996b, pp. 1416-1417)

Rav, without using the language of somatic markers, similarly accounts for the origin of the feeling of necessity.

> But whence comes the feeling of safety and confidence in the soundness of the schemes which formal logic incorporates? To an evolutionary epistemologist, logic is not based on conventions; rather, we look for the biological substrata of the fundamental schemes of inference. Consider for instance *modus ponens*:

$$\begin{array}{c} A \rightarrow B \\ \underline{\quad A \quad} \\ \therefore B \end{array}$$

> If a sheep perceives only the muzzle of a wolf, it flees already for its life. Here, 'muzzle→wolf' is 'wired' into its nervous system. Hence the mere sight of a muzzle—any muzzle of a wolf, not just the muzzle of a particular wolf—results in 'inferring' the presence of a wolf. Needless to say such inborn behavioral patterns are vital. ... The necessary character of logic, qua codified logico-operational schemes, thus receives a coherent explanation in view of its phylogenetic origin. (Rav, 1989, p. 63, 2006, p. 83)

I hypothesise that when humans developed language sufficiently to express abstract general rules in words, the rules they first articulated also existed as behavioural patterns (like that of the sheep) tied to somatic markers. The first human to say "If you see a wolf, run!" was articulating a general rule that when applied in *modus ponens* activated the somatic marker of necessity. This marker remained associated with *modus ponens* in other contexts, simply because evolution does not select out attributes that have survival value.

WHAT IS DEDUCTIVE REASONING, THAT HUMANS CAN REASON DEDUCTIVELY?

I conclude by returning to the question “What is deductive reasoning, that humans can reason deductively?” The answers given above can now be reinterpreted in light of the characteristics of humans that allow us to reason deductively.

Modus ponens

Modus ponens encapsulates a way of relating abstract categories back to specific cases. Language allows the relationship to be expressed, and allows *modus ponens* to be applied to abstract categories that are not based on direct experience. Because human beings evolved in a world in which relating abstract categories back to specific cases is useful, we can reason deductively, and because we can use language, we can learn to do so in abstract contexts. This suggests that the main challenge in teaching people to reason deductively in very abstract contexts like mathematics is not teaching them deductive reasoning. Deductive reasoning comes with being human. However, the abstractions of mathematics are not the context in which humans came to reason deductively, so learning to reason deductively in such contexts requires learning to cope with abstraction better (perhaps through better representations).

Necessary conclusions

Deductive reasoning comes with a feeling of necessity. But necessity is not a property of deduction, it is a property of deductive reasoning being done by people who have learned to feel the necessity of deductive conclusions. We cannot assume that students automatically have the somatic maker that makes them feel necessity in abstract contexts like mathematics. I believe that all humans do, as a result of our evolutionary history. That would account for Galotti, Komatsu, and Voelz’s (1997) finding that children shows signs of associating deduction with certainty early in their schooling. But feeling necessity in mathematics might involve creating a new somatic markers, which would account for the children’s lack of complete confidence. In either case, mathematics educators must explore how such a somatic marker is activated, and how it interacts with other somatic markers.

The idea of a somatic marker for necessity provides a new viewpoint from which to reexamine Fischbein’s (1982) process of elaborating new intuitions.

> A new “basis of belief”, a new intuitive approach, must be elaborated which will enable the pupil not only to understand a formal proof but also to believe (fully, sympathetically, intuitively) in the a priori universality of the theorem guaranteed by the respective proof. (Fischbein, 1982, p. 17)

Fischbein suggested that preformal proving (that is, deductive reasoning in less abstract contexts) might help in the development of such a basis of belief, but if the somatic marker for necessity is already present then the task is not to develop it, but rather to understand what might interfere with it when encountering formal proof.

Further research in this direction is needed, supported by a theoretical framework that sees deductive reasoning in human terms.

CONCLUSION

The biological bases for deductive reasoning have two important implications for teaching proof. We do not need to begin by teaching students how to reason deductively. And we do not have to teach them the feeling of necessity. However, we must begin in contexts where abstraction is not an obstacle to reasoning, and we must be attentive to other somatic markers (for example associated with feeling of mathematics anxiety) that will interfere with feeling necessity.

NOTES

1. The task is to determine which of four two-sided cards need to be turned over to verify a general rule. The task exists in many versions, based on the original in which the cards have a letter on one side and a number on the other and the rule is "If a card has a vowel on one side, then it has an even number on the other side.". The visible sides of the cards show one vowel, one consonant, one even number and one odd number and the correct answer is to pick the vowel and the odd number, corresponding to the logical proposition P & not Q.

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